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ON SYSTEMS WITH 'LEIER CONSTRAINT' IN THE CENTRAL NEWTONIAN FORCE FIELD

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Abstract

We consider the system that moves in the Newtonian central force field and consists of a rigid body and a particle. The particle coasts along on the cable with both ends placed in the body. We call such cable a 'leier'. (Dutch maritime term 'leier' means the rope with both fixed ends). We assume the system mass center describes circular orbit and the cable length is small in comparison with orbit radius. We study the particle motion near the body, the system relative equilibria, the capture by the leier of a free-moving particle.

Key words

Leier constraint, tethered system, space garbage.

1 Introduction

Space tethered systems is one of the most interesting topics in dynamics. For the first time the motion of a particle tethered to a spacecraft has been studied in [Beletsky & Novikova, 1969; Beletsky, 1969]. Presently there are hundreds papers devoted to various aspects of the motion of such couple. In this paper we suggest some generalization of the classic tether system . We consider a pair composed of a particle and a dumbbell-shaped rigid body. The particle is tethered to the body by a weightless nonstretchable cable. Both cable ends is located in the body. In the most cases we assume the cable is fixed to the dumbbell endpoints. The particle coasts along on the cable. We call such pair 'a system with leier constraint'. (The Dutch maritime term 'leier' means the rope with both fixed ends.) We assume the mass center of the considered system describes a circular orbit in the Central Newtonian Force Field, the cable length is small in comparison with the orbit radius and the cable does not leave the orbit plane. We study the following problems.

Firstly, we assume the particle does not influence the dumbbell motion. We describe the particle motion if the dumbbell axis is directed to the attracting center or on a tangent to the orbit. Note that the cable can be tense (constrained motion) or nontense (free or unconstrained motion). We deduce the condition for constrained motion and classify the periodical nonimpactive trajectories of the particle that include segments of constrained and free motions.

Secondly, we assume the particle mass is not small. In this case we deduce the motion equations of a system and condition for constrained motion. We also find 16 relative equilibria of the system and study their stability. We claim that the dumbbell can be stabilized somehow by selection of the particle mass and the cable length.

Thirdly, we consider the body resting in the orbital frame of reference is equipped with sufficiently long spar. Two sliders with bobbins fixed to them can move along the spar. The cable can be winded on the bobbins. We claim that almost always the gripper coasting along on the cable can grasp the particle freely moving in the vicinity of the body without discontinuities in relative velocity and acceleration. In other words, we suggest the algorithm for gently (non-impactive) capture of space garbage by 'the leier constraint'. We describe such capture and suggest some scheme for reducing the necessary cable length.

Fourthly, we study the dumbbell motion being forced by the particle in some particular cases.

2 Designations and parameters.

Consider a mechanical system consisting of the body m_1m_2 and a particle with mass m_3 (figure 1). In the general case assume that the body is dumbbell-shaped, i.e. it is composed of particles with masses m_1 and m_2 connecting by weightless rod of length $2c$. Without loss of generality, $m_1 \leq m_2$. Suppose the particle m_3 coasts along on the cable with ends fixed to m_1 and m_2. This cable can be called 'a leier'. Denote by $2a$ the cable length. Let C be the mass center for considered system and O_1 be the attracting center. Suppose C moves along the circular orbit, i.e. $O_1C = r = \mathrm{const}$ and the particles m_1, m_2, m_3 do not leave the plane of this orbit. Moreover assume that $a \ll r$. Ev-

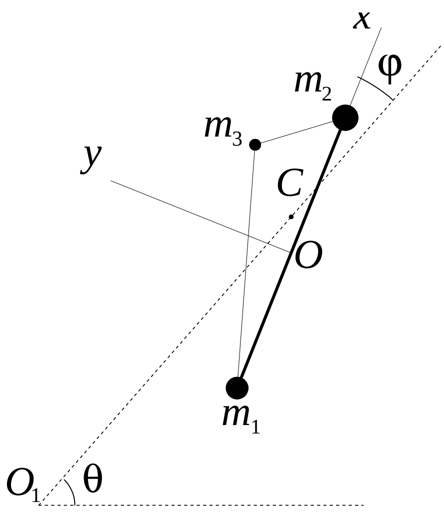

Figure 1.

idently, the particle m_3 cannot leave the ellipse with foci in the dumbbell endpoints. The ellipse has eccentricity $e = c/\,a$ and semi-axises a and $b = \sqrt{a^2 - c^2}$. Let Oxy be a coordinate system with the origin in the dumbbell midpoint. Clearly, if x and y are the coordinates of the particle m_3 the inequality

$$x^2 + dy^2 - a^2 \leq 0; \quad d = a^2\big/\,b^2 \qquad (1)$$

is valid. The motion of m_3 is called the constrained one if (1) is equality. In this case the coordinates of m_3 can be determine by formulae

$$x = a\cos\gamma; \quad y = b\sin\gamma, \qquad (2)$$

where γ is the eccentric anomaly of the mentioned ellipse. If m_3 moves inside the ellipse then we say that the motion is the unconstrained one (or the free one).

