MULTIESTIMATION SCHEME FOR ADAPTIVE CONTROL OF THREE TANK SYSTEM

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Abstract: This paper deals with a one possibility of improvement of a self-tuning controller reliability and performance. A simple estimation scheme is replaced by so-called a multiestimation scheme and the self-tuning controller is then synthesized from this scheme. A higher level switching structure between various estimation schemes is used to supervise the reparametrization of the self-tuning controller in real time. The basic usefulness of the proposed scheme is to improve the accuracy of estimated parameters of the controlled system and then better transient response is obtained.

Keywords: Self-tuning control, Recursive estimation, ARX models, ARMAX models, Recursive identification algorithms, Multiestimation scheme.

1. INTRODUCTION

A self-tuning controller is based on recursive estimation of parameters of the controlled plant. These parameters are then used in controller synthesis. Unknown disturbances, non-modeled dynamics and abrupt changes in parameters of controlled system can cause a simple recursive identification scheme leads to inadequate estimations. The controller based on these estimated parameters can give poor performance.

In order to deal with this problem several suggestion were proposed and can be found in literature. In some of these approaches the modification of an existing recursive identification algorithms is proposed in order to improve theirs behavior (Hill, and Ydstie, 2004) or off-line version of identification algorithm can be used in identification part of self-tuning controller, due to the fact that off-line version produced more accurate parameter estimates than its recursive counterparts (Jiang and Zhang, 2004). Another possibility deals with using so-called multiple models approach (Naredra and Balakrishnan, 1994; Naredra and Balakrishnan, 1997). The models are established for different operating conditions. Each of them has corresponding linear controller. A supervisor then determines form process data which model best represents the process at a particular time, and then switches in its associated controller.

In this paper the similar approach to multiple models is used and is based on work presented in (Ibeas, *et al.*, 2002). A simple estimation scheme in identification part of self-tuning controller is replaced by so-called a multiestimation scheme and the self-tuning controller is than synthesized from this scheme. The scheme contains the supervisor which chooses the active controller and determined the switching time between controllers. The basic usefulness of the proposed scheme is to improve the transient response.

The paper is organized as follows: in Section 2 the basic linear model structures are described, Section 3 summarize several well-known recursive identification procedures and forgetting factors, Section 4 gives the experimental results, finally Section 5 concludes paper.

2. MODEL STRUCTURE

The basic step in identification procedure is the choice of suitable type of the model. The structure of the model should sufficiently describe the dynamics of given plant and purposes for which model is build.

All linear models can be derived from general linear model (Ljung, 1987) by its simplification. In this work, the three basic linear models are taken into consideration. These are ARX (*AutoRegressive with eXogenous input)*, ARMAX (*AutoRegressive Moving Average with eXogenous input)*, OE (*Output Error)* models.

These models are used in identification part of proposed multiestimation scheme for description of the dynamics of given plant.

3. RECURSIVE PARAMETER ESTIMATION

Recursive identification algorithm is an integral part of STC and play important role in tracking time-variant parameters. The recursive parameter estimation algorithms are based on the data analysis of the input and output signals from the process to be identified. Many recursive identification algorithms were proposed (Ljung, 1987, Söderström and Stoica, 1989; Wellstead and Zarrop, 1991). In this part several well-known recursive algorithms are briefly summarized.

3.1 RLS

Recursive least square method (RLS) can be used for parameter estimate of ARX model. Standard RLS algorithm assumes that the parameters of the model process are constant. In many cases, however, the estimator will be required to track changes in a set of parameters. To cope with tracking the time-variant parameters some adjustment mechanism must be introduced in the basic equations. Several implementations have been proposed (Kulhavý, 1987; Ljung, 1987; Söderström and Stoica 1989; Kulhavý and Zarrop 1993; Corriou, 2004; Wellstead and Zarrop 1991). In this work, adaptive directional forgetting is taken into consideration.

