THE BEHAVIOR OF AN AERODYNAMIC PENDULUM WITH VERTICAL AXIS OF ROTATION

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Abstract

Parametric analysis of the behavior of an aerodynamic pendulum is carried out, motivated by the study of a small wind power generator with a vertical axis. A mathematical model for the free rotation of this pendulum is constructed, leading to a system of nonlinear ODEs and transcendental algebraic equations. A qualitative analysis of the phase portrait is carried out: all equilibrium solutions are found, their stability is studied, characteristics of a stable rotational regime are determined; and domains of attraction for equilibrium solutions and for the rotational regime are also found. The mathematical model is used to study the operational regimes of the system "wind turbine + generator". Estimations of the trapped power as a function of the external load in the circuit are obtained; optimal values for the power and the load are found. A pitch angle control mechanism is proposed in order to increase this power.

Key words

Quasi-steady model, phase portrait, parametric analysis, rotational mode, power, control.

1 Notation

β = pitch angle (the angle between the plate and the plane orthogonal to the holder);
r = length of the holder;
$2l$ = width of the plate (chord);
h = height of the plate;
$S = 2lh$ = surface area of the plate;
J_O = moment of inertia of the pendulum about the axis of rotation;
J_B = moment of inertia of the pendulum about the vertical central axis of the plate;
m_1, $m_2 = 0$, i = diagonal elements of the central tensor of apparent additional masses of the plate;
$J = J_O + i + m_1 r^2 \sin\beta$ = the moment of inertia of the system about the rotation axis;
ρ = density of air;
$\mathbf{V}$ = wind velocity vector;
$\mathbf{U} = \mathbf{V}_N - \mathbf{V}$, where $\mathbf{V}_N$ is the velocity of the center of plate N;
α = the effective angle of attack (this is the angle between the vector $\mathbf{U}$ and the plate, it is called "angle of attack" below);
$e(\alpha) = AF / r$ = shift of the center of pressure F;
$C_x(\alpha)$, $C_y(\alpha)$ = coefficients of the drag and the lift forces;
$X = 0.5C_x(\alpha)\rho SU^2$, $Y = 0.5C_y(\alpha)\rho SU^2$ = magnitudes of the drag and the lift;
C = coefficient of electro-mechanic coupling;
ϑ = angle between the holder and the vector $\mathbf{V}$;
$\dot{\vartheta}$ = angular velocity of the rotation of the pendulum (here a dot denotes the derivative with respect to time);
$\tau = Vt / r$ = dimensionless time; $u = U / V$;
$\vartheta' = \omega = r\dot{\vartheta} / V$ (prime denotes the τ-derivative);
$A = \dfrac{J}{\rho l h r^3}$; $A_1 = \dfrac{J_B}{\rho l h r^3}$; $a = \dfrac{1}{A}$; $a_1 = \dfrac{1}{A_1}$; $b = \dfrac{m_1}{\rho l h r}$;
$c = \dfrac{C}{V\rho l h r^2}$ = dimensionless parameters.

2 Introduction

Mankind has been using wind power for ages. 1920s and 1930s saw a sharp increase of interest in wind power stations, both in science and engineering, against the backdrop of rapid development in research of propelled aviation (for both airplanes and helicopters). A recurrence of this interest became a feature of the last decades, as more attention is paid to ecology and sustained development. We can now observe increased numbers of new wind power stations and more use of wind energy, as well as larger amounts and volumes of research publications on the subject. While a comprehensive review of such literature is beyond the scope of the present work, we should mention a few examples [Gorelov, 2003; Kharitonov, 2006; Klimas and Sheldahl, 1978; Leishman, 2002; Paraschivoiu, 1983]. While most authors engage in detailed descriptions of the units, we would like to present a sufficiently simple model which can be used not only to adequately describe the observed phenomena and effects, but also to optimize the parameters of the construction.