Let $\mu{=}(m_2{-}m_1)\,/\,(m_2{+}m_1)$ and $\nu{=}m_3\,/(m_2{+}m_1)$. Denote by φ the angle between O_1C and the rod. It is clear that the dimensionless parameters μ, ν, e and the variables φ, γ determine the considered system dynamics completely in the case of constrained motion.

3 The motion of the small particle about the resting dumbbell.

Let the mass of m_3 be so small that it does not influence the dumbbell motion, i.e. $\nu \ll 1$. Suppose the dumbbell is at rest with respect to O_1C. It is possible only if $\varphi = 0$, $\varphi = \pi$ (the 'vertical' dumbbell) or $\varphi = \pm\pi\,/\,2$ (the 'horizontal' dumbbell) [Beletsky, 1966]. If the dumbbell is located 'vertically' then the equation of constrained motion for the particle m_3 is

$$\left(1{-}e^2\cos^2\gamma\right)\gamma'' + \frac{e^2}{2}\gamma'^2\sin 2\gamma + 3\sin\gamma(\cos\gamma - \mu e) = 0, \quad (3)$$

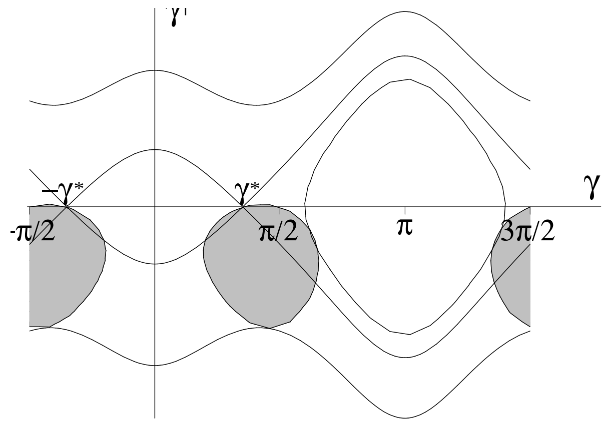

Figure 2.

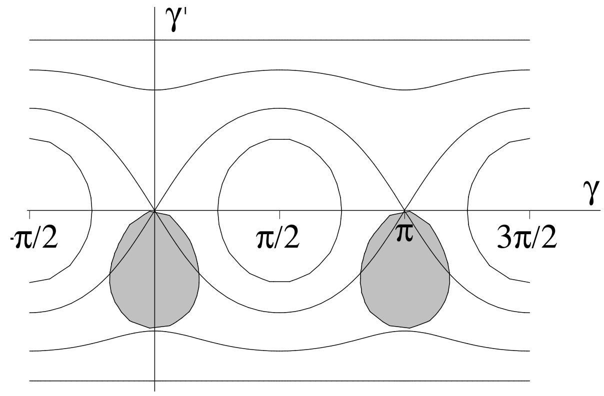

Figure 3.

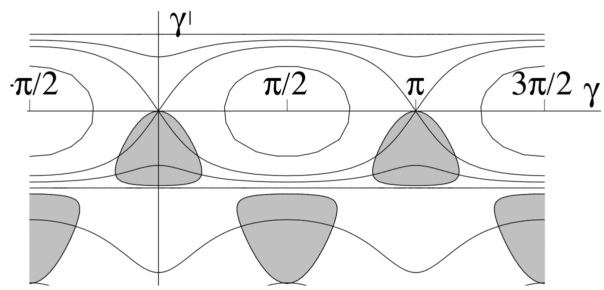

Figure 4.

where the prime '′' denotes the derivative with respect to 'dimensionless time' τ, $\tau = G^{1/2}M^{1/2}r^{-3/2}t$, G is the gravity constant, M is the attracting center's mass. (3) has Jacobi's integral

$$(1 - e^2\cos^2\gamma)\gamma'^2 - 3\cos\gamma(\cos\gamma - 2\mu e) = h_1 = \text{const}$$

It can be proved that in the considered case the constrained motion is possible only if

$$\begin{aligned}\sqrt{1-e^2}\gamma'^2 + 2\left(1 - e^2\cos^2\gamma\right)\gamma' + \\ + 3\sqrt{1-e^2}\cos\gamma(\cos\gamma - e\mu) \geq 0\end{aligned} \qquad (4)$$