3.2 RPLR

Recursive pseudolinear regression method (RPLR) is used for parameter estimations of ARMAX and OE model. Formally it takes the same form as RLS (Ljung, 1987; Söderström and Stoica 1989; Nelles, 2001). However, the regression and parameter vectors are different

3.3 RIV

It can be shown that if the process does not meet the noise assumption made by the ARX model, the parameters are estimated biased and nonconsistent. This problem can be avoided using instrumental variable method (Söderström and Stoica, 1989). In this work two types of instrument are taken into consideration. These are model independent instrument (RIV1) and model dependent instrument (RIV2) (Branica, *et al.*, 2002).

3.4 RPEM

The recursive prediction error method (RPEM) allows the online identification of all linear model structure. Since all model structure except ARX are nonlinearly parameterized, no exact recursive algorithm can exist; rather some approximations must be made (Moore and Boel 1986; Moore and Weiss 1979; Söderström and Stoica 1989).

4. PARALLEL MULTIESTIMATION SCHEME

Recursive estimation parameter algorithms play an important role in tracking time variant parameters of the process dynamic model and are fundamental part of self-tuning controller. If the estimation algorithm starts running with estimated vector far away from real plant parameter vector, than the transient will have large deviations from desired output resulting in a bad performance. A parallel multiestimation scheme has chosen in order to improve the transient response of the adaptive system. The architecture of multiestimation scheme is depicted in Fig 1.

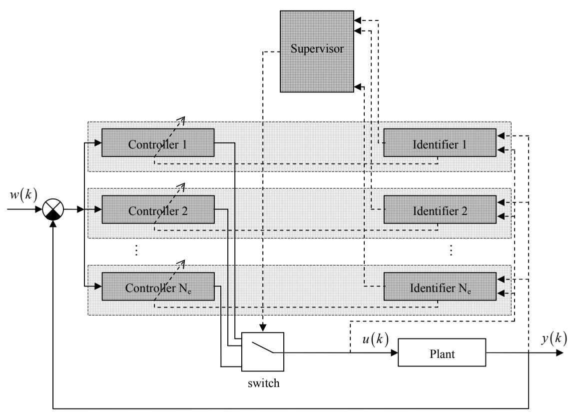

Fig. 1. Multiestimation scheme

The scheme consists of N_e pairs of identification algorithm-adaptive controller. All N_e estimation algorithms are performed at each sampling time t_k (running in parallel) (i.e. at each sampling time t_k

every algorithm gives estimated plant parameter vector $\hat{\boldsymbol{\Theta}}_i(k)$ and estimated plant output $\hat{y}_i(k)$, $i \in \mathrm{K} = \{1, 2, \ldots N_e\}$ based on past plant input and output measurements).

Each algorithm is different from each other in what is concerned with the estimated parameter vector initialization and/or the kind of the estimation algorithms and integrate the so-called multiestimation scheme.

There also exist N_e adaptive controllers (only one being in operation at each time) such that i-th adaptive controller is parametrized at every time instant t_k by i-th estimation algorithm. Thus, every pair identification algorithm-adaptive controller is indexed with only one integer $i \in \mathrm{K}$. Denote by c_k the integer that defines the controller (parametrized by its respective identification algorithm) which is active (i.e. connected to the plan for control purposes) at time t_k. The supervisor chooses the active controller from the set of possible controllers $\mathrm{K} = \{1, 2, \ldots N_e\}$ base on the prediction error of each identification algorithm and output jump associated with the controller switching. The active controller is then parametrized at time kT_0 by its respective identification algorithm and is connected in feedback to the plant.

4.1 Adaptive controller

The algebraic method is utilized to design all the controllers used in the proposed multiestimation scheme. The controllers are based on minimization of quadratic criterion with controller output signal penalization – LQ controller. The minimization of quadratic criterion is realized by spectral factorization (Bobál, *et. al.,* 2005). The structure of such controller is depicted in Fig. 2.

The control law is given by:

$$K(q,k)P(q,k)u(k) = R(q,k)w(k) - Q(q,k)y(k) \qquad (1)$$

where polynomials of the controller $(P(q,k), R(q,k), Q(q,k)) \in \{(P_i(q,k), R_i(q,k), Q_i(q,k))\}$, i.e. at each time, the polynomials of the controller are defined by $(P_i(q,k), R_i(q,k), Q_i(q,k))$ for some $i \in K$ and then, the control input is generated by the corresponding i-th controller $u(k) = u_i(k)$.