In our approach, an aerodynamic pendulum is considered as a model for the principal element of the wind-capturing unit of a wind generator for a Vertical Axis Wind Turbine (VAWT). A VAWT has a number of advantages over the horizontal axis wind turbine. For a

VAWT an orientation device is not required; the generator can be set on the ground; the construction is simpler; and gyroscopic loads are lower. The invention of Darrieus turbine in 1926 in France can be marked as the beginning of the modern age of scientific research on VAWT. Since then the VAWT have been actively studied in many countries (see for example [Dosaev, Kobrin, Lokshin, Samsonov and Seliutsky, 2007; Gorelov, 2003; Klimas and Sheldahl, 1978; Lokshin and Samsonov, 1996; Lokshin and Samsonov, 1998; Paraschivoiu, 1983]). Some problems of motion of the aerodynamic pendulum were discussed at the 1st and 2nd ENOC ([Lokshin, Okunev, Ryzhova and Samsonov, 1996; Parshin and Samsonov, 1994; Parshin and Samsonov, 1994]).

We consider an aerodynamic pendulum with a vertical rotation axis in a steady horizontal wind flow as the model of the wind-receiving element of a straight winged VAWT.

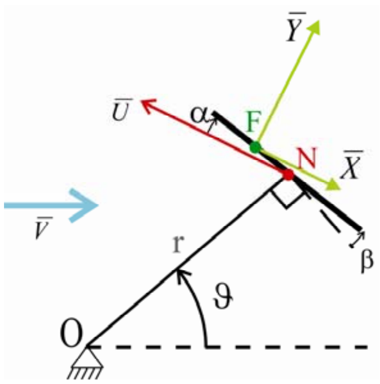

Fig 1. The aerodynamic pendulum (top view)

Assume that the pendulum consists of a thin rectangular flat plate and a horizontal weightless holder ON, which is much longer than the width of the plate. The plate is attached to the holder at its geometric center N, so that the plate stays vertical and forms the angle β with the vertical plane orthogonal to the holder (Fig. 1). In the first part of this paper we assume that $\beta \equiv const$, so that the plate and the holder form a single rigid body that can rotate about the fixed vertical axis O. The center of mass of the plate coincides with its geometric center N.

Suppose that the flow around the plate is planar. Assume that the aerodynamic force acting on the plate consists of two components. The quasi-steady component is determined in stationary wind tunnel experiments (e.g. Fig. 2, where $e(\alpha)$ is given for $r = 2l$) and the non-steady component is described by the tensor of apparent additional masses.

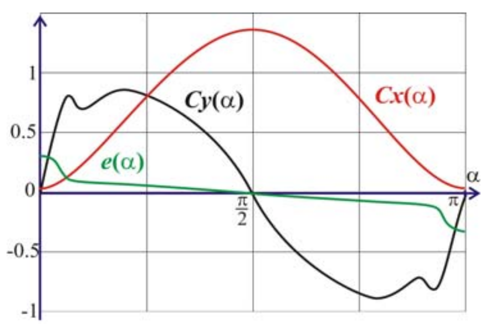

Fig. 2. Functions $C_x(\alpha)$, $C_y(\alpha)$, $e(\alpha)$ for flat aspect-ratio-8 plate ([Tabachnikov, 1974])

3 The mathematical model

Since the system has just one degree of freedom, we take ϑ as the generalized coordinate. Then the dimensionless Lagrange equations for the system "plate + continuum" are as follows [Dosaev, Kobrin, Lokshin, Samsonov and Seliutsky, 2007; Lokshin, Okunev, Ryzhova and Samsonov, 1996; Lokshin, Privalov and Samsonov, 1986; Lokshin and Samsonov, 1996; Lokshin and Samsonov, 1998;]:

$$\begin{aligned} &\vartheta' = \omega \\ &A\omega' = u^2 (C_y(\alpha)(\sin(\alpha+\beta) - e(\alpha)\cos\alpha) - \\ &\quad - C_x(\alpha)(\cos(\alpha+\beta) + e(\alpha)\sin\alpha)) + \\ &\quad + b(\omega\cos(\vartheta + 2\beta) - 0.5\omega^2 \sin(2\beta)) - c\omega \end{aligned} \qquad (1)$$

Here u and α are determined from the kinematic relations:

$$\begin{aligned} u\cos\alpha &= \omega\cos\beta + \sin(\vartheta+\beta) \\ u\sin\alpha &= -\omega\sin\beta + \cos(\vartheta+\beta) \end{aligned} \qquad (2)$$

Terms, containing the coefficient b, are due to the effect of apparent additional masses. The term $-c\omega$ models the load upon the axis of rotation due to the operation of the generator.