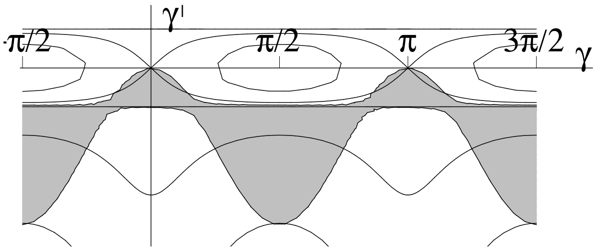

Figure 5.

The phase portrait for (3) is represented in figure 2, where the areas of unconstrained motion are shadowed.

Let us remark that in the considered case there are two stable ($\gamma = 0$ and $\gamma = \pi$) and two unstable ($\gamma = \pm\gamma^* = \pm\arccos e\mu$) equilibria of the particle m_3. The stable equilibria correspond with the situations when m_1, m_2, m_3 belong to O_1C. The dumbbell and the particle move with the nontense cable if the equilibria are unstable.

If the dumbbell is stabilized in the 'horizontal' equilibria then the motion equation for the particle m_3 is

$$\left(1-e^2\cos^2\gamma\right)\gamma''+\frac{e^2}{2}\gamma'^2\sin2\gamma-\frac{3}{2}(1-e^2)\sin2\gamma=0 \quad (5)$$

(5) has Jacobi's integral

$$(1-e^2\cos^2\gamma)\gamma'^2+3(1-e^2)\cos^2\gamma = h_2 = \text{const.} \quad (6)$$

In this case there are two stable ($\gamma = \pm\pi/2$) and two unstable ($\gamma = 0$ and $\gamma = \pi$) equilibria of the particle m_3. If the equilibria are stable then m_1, m_2, m_3 are vertexes of some isosceles triangle. If the equilibria are unstable then the cable is nontense and m_1, m_2, m_3 belong to a 'horizontal' straight line crossing O. It can be proved that in the considered case the constrained motion is possible only if

$$\sqrt{1-e^2}\gamma'^2+2\left(1-e^2\cos^2\gamma\right)\gamma'+3\sqrt{1-e^2}\sin^2\gamma\geq0 \quad (7)$$

From this inequality it follows that in our case the areas of unconstrained motion have various forms at various values of e. So for $e \leq \sqrt{2/3}$ we have the areas shadowed in figure 3, for $\sqrt{2/3} < e < \sqrt{3}/2$ we have the areas shadowed in figure 4, and for $e > \sqrt{3}/2$ we have the areas shadowed in figure 5.

It is easy to prove that the majority of trajectories for m_3 free motion crosses the ellipse transversely. It follows from the law of the relative motion near the orbital station (see for example [Beletsky, 1972]). Thus in the general case an impact against the cable is a result of the particle's free motion [Beletsky, 1995]. However there exists a set of nonimpactive periodical trajectories that include the segment of free motion [Rodnikov, 2006b]. These trajectories can be divided into two types. The trajectory of the first type is depicted in figure 6. It includes the segment of constraint motion (the upper arc B_2B_1) and the segment of free motion (the lower arc B_1B_2). We call such trajectory 'the oval'. The trajectory of the second type is depicted in figure 7. It includes the segment of constraint motion (the arc $A_1B_2B_1A_2$) and the segment of free motion (the 'hilly' curve B_1B_2). We call such trajectory 'the horned sickle'. Note some common properties of the described nonimpactive trajectories. Firstly, both types exist for all stationary motions of the dumbbell, i.e. at $\varphi = 0, \pm\pi/2, \pi$. Secondly, B_1 (the point of 'leaving from constraint') and B_2 (the point of 'landing to constraint') are symmetric with respect to vertical axis of the ellipse. Thirdly, the 'oval' trajectory can be both convex and concave. Note also that there exist the other trajectories that include the segments of free motion with nonimpactive 'landing to constraint'. These trajectories are not periodic and symmetric as they result in impact against the cable. They exist only if the dumbbell is 'horizontal' for a denumerable set of eccentricity e values.