The coefficients of polynomials of i-th controller $(P_i(q,k), R_i(q,k), Q_i(q,k))$ at each sampling time t_k are calculated base on parameters estimate provided by i-th recursive identification algorithm (only parameters of deterministic part of the estimated models are utilized for controller synthesis).

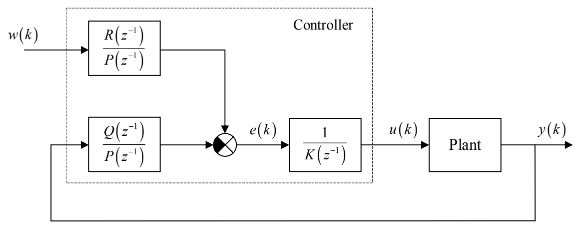

Fig. 2. Controller structure (2DOF)

Now, it is necessary to explain how to choose the current adaptive controller (active controller at t_k) from family of parallel controllers such that the adaptation transients are practically acceptable. Thus, the adaptive controller is reparametrized by its corresponding recursive identification algorithm during appropriate time intervals. A supervisor calculates the switching times between the various estimators what is used as a mechanism to on-line reparametrize the basic adaptive controller in operation to generate control input.

4.2 Supervisor

Supervisor evaluates the performance of possible controllers connected to the plant with aim of choosing the current controller from the set of parallel controllers. The proposed specific performance index takes the form:

$$\begin{aligned} J_i(k) = \alpha_k \sum_{\ell=k-M}^{k} \lambda(\ell)^{k-\ell} \left(y(\ell) - \hat{y}_i(\ell)\right)^2 \\ + \beta_k \left(\hat{y}_{c_{k-1}}(k+1) - \hat{y}_i(k+1)\right)^2 \end{aligned} \qquad (2)$$

where

$$\begin{aligned} \hat{y}_{c_{k-1}}(k+1) = \big[&-y(k), -y(k-1), \ldots, -y(k-n+1), \\ &u_{c_{k-1}}(k), u(k-1), \ldots, u(k-n+1)\big]\hat{\boldsymbol{\Theta}}_{c_{k-1}}(k) \end{aligned} \qquad (3)$$

and

$$\begin{aligned} \hat{y}_i(k+1) = \big[&-y(k), -y(k-1), \ldots, -y(k-n+1), \\ &u_i(k), u(k-1), \ldots, u(k-n+1)\big]\hat{\boldsymbol{\Theta}}_i(k) \end{aligned} \qquad (4)$$

with $c_k \in K$ denoting the identifier-controller pair in operation at sample k. M is an integer large enough to give sense to the performance evaluation. Performance index consists of two parts. The first one takes into account the accuracy of each identification algorithm, where forgetting factor λ_ℓ established the effective memory of the index in rapidly changing environments. The second one weights the output jump associated with the controller switching. If the switching between two controllers causes an abrupt variation of the plant output, then this second term acts either to avoid switching or to make the system to switch to a controller closer to the current one in operation in order to reduce the output variation of the plant. Weights α_k, β_k determine the contribution of each term to the global index and are such that $\alpha_k, \beta_k \geq 0, \alpha_k + \beta_k = 1 \quad \forall k \geq 0$.

The switching rule for the basic adaptive controller reparametrization is obtained from the performance index as follows. Denote the switching sampling time sequence by $\{t_1, t_2, \ldots, t_\pi\}$ (where π is number of switching), then the identifier c_{N_i} that parametrized the basic adaptive controller at time $t_i = N_i T_0$, $i = 1, 2, \ldots, \pi$ is determined by this equation

$$c_{N_i} \in \left\{ j \in K \middle| J_j(N_i) = \min\left\{ J_r(N_i) \quad with\ r \in K \right\} \right\} \qquad (5)$$

where the switching sampling sequence is such that $t_{i+1} - t_i \geq \tau_D$ where τ_D is a dwell time (i.e. active controller has to be in operation at least for time τ_D). The dwell time appears because of stability consideration (Ibeas, *et al.*, 2002). The c-mapping is constant between two switching sampling time (i.e. the same identifier parametrizes the adaptive controller).