4 Equilibrium positions

Consider equilibrium solutions of the system (1). Using (2) we obtain that the following conditions are to be satisfied in a stationary point (values of variables in a stationary point are marked with "0"):

$$\begin{aligned} &\omega_0 = 0\,;\ u_0 = 1\,;\ \vartheta_0 + \alpha_0 + \beta = \pi/2\,; \\ &C_\tau(\alpha_0) = -C_y(\alpha_0)(\sin(\alpha_0+\beta) - e(\alpha_0)\cos\alpha_0) + \\ &+ C_x(\alpha_0)(\cos(\alpha_0+\beta) + e(\alpha_0)\sin\alpha_0) = 0 \end{aligned} \qquad (3)$$

For example, if we take the aerodynamic functions from Fig. 2, $C_\tau(\alpha)$ is as follows (Fig. 3):

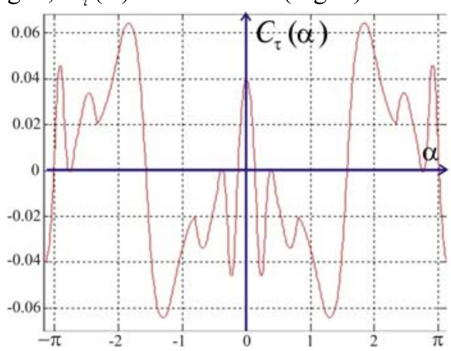

Fig. 3. $C_\tau(\alpha)$ for $\beta = 0$, $e(\alpha) \equiv 0$

To study the stability of equilibrium positions consider linearized equations:

$$\begin{cases} \Delta\vartheta' = \Delta\omega \\ A\Delta\omega' = -p\Delta\omega - q\Delta\vartheta\text{, where} \end{cases}$$

$$p = -C_\tau'(\alpha_0)\sin(\beta+\alpha_0) - b\sin(\alpha_0-\beta) + c\,;$$

$$q = -C_\tau'(\alpha_0)\,.$$

If $p > 0$, $q > 0$, then the equilibrium solution of the nonlinear system (1) is asymptotically stable. If $p < 0$ or $q < 0$, it is unstable.

Consider the configuration for which the plate is orthogonal to the holder (i.e. $\beta = 0$), then the position "up stream" (which corresponds to the point $(\pi, 0)$ of the phase plane) is one of the steady states, that is easy to notice from (3), taking into account aerodynamic properties of a symmetric plate. Let's study the stability of this particular equilibrium in assumption that $e(\alpha) \equiv 0$. We have:

$p = C'_\tau(-\pi/2) + b + c$;

$q = -C'_\tau(-\pi/2)$.

$C'_\tau(-\pi/2) = C'_y(\pi/2) + C_x(\pi/2)$.

So for the plate which aerodynamic functions are shown at Fig. 2, we obtain that $q > 0$ (see Fig. 3) and the sign of p depends on the value of the sum $(b+c)$. When $0 \le b + c < -C'_\tau(-\pi/2)$, the point $(\pi, 0)$ is a source. When $-C'_\tau(-\pi/2) < b + c$ it is a sink or a stable node.

If $b + c = -C'_\tau(-\pi/2)$, the point $(\pi, 0)$ is a weak focus. To solve the question of the stable cycle existence we find the sign of the 1st Liapunov value L_1 for this point (according to the technique described in [Bautin and Leontovich, 1990]). After calculation we obtain the following expression:

$$L_1 = -\frac{1}{16}\frac{C'''_\tau(-\pi/2)}{\sqrt{-C'_\tau(-\pi/2)}}\left(\frac{\sqrt{A}}{-C'_\tau(-\pi/2)} + \frac{1}{\sqrt{A}}\right)$$

One could see: when $C_\tau{}'(-\pi/2) < 0$ and $C'''_\tau(-\pi/2) > 0$ (that is satisfied in our case, Fig. 3), we have $L_1 < 0$. Hence the considered weak focus is stable, and such a value $\tilde{p}$ exists, that if $\tilde{p} < p < 0$, then there is a stable cycle in the neighborhood of the point $(\pi, 0)$. In other words for some values of the sum $(b+c)$ there is a stable cycle around the point $(\pi, 0)$. It is also in agreement with numerical calculations of face trajectories.