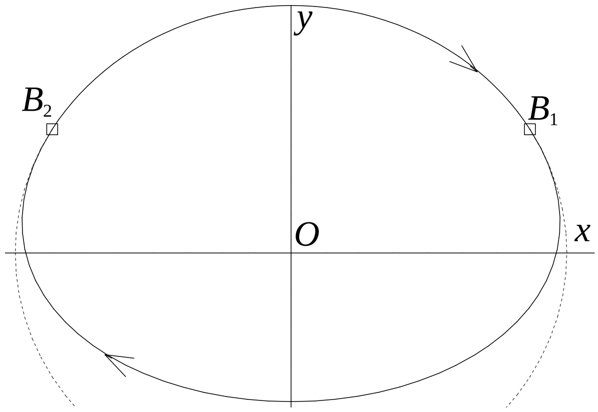

Figure 6.

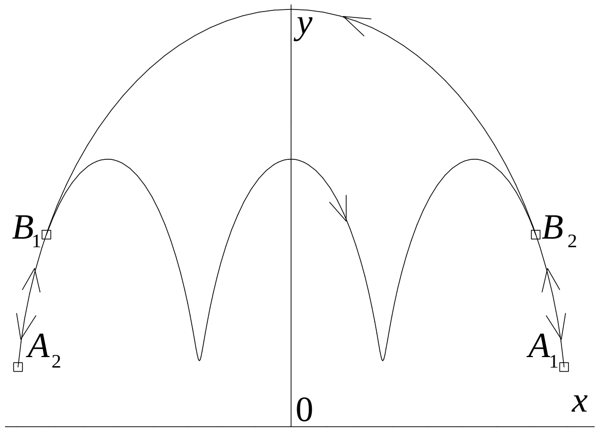

Figure 7.

4 Relative equilibria of the dumbbell with the counter-balance.

Suppose m_3 is not small. In this case the particle is an original counter-balance for the dumbbell. Lagrangian for relative motion of the considered system has a form [Rodnikov, 2004; Rodnikov, 2006a]

$$L = L_2 + L_1 + L_0, \tag{8}$$

where

$$L_2 = \frac{1}{2}\left\{\varphi'^2 + \kappa\left[\left(1 - 2e\mu\cos\gamma + e^2\cos^2\gamma\right)\varphi'^2 + \right.\right.$$

$$\left.\left.+2\sqrt{1-e^2}\left(1 - e\mu\cos\gamma\right)\varphi'\gamma' + \left(1 - e^2\cos^2\gamma\right)\gamma'^2\right]\right\},$$

$$L_1 = \kappa e\cos\gamma\left(e\cos\gamma - 2\mu\right)\varphi',$$

$$L_0 = \frac{3}{2}\cos^2\varphi + \kappa\left\{\frac{9}{8}e^2\cos 2\varphi - \frac{3}{2}e\mu\cos\gamma + \frac{3}{8}e^2\cos 2\gamma - \right.$$

$$-\frac{3}{4}e\mu\left[\left(1-\sqrt{1-e^2}\right)\cos(2\varphi-\gamma) + \left(1+\sqrt{1-e^2}\right)\cos(2\varphi+\gamma)\right] +$$

$$\left.+\frac{3}{16}\left[\left(1-\sqrt{1-e^2}\right)^2\cos(2\varphi-2\gamma) + \left(1+\sqrt{1-e^2}\right)^2\cos(2\varphi+2\gamma)\right]\right\}$$

$$\kappa = \frac{\nu}{e^2(1-\mu^2)}.$$

Hence we have Jacobi's integral

$$L_2 - L_0 = h. \tag{9}$$

The constrained motion is possible only if

$$\begin{gathered}\sqrt{1-e^2}(1 - e\mu\cos\gamma)(\varphi' + 1)^2 + \\ +2(1 - e^2\cos^2\gamma)(1 + \varphi')\gamma' + \sqrt{1-e^2}\gamma'^2 - \\ -\frac{3}{2}(1-e^2)\sin 2\gamma \sin 2\varphi + \frac{\sqrt{1-e^2}}{2}[3\cos 2\varphi\cos 2\gamma + \\ +1 - e\mu\cos\gamma(1 + 3\cos 2\varphi)] \geq 0\end{gathered} \tag{10}$$

We claim that there exist 12 or 16 relative equilibria of the system with Lagrangian (8). These equilibria can be divided into 4 groups.

The first group includes four 'vertical-vertical' equilibria determined by formulae $\varphi_1 = 0, \gamma_1 = 0$; $\varphi_2 = \pi, \gamma_2 = 0$; $\varphi_3 = 0, \gamma_3 = \pi$; $\varphi_4 = \pi, \gamma_4 = \pi$. In this case m_1, m_2 and m_3 belong to straight line crossing the attracting center. The first and the second equilibria are stable if $\nu(\mu + e^2\mu - 2e) < e(1 - \mu^2)$ and unstable otherwise. The third and the fourth equilibria are stable at all values of e,μ, ν.