Then the control law has the form:

$$K(q,k) P_{c_k}(q,k) u(k) = R_{c_k}(q,k) w(k) - Q_{c_k}(q,k) y(k) \quad (6)$$

5. EXPERIMENTAL RESULTS

The proposed multiestimation scheme was tested on laboratory model DTS200 "Three-Tank-System" (produced by Amira GmbH Duisburg, Germany). The each controller used in multiestimation scheme is based on minimization of quadratic criterion with controller output signal penalization. The minimization of quadratic criterion is realized by spectral factorization (Bobál, *et. al.,* 2005). The experimental results are compared to simple self-tuning controller.

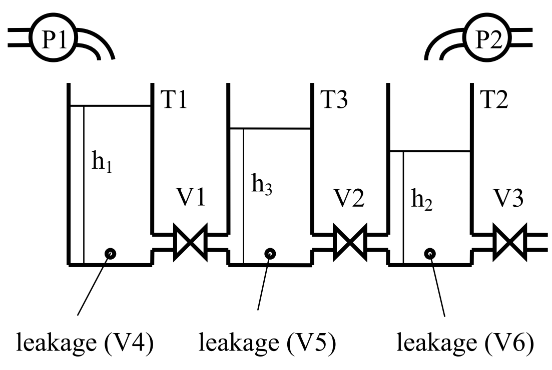

Fig. 3. Schema of three-tank system

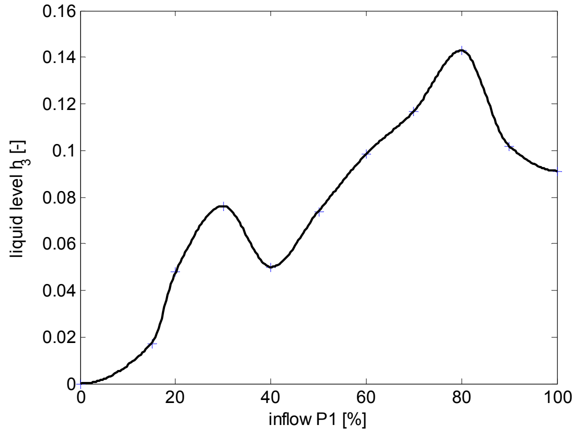

Fig. 4. Static characteristic of the system

The schema of laboratory model DTS200 - three-tank system is depicted in Fig. 3.

The system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, measurement of liquid level and others elements. The schema of the system is shown in Fig. 3. The pump P1 controls the inflow to tank T1 while the pump P2 controls the liquid inflow to tank T2. The characteristic of the flow between tank T1 and tank T3 can be affected by valve V1, flow between tanks T3 and T2 can be affected by valve V2 and the outflow of the tank T2 can be affected by valve V3. The system also provides the capability of simulating leakage from individual tanks opening the valves V4, V5 and V6.

The valves states did not change during the experiments and were positioned as follows: valves V1, V2, V3 and V6 were fully opened and valve V4 (leakage of tank T1) and valve V5 (leakage of tank T3) were approximately in the midpoint of their control range.

The input and output signals were chosen as follows: The controlled value during the experiments was liquid level of the tank T3. The control signal was voltage of the motor of the pump P1.

The static characteristic of the system is depicted in Fig. 4. From this figure can be seen that the controlled system is nonlinear.

5.1 Multiestimation scheme

The same initial conditions for system identification were used for all the types of recursive algorithms used in proposed multiestimation scheme. The adaptive controllers used in multiestimation scheme are the same type. The initial parameter estimates were chosen to be

for ARX, OE model

$$\hat{\boldsymbol{\Theta}}(k) = [0.1, 0.2, 0.3, 0.4]^T \qquad (7)$$

for ARMAX model

$$\hat{\boldsymbol{\Theta}}(k) = [0.1, 0.2, 0.3, 0.4, 0.1, 0.2]^T \qquad (8)$$

Each recursive identification methods used in proposed multiestimation scheme were performed with introduced adaptive directional forgetting factor. Initial values for adaptive forgetting were chosen to be
$\varphi(0)=1,\ \rho(0)=0.99,\ \upsilon(0)=10^{-6},\ \lambda(0)=0.001$.