5 Existence of auto-rotation

Existence of rotational modes for the pendulum motion is of special interest, because a stable rotational mode can serve as an approximation to the operating regime of a wind turbine.

A rotational mode of motion corresponds to a periodic trajectory of the system (1)-(2) encircling the phase cylinder. For such a trajectory, the following property holds: any value ϑ_1 satisfies the equation $\int\limits_{\vartheta_1}^{\vartheta_1+2\pi} \omega'(\vartheta, \omega(\vartheta))d\vartheta = 0$, meaning that $\omega|_{\vartheta=\vartheta_1} = \omega|_{\vartheta=\vartheta_1+2\pi}$.

The following claim takes place for the system (1)-(2) (see [Bautin and Leontovich, 1990]):

►Let the following equation holds for some value $\Omega_0 = const \ne 0$:

$$\int\limits_{\vartheta_1}^{\vartheta_1+2\pi} \omega'(\vartheta, \Omega_0)d\vartheta = 0 \qquad (4)$$

Then for a sufficiently large value of A the system (1)-(2) has a periodic phase trajectory initiated from the straight line $\omega = \Omega_0$.◄

For $e(\alpha) \equiv 0$ it can be shown that there are at least two values Ω_0 (with different signs) satisfying (4), if quantities β and c satisfy the inequality

$$3[\cos 2\beta \cdot \int\limits_0^{\pi/2} (C_y(\vartheta)\sin 2\vartheta - C_x(\vartheta)\cos 2\vartheta)d\vartheta - \int\limits_0^{\pi/2} C_x(\vartheta)d\vartheta] - \pi c > 0 \qquad (5)$$

For specific functions $C_x(\alpha)$, $C_y(\alpha)$ (as in Fig. 2) (5) assumes the following form:

$$3[1.237\cos 2\beta - 1.196] - \pi c > 0 \qquad (6)$$

Thus, (6) is a sufficient and realizable condition for existence of rotational modes in the motion of the pendulum in question, with large moment of inertia (when $e(\alpha) \equiv 0$). Moreover, these rotational modes will be attracting.

Consider the equality (4) as an equation for Ω_0. If $\beta = 0$, an approximate solution can be found for the case $\Omega_0 > 1$: $\Omega_0^2 \approx 0.5C'_y(0)/C_x(0)$ (see [2]). It can be shown, that Ω_0 is an even function of β for small β (for $b = 0$ and $e(\alpha) \equiv 0$).

6 Numerical analysis of rotational modes and domains of attraction for $\beta = 0$ and various values of c

For the plate described above (Fig. 2) in the case $\beta = 0$, let the system be characterized by the fixed parameters A and b. Here we take the following values $A = 8.14$, $b = 0.1\,\pi/2$. For example, these values correspond to the case, when $r = 1.2\,\mathrm{m}$, $l = 0.06\,\mathrm{m}$, $h = 0.96\,\mathrm{m}$, $J_o = 1\,\mathrm{kg}\cdot\mathrm{m}^2$, $i = 0.0125\,\mathrm{kg}\cdot\mathrm{m}^2$, $m_1 = 0.0136\,\mathrm{kg}$, $\rho = 1.25\,\mathrm{kg/m}^3$. As estimates for m_1 and i we take: $m_1 = \rho\pi h l^2$, $i = 0.0625\rho\pi h^4 l$. Let $V = 10\,\mathrm{m/s}$.

Consider some features of the phase portrait of the system (1)-(2). Let's start by looking for periodic trajectories.