The second group includes four 'horizontal-horizontal' equilibria determined by formulae $\varphi_5 = \pi/2, \gamma_5 = 0$; $\varphi_6 = -\pi/2, \gamma_6 = 0$; $\varphi_7 = \pi/2, \gamma_7 = \pi$; $\varphi_8 = -\pi/2, \gamma_8 = \pi$. In this case m_1, m_2 and m_3 belong to straight line that is perpendicular to O_1C, the cable is not tense.

The third group includes four 'vertical-horizontal' equilibria $\varphi_9 = 0, \gamma_9 = \arccos e\mu$; $\varphi_{10} = \pi, \gamma_{10} = \arccos e\mu$; $\varphi_{11} = 0, \gamma_{11} = -\arccos e\mu$; $\varphi_{12} = \pi, \gamma_{12} = -\arccos e\mu$. In this case dumbbell is 'vertical', the particles m_1, m_2, m_3 are a vertexes of some triangle, the cable is not tense.

If $\mu = 0$ $(m_1 = m_2)$ then the fourth group includes the equilibria $\varphi_{13} = \pi/2, \gamma_{13} = \pi/2$; $\varphi_{14} = -\pi/2, \gamma_{14} = \pi/2$; $\varphi_{15} = \pi/2, \gamma_{15} = -\pi/2$; $\varphi_{16} = -\pi/2, \gamma_{16} = -\pi/2$; that can be called 'horizontal-vertical'. Here dumbbell is 'horizontal', the particles m_1, m_2, m_3 are a vertexes of an isosceles triangle, the cable is tense.

If $\mu \neq 0$ 'horizontal-vertical' equilibria do not exist. Nevertheless, if

$$\nu(\mu + e^2\mu - 2e) < e(1 - \mu^2)$$

then the fourth group includes 4 equilibria which can be called 'inclined-vertical'. They are found by the following procedure. Separate the area of admissible e, μ, ν into surfaces

$$\nu = \frac{e(1-\mu^2)\sqrt{(1+k)(2-e^2+ke^2)}}{2\left[(1+ke^2)\mu - e\sqrt{(1+k)(2-e^2+ke^2)}\right]}$$

Let φ^* be an acute angle. If $k = \cos 2\varphi^*$ then the formulae

$$\varphi_{13} = \varphi^*, \quad \gamma_{13} = -\arccos\frac{\cos\varphi^*}{\sqrt{1 - e^2\sin^2\varphi^*}}$$

$$\varphi_{14} = \varphi^* - \pi, \quad \gamma_{14} = -\arccos\frac{\cos\varphi^*}{\sqrt{1 - e^2\sin^2\varphi^*}}$$

$$\varphi_{15} = -\varphi^*, \quad \gamma_{15} = \arccos\frac{\cos\varphi^*}{\sqrt{1 - e^2\sin^2\varphi^*}}$$

$$\varphi_{16} = \pi - \varphi^*, \quad \gamma_{16} = \arccos\frac{\cos\varphi^*}{\sqrt{1 - e^2\sin^2\varphi^*}}$$

are valid. Note also that the cable is tense for such equilibria.

The equilibria from the second group, the third group or the fourth group are unstable. Fixing the particle m_3 on the cable we can stabilize 'horizontal-vertical' and 'inclined-vertical' equilibria if

$$(1 - 2e^2 + \mu^2 e^4)\nu - e^2(1 - \mu^2) > 0$$

The last inequality can be deduced from the Beletsky theorem [Beletsky, 1966].

5 The algorithm for a capture of a free particle.

Let the body be at rest in the orbital frame of reference again. For example, the dumbbell can be oriented 'vertically'. Suppose the body is equipped with a sufficiently long spar. Two sliders can move on the spar. The cable can be winded on the bobbins fixed to the sliders. The gripper coasts along on the cable. We

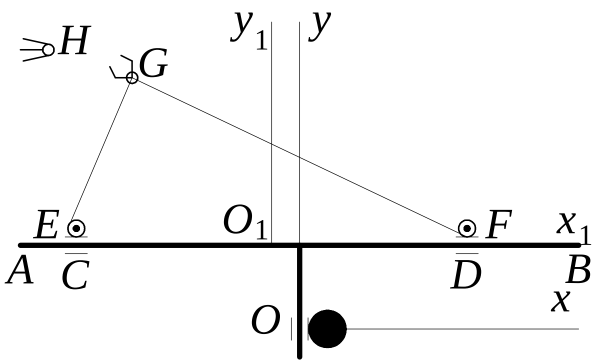

Figure 8.