First assume $c = 0$. It is possible to show numerically, that (4) has a unique nonzero root Ω_0 (for sufficiently large A, the lines $\omega = \pm\Omega_0$ give birth to attracting periodic trajectories) and that there are no other periodic trajectories, not just originating from straight lines, but also originating for other reasons. Besides, for $c = 0$ there exists an unstable cycle $\mathbf{\Gamma}$. Inside this cycle $\mathbf{\Gamma}$ there are 9 rest points, two unstable cycles and one stable one. Outside $\mathbf{\Gamma}$ there is only one equilibrium point – the saddle point $(0, 0)$. All phase points lying outside of the cycle $\mathbf{\Gamma}$ belong to the domain of attraction of one of the periodic trajectories found.

As the coefficient c increases, the unstable cycle $\mathbf{\Gamma}$ expands and bifurcates into two unstable periodic trajectories, and this bifurcation is not described by the origination of periodic trajectories from straight lines.

To explain of the further evolution of the phase portrait with the parameter c we give a qualitative picture of the dependence between the values of the coefficient c and the values $\omega(0) = \omega|_{\vartheta=0}$ at the periodic phase trajectories (Fig. 3). We give a schematic picture of the zones of attraction of steady periodic trajectories. Unstable periodic trajectories (thin lines) limit domains of attraction of the stable ones (bold lines).

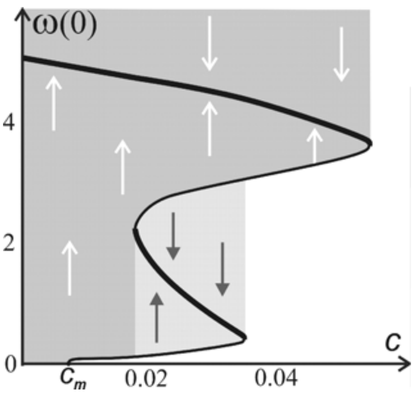

Fig. 3. Diagram of values $\omega|_{\vartheta=0}$ at periodic trajectories as functions of c

Note that the similar figure was obtained [Lokshin, Okunev, Ryzhova and Samsonov, 1996] for aerodynamic pendulum for which the plate is fixed along the holder.

Fig.4 shows, how does the value c_m (which is responsible for the appearance of the low-ω unstable periodic trajectory) depend on the parameter b (dimensionless additional masses).

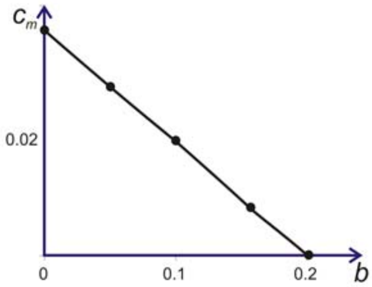

Fig. 4. Diagram of value c_m as function of b

7 Average trapped power

Denoting the dimensionless aerodynamic torque in the system (1) by $q(\vartheta,\omega)$, one can rewrite the equations as follows: $\vartheta' = \omega$; $A\omega' = q(\vartheta,\omega) - c\omega$.

Introduce the function $f(\Omega) = \int_0^{2\pi} q(\vartheta,\Omega)d\vartheta / 2\pi$. Then the equation (4) can be rewritten as

$$f(\Omega_0) - c\Omega_0 = 0 \tag{7}$$

Consider the stable periodic trajectory of the system (1)-(2) that is originated from the straight line $\omega = \Omega_0 > 0$. Average trapped power in the corresponding regime is $P = \rho V^3 lh \int_0^{2\pi} \omega q(\vartheta,\omega)d\vartheta / 2\pi$. Its magnitude is determined by the parameters β and c . Let us estimate this power as $\tilde{P} = \rho V^3 lh \Omega_0 f(\Omega_0)$. The value Ω_0 and the behavior of the function $f(\Omega)$ are also determined by β and c , so $\tilde{P}$ is a function of β and c . It is interesting to find the maximum of the function $p(c,\beta) = \Omega_0(c,\beta) f(\Omega_0(c,\beta))$. Using (7), it can be written as follows

$$p(c,\beta) = c\Omega_0^2(c,\beta) \tag{8}$$

Fix β and determine the value of c for which the maximum of $p(c,\beta) = \Omega_0 f(\Omega_0)$ is achieved. As Ω_0 , for fixed β , is a function of c as given by (7), the search for the optimal c can be replaced by searching for the optimal value Ω^* of variable Ω_0 . This value is determined from the equation

$$f(\Omega^*) + \Omega^* \left.\frac{df}{d\Omega}\right|_{\Omega=\Omega^*} = 0 \tag{9}$$

Then c^* , the optimal value of the coefficient c , is given by the expression: $c^* = f(\Omega^*) / \Omega^*$.

For example, for $\beta = 0$ in the system with fixed parameters as outlined above, a numerical solution of (9) gives $\Omega^* \approx 4.18$, that is $\dot{\vartheta}^* = 34.8$ 1/s. At this regime we have $C^* \approx 0.056\ \ \mathrm{kg \cdot m^2/s}$, $P \approx 68\mathrm{W}$.

Alternatively, the equation (9) allows for an approximate analytical solution (for $\Omega^* > 1$) at $\beta = 0$ [2]:

$$(\Omega^*)^2 = \frac{C_y'(0)}{6C_x(0)} \tag{10}$$

In the considered case we have $C'_y(0) = 4.1826$, $C_x(0) = 0.04$, and the formula (10) gives the value $\Omega^* = 4.17$, corresponding to $\dot{\vartheta}^* = 34.75$ 1/s.

A direct search for c^* by determining the values of P for various values of c via numerical integration of the equations of motion gives $\Omega^* \approx 4.14$, $C^* \approx 0.058\,\mathrm{kg \cdot m^2/s}$, $P \approx 70\mathrm{W}$. Comparison of the results shows that in practical tasks it is sufficient to maximize the function $\tilde{P}$.

Note: numerical calculations showed that the domain of attraction of the obtained stable periodic mode is limited by an unstable periodic trajectory in the neighborhood of the straight line $\omega = 3.5$.

The approximate values of maximum power at the rotational mode, obtained numerically, are given in the Table 1 for several values of pitch angle β and for wind speed $V = 10$ mps.

From the data of the Table 1 we can conclude that the output power depends essentially on the pitch angle. Note that for values of β large enough, the sufficient condition (6) of the existence of a rotational mode becomes unrealizable. Numerical integration of the equations of motion showed that for $\beta = -7°$ and $c = 0$, an attracting periodic trajectory still exists, whereas for $\beta = \pm 10°$ there no longer are any periodic trajectories.

Table 1. Estimations of maximum trapped power

Angle β [degree]	C^* [$\mathrm{kg \cdot m^2/s}$]	$\dot{\vartheta}^*$ [1/s]	P^* [W]
-5	0.011	23.0	6
-4	0.022	25.1	14
-3	0.031	29.3	27
-2	0.040	34.0	46
-1	0.054	34.4	65
0	0.058	34.5	70
1	0.049	34.4	59
2	0.038	34.3	38
3	0.010	29.6	8

5 Pitch angle control

We investigate the possibility of increasing the average trapped power by varying the pitch angle β. For simplification we consider the case $m_1 = 0$. Rewrite the equations of motion for the case of variable β:

$$\begin{aligned}&\vartheta' = \omega;\\&\omega' = au^2 M(\vartheta,\omega,\beta) - ac\omega + a\Phi(\beta,\sigma);\\&\beta' = \sigma;\\&\sigma' = -a_1 u^2 M_1(\vartheta,\omega,\beta)e(\alpha) - au^2 M(\vartheta,\omega,\beta) + ac\omega - (a+a_1)\Phi(\beta,\sigma),\end{aligned} \tag{11}$$

where

$$\begin{aligned}&M = C_y(\alpha)\sin(\alpha+\beta) - C_x(\alpha)\cos(\alpha+\beta);\\&M_1 = C_y(\alpha)\cos\alpha + C_x(\alpha)\sin\alpha.\end{aligned} \tag{12}$$

$\Phi(\beta,\vartheta)$ is the control torque, it is assumed to have the following form $\Phi(\beta,\vartheta) = K_1(\beta-\beta^*) + K_2(\sigma-\sigma^*)$. Here K_1 and K_2 are constant values; $\beta^*(\vartheta)$ and $\sigma^*(\vartheta)$ are the assigned functions which we want to support using the control. Now we discuss the way of choosing $\beta^*(\vartheta)$ and $\sigma^*(\vartheta)$.