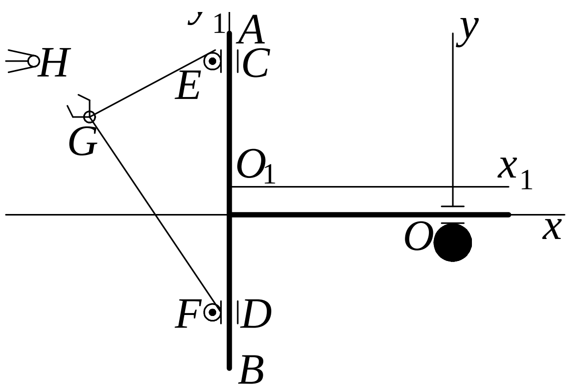

Figure 9.

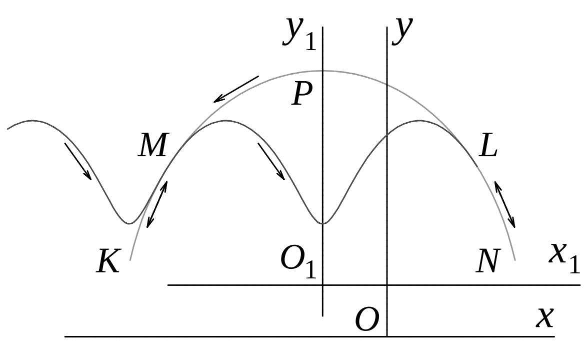

Figure 10.

consider the case of ‘horizontal’ spar (figure 8) and the case of ‘vertical’ spar (figure 9). In these figures O is the body, AB is the spar, C and D are the sliders, O_1 is midpoint of a segment CD, E and F are the bobbins, G is the gripper, axes Oy directs to the attracting center. Note that the depicted schemes are conditional. The bobbins can be fixed to different spacecrafts moving along concentric orbits (the system similar to ‘the space elevator’) or along the same orbit (a so-called ‘monkeys-bridge’ [Burov, 2003]). Suppose we have

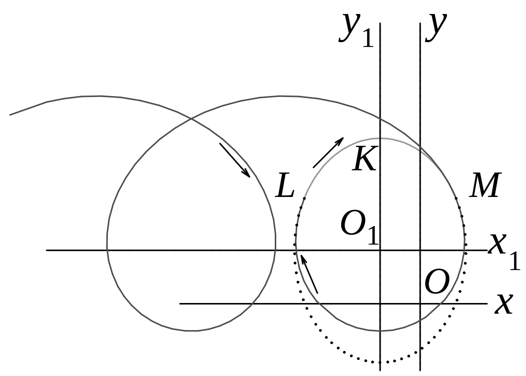

Figure 11.

detected an uncontrolled particle H that approaches the body O. Clearly, if the sliders do not move and the bobbins do not rotate then the gripper cannot leave the ellipse with foci in C and D. Using the formulae from the [Beletsky, 1972] and from previous chapters, we can determine the positions for the points C and D and the length of the cable’s unwound part so that the part of the particle’s trajectory inside the ellipse is a free-motion segment of an non-impactive periodic trajectory. Evidently, there are two ways to force H to move periodically, namely ‘the libratory’ type (figure 10) and ‘the rotary’ type (figure 11). Thus the following sequence of steps leads to the such capture of the particle H.

- The gripper’s initial position (the point K) is calculated from the orbital parameters of H; the gripper is kept in this position for a some time.
- The gripper coasts along on the tense cable with fixed sliders and bobbins until the gripper meets the H (the point M); then the gripper ‘leaves the constraint’, i.e. the cable weakens. ‘The libratory motion’ begins with zero velocity and a certain initial impulse is necessary for ‘the rotary’ motion.
- After the gripper meets the H, they move together freely and at point L the couple $G - H$ nonimpactively ‘retrieves’ the constraint.
- Constrained motion of the couple continues (the bobbins are fixed and the cable is stretched) till the point M is not achieved (trajectory $L - N - L - P - M - K - M$ or $L - K - M$); then the couple again moves freely from M to L, etc.