Assume c is fixed. Then due to (8) the task of increasing of the average trapped power is reduced to increasing of the average angular velocity of the rotational mode, which in its turn is reduced to the increase of the torque. We would like to increase the aerodynamic torque $M(\vartheta,\omega,\beta)$. To simplify the realization of the control we choose β to be a function of ϑ only.

We illustrate this approach to selection of β in a simplified setting, where we assume $\omega = \Omega$, with $\Omega = const$ is the average angular velocity of the rotational mode obtained. It can be shown that the dimensionless aerodynamic torque can be written as follows:

$$M = \sqrt{(\Omega+\sin\vartheta)^2 + \cos^2\vartheta}\,(C_y(\alpha)\cos\vartheta - C_x(\alpha)(\Omega+\sin\vartheta))$$

where $\alpha = -\beta + arctg\dfrac{\cos\vartheta}{\Omega+\sin\vartheta}$.

First we search for the value $\alpha = \alpha^*$ where the maximum of M is achieved. Setting to zero the derivative of M with respect to α, we obtain: $C_y'(\alpha)\cos\vartheta - C_x'(\alpha)(\Omega+\sin\vartheta) = 0$.

Assume that $C_y'(\alpha) \approx C_y'(0)$; $C_x'(\alpha) \approx C_x''(0)\alpha$. These simplifications are based on the specific properties of the aerodynamic functions: $C_y(\alpha)$ is close to a linear function and $C_x(\alpha)$ is close to a quadratic function when $|\alpha| < \alpha_m << 1$ (for the functions shown in Fig. 2 $\alpha_m = 0.2$). With these assumptions the solution of the last equation is given by the formula:

$$\alpha^* = \frac{C_y'(0)\cos\vartheta}{C_x''(0)(\Omega+\sin\vartheta)} \tag{13}$$

If the condition $|\alpha^*| > \alpha_m$ holds, we set α^* equal to $-\alpha_m$ or α_m, respectively.

Now we can approximately solve for the desired β^* via the formula:

$$\beta^*(\vartheta) = -\alpha^*(\vartheta) + \frac{\cos\vartheta}{\Omega+\sin\vartheta} \tag{14}$$

This function is piecewise differentiable, so $\sigma^*(\vartheta)$ can be viewed as a piecewise continuous function.

Let's consider an example, for which $C = 0.17\,\mathrm{kg\cdot m^2/s}$, $\Omega = 3.2$. We choose $K_1 = 1000$, $K_2 = 20$, and we assume, that the value of $\Phi(\beta,\vartheta)$ is bounded by 1, i.e. if $K_1(\beta-\beta^*) + K_2(\sigma-\sigma^*) > 1$, we take $\Phi(\beta,\vartheta) = 1$, and if $K_1(\beta-\beta^*) + K_2(\sigma-\sigma^*) < -1$, then we take $\Phi(\beta,\vartheta) = -1$. At the obtained rotational mode, the average trapped power is approximately equal to 115W, the power input to control is about 30W. The corresponding periodic trajectory of the system (11)-(12) is shown in Fig. 4.

The pitch angle and the angle of attack as functions of the angle ϑ in this regime are shown in Fig. 5. The domain of attraction of the obtained trajectory is bounded below by an unstable periodic solution with $\omega \approx 2.75$.