This algorithm possesses the following important features. Firstly, at any instant of time during the motion the velocity and the acceleration of the object H change continuously; secondly, the gripper goes by inertia (an initial impulse may be required); thirdly, the process of joining should not be instantaneous. We claim that the capture is impossible only for two types of the particle orbits. The gripper cannot reach the particle if its orbit is circular and if $3(a_0 - r) = \pm 4a_0\varepsilon$, where a_0 and ε are the major semi-axis and eccentricity of the par-

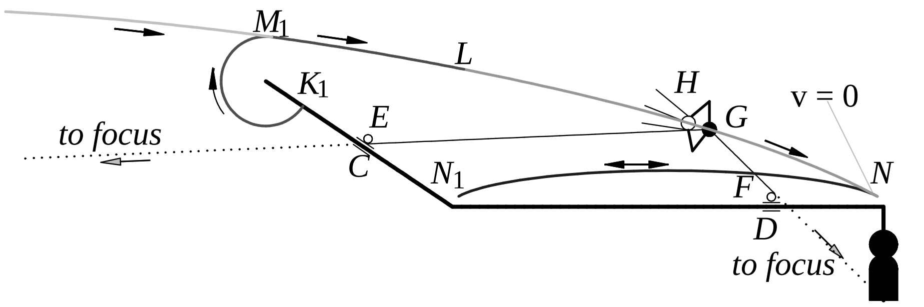

Figure 12.

ticle's orbit. The capture details is described in [Rodnikov, 2006c].

Let us remark that for many orbits a very long cable may be required. The most complicated case arises for 'the libratory' type. Note however that the ends of the cable do not need to be fixed to the foci of the ellipse. Only one condition must be held: during the motion with the tense cable the branches of the cable must be always directed toward these foci. We suggest an updated algorithm that allows to reduce the required cable length in many times. This algorithm can be represented by the following sequence of steps (figure 12):

- The instant of time when the gripper is pushed from the spar (the point K_1) is so chosen that the gripper arrives at the point M_1 simultaneously with the particle (the particle's and gripper's velocities must coincide at this point).
- The particle and the gripper travel together and land on the constraint at the point L. The capture should be realized during this motion. The branches of the non-tense cable trace the motion of the point G.
- The particle and the gripper continue their constrained joint travel along an arc of the ellipse until they reach the point N. The sliders are moving and the bobbins are rotating. The branches of the cable are directed toward the foci of the ellipse.
- The sliders and bobbins are fixed. A libratory motion along the arc of the ellipse between the points N and N_1 occurs.

6 On the dumbbell motion being forced by the particle

Suppose the mass m_3 is small, i.e. $\nu \ll 1$, but the dumbbell is not at rest. It is easily shown that if the dumbbell's initial position is quasi-horizontal then the particle motion along the leier force the upturning of the dumbbell. The further motion of the dumbbell belongs to one of three types. There are 'the libratory motion' about the 'vertical' equilibria, 'the rotary motion' about mass center, the complicated 'tumbling motion' consisting of libratory and rotary segments. Let us remark that the dumbbell tends to librations about its 'horizontal' equilibria for some singular initial values of (γ', γ).

It is not hard to prove that if $h < \frac{3}{8}\kappa(5e^2 - 2)$ then only 'the libratory motion' is possible. Here h is the constant of Jacobi's integral (9). For instance, the libratory motion is observed for any initial value of φ and zero initial velocities if initial value of γ is about $\pm\pi/2$. It can be shown numerically that 'the rotary motion' is guaranteed only if the initial value of $|\gamma'|$ is sufficiently big.

Note that 'the tumbling motion' is a set of right-hand and left-hand rotations with close to flat angles. Consider a single rotation from this set. Let $\varphi_1', \varphi_1, \gamma_1', \gamma_1$ be values of $\varphi', \varphi, \gamma', \gamma$ in the beginning of this rotation. It is clear that $\varphi_1' \approx 0$ and $\varphi \approx \pm\pi/2$.(Without loss of generality it can be assumed that $\varphi \approx -\pi/2$). It is obvious that the motion in the vicinity of 'horizontal' equilibria determine the direction of the dumbbell's further rotation. Substituting $-\pi/2 + \sqrt{\kappa}\psi$ for φ in the dumbbell's motion equation we obtain

$$\psi'' - 3\psi + \sqrt{\kappa}D = 0, \tag{11}$$

where
$D = \sqrt{1-e^2}\gamma'' - e^2 \sin 2\gamma \cdot \gamma' - \dfrac{3}{2}\sqrt{1-e^2}\sin 2\gamma,$
Here we are restricted to a case of symmetric dumbbell ($\mu = 0 \Leftrightarrow m_1 = m_2$) and neglect the terms of order higher than $\sqrt{\kappa}$. Using (5) and (6) we get $D = f(\gamma, h_2)$, where $\gamma = \gamma(\tau, \gamma_1, h_2)$ is the solution of (6) and h_2 depends on γ_1 and γ_1'.