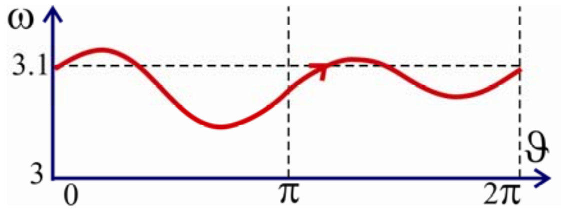

Fig. 4. Attracting periodic trajectory for variable β

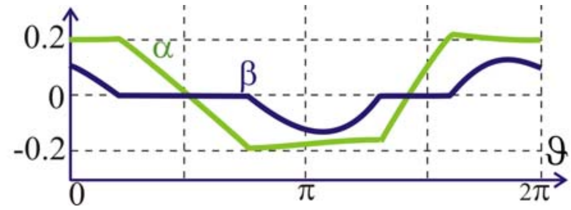

Fig. 5. Dependence of angles α and β on the angle ϑ

Conclusion

In this paper, an aerodynamic pendulum is considered as a model for the principal element of the wind-capturing unit of a wind generator for a Vertical Axis Wind Turbine VAWT.

Existence of auto-rotational regimes is studied analytically and numerically.

Numerical analysis of the phase portrait is performed. Domains of attraction of stable periodic trajectories are founded.

Estimates of the magnitude of the power of a VAWT, neglecting losses in the electric generator, are obtained numerically for several values of the pitch angle.

The possibility of using the variable pitch angle as the control function is considered.

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References

Bautin, N.N., Leontovich, E.A. (1990) Methods and techniques of the qualitative research of the dynamic systems in the plane (in Russian). Science, Moscow

Dosaev, M.Z., Kobrin, A.I., Lokshin, B.Y., Samsonov, V.A., Seliutsky, Y.D. (2007) Constructive Theory of Small-Scale Wind Power Generators. Study guide. Parts I-II. Publishers of the mechanical and mathematical faculty of the MSU, Moscow

Gorelov, D.N. (2003) Problems of Darrieus wind turbine aerodynamic (in Russian). J Termophysics and aeromechanics 1: 47-51

Kharitonov, V.P. (2006) Autonomous wind-electrical plants (in Russian). Publishers of the Research Institute of Agricultural Electrification, Moscow

Klimas, P.C., Sheldahl, R.E. (1978) Four Aerodynamic Prediction Scheemes for Vertical-Axis Turbines: A Compendium. Issued by Sandia Laboratories, operated for United States Department of Energy by Sandia Corporation

Leishman, J.G. (2002) Challenges in Modelling the Unsteady Aerodynamics of Wind Turbines. J Wind Energy 5: 85-132

Lokshin, B.Ya., Okunev, Yu.M., Ryzhova, V.E., Samsonov, V.A. (1996) Parametric Analysis of Motion of the Aerodynamical Pendulum. Proceedings of the 2nd European Nonlinear Oscillations Conference (ENOC) vol 1, Prague, pp 261-264

Lokshin, B.Ya., Privalov, V.A., Samsonov, V.A. (1986) Introduction to the problem of the rigid motion in resistant environment. Study guide (in Russian). Publishers of the MSU, Moscow

Lokshin, B.Ya., Samsonov, V.A. (1996) Calculation-analytic research of the aerodynamic pendulum behavior (in Russian). Vestnic of the MSU. Series 1. Mathematics, mechanics. 6. pp 50-55

Lokshin, B.Ya., Samsonov, V.A. (1998) On heuristic model of aerodynamical pendulum (in Russian). J Fundamental and Applied Mathematics 3 vol 4: 1047-1063

Paraschivoiu, I. (1983) Predicted and Experimental Aerodynamic Forces on the Darrieus Rotor. Journal of Energy 6 vol 7: 610-615

Parshin, D.Ye., Samsonov, V.A. (1994) Aerodynamical pendulum airflow: qualitative analisys. Programme and abstracts of the 1st European Nonlinear Oscillations Conference (ENOC), p 116

Parshin, D.Ye., Samsonov, V.A. (1994) Aerodynamical pendulum as a model of the vertical axis wind turbine. Programme and abstracts of the 1st European Nonlinear Oscillations Conference (ENOC), p 116

Prandtl, L. (1942) Fuhrer durch die Stromungslehre. Gottingen

Sabinin, G.Kh. (1931) Theory and dynamical calculation of wind rotors (in Russian). Transactions of CAHI 104: 1-71

Tabachnikov, V.G. (1974) Stationary characteristics of wings at small speeds for full-range of angles of attack (in Russian). Transactions of CAHI 1621: 79-92