Solutions of (11) have a form $\psi(\tau) = p(\tau) + q(\tau)$, where $p(\tau)$ is a periodic function and

$$q(\tau) = \frac{1}{2\sqrt{3}}\left(C_1 e^{\tau\sqrt{3}} + C_2 e^{-\tau\sqrt{3}}\right),$$
$$C_1 = \kappa^{-1/2}(\sqrt{3}\varphi_1 + \sqrt{3}\pi/2 + \varphi_1') - \sqrt{\kappa}A,$$

$$A = \int_0^{+\infty} e^{-\xi\sqrt{3}} f\left(\gamma(\xi, \gamma_1, h_2), h_2\right) d\xi$$

Clearly, if $C_1 > 0$ then the dumbbell will turn counterclockwise and if $C_1 < 0$ then the dumbbell will turn clockwise. Certainly, this criteria is valid only if the inequality (10) is fulfilled during the considered rotation. If $C_1 = 0$ then the dumbbell remain in the vicinity of horizontal equilibria, i.e. we have an asymptotic motion of the dumbbell.

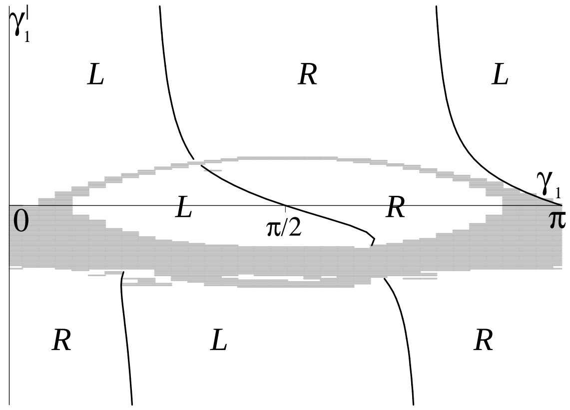

Figure 13.

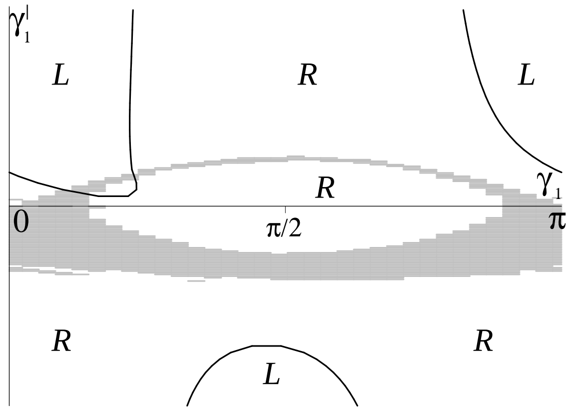

Figure 14.

In particular, if $\varphi_1' = \varphi_1 = 0$, i.e. the dumbbell is precisely horizontal in the beginning of considered rotation, then the equality $A = 0$ is the equation of a curve dividing the plane (γ_1, γ_1') into the areas of left-hand (L) and right-hand (R)rotations (figure 13. Here $e = 1/2$).

The similar curve for $\kappa = 0.01$, $\varphi_1' = 0$, $\varphi_1 = -90^0 3'$ is depicted in figure 14. In figures 13,14 the shadowed area corresponds to the motion with the weakened cable.

Finally note that the infinite integral A is reduced up to definite. For instance, if $h_2 > 3(1 - e^2)$ and $\gamma_1' > 0$ then we have

$$A = \frac{1}{1 - e^{-T\sqrt{3}}} \int_{\gamma_1}^{\pi+\gamma_1} \exp\left(-\sqrt{3}\int_{\gamma_1}^{\gamma} \frac{d\xi}{\sqrt{r(h_2,\xi)}}\right) \frac{f(\gamma, h_2)}{\sqrt{r(h_2,\gamma)}} d\gamma,$$

where

$$r(h_2, \gamma) = \frac{h_2 - 3(1 - e^2)}{1 - e^2 \cos^2 \gamma}, \quad T = \int_0^\pi \frac{d\gamma}{\sqrt{r(h_2, \gamma)}}$$

7 Conclusion

In this paper the motion in the Newtonian Central Force Field of the system with ‘the leier constraint’, i.e. the system consisting of the dumbbell-shaped rigid body and the particle coasting along on the cable with ends fixed to the dumbbell is considered.

Firstly, the motion of a small particle along the leier fixed to the dumbbell resting in the orbital frame of reference is studied.

Secondly, the relative equilibria of the system are found. The conditions of their stability is deduced.

Thirdly, the algorithm for the capture of the freely moving particle by the gripper coasting along on the leier is suggested.

Fourthly, the initially quasi-horizontal dumbbell’s rotation being forced by the small particle is studied.

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