Sampling period is $T_0 = 3s$, $M = 10$, weights $\alpha_k = 0.1$, $\beta_k = 0.9$, forgetting factor $\lambda_k = 0.97$ and minimum number of dwell samples between switching $N_D = 2$.

The multiestimation scheme consists of seven pairs of estimator-adaptive controller. The following recursive estimation algorithms were used in proposed scheme: RLS, RIV1, RIV2, RPLR_OE, RPLR_ARMAX, RPEM_OE, RPEM_ARMAX, respectively.

In this case, the weights were chosen so that the variance of the plant output caused by switching between controllers should be restricted.

The third pair estimator-adaptive controller (i.e. RIV-adaptive controller) is active for first twenty time instant after that the multiestimation scheme is started.

Experimental results of adaptive control with multiestimation scheme can be seen in Fig. 5.

From Fig. 5. can be seen that the basic requirement to ensure permanent zero control error were satisfied. Controlled variable achieved the desired value faster than simple self-tuning controller and the course of plant output is also smoother.

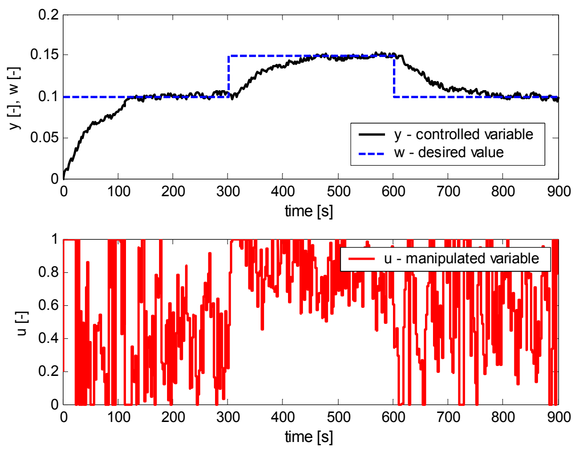

Fig. 5. Experimental results: Multiestimation scheme

From the course of manipulated variable (lower part of Fig. 5) can be concluded that despite the fact that the weights were chosen in order to limit variance in controller output the course of manipulated variable contains large changes but the course of controlled variable is satisfactory.

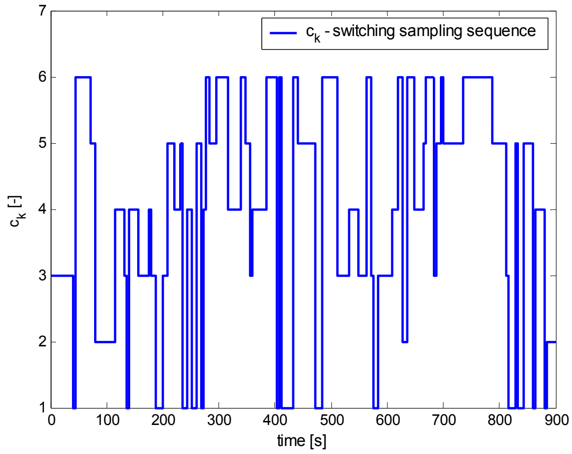

Fig. 6. Switching sampling sequence

Switching sampling sequence between pairs of estimator-adaptive controller is shown in Fig. 6. Vertical axis describes the index of active pair estimator-adaptive controller in each time instant. From this figure can be seen that active pair estimator-adaptive controller is changed frequently despite the fact that the weights were chosen in order to limit the number of switching between the pairs of estimator-adaptive controller.

5.2 *Self-tuning controller*

In this case only the first pair estimator-adaptive controller is utilized from multiestimation scheme (i.e. parameter estimate of controlled plant model is provided by recursive least squares method and controller synthesis is performed on these parameter estimate at each sampling time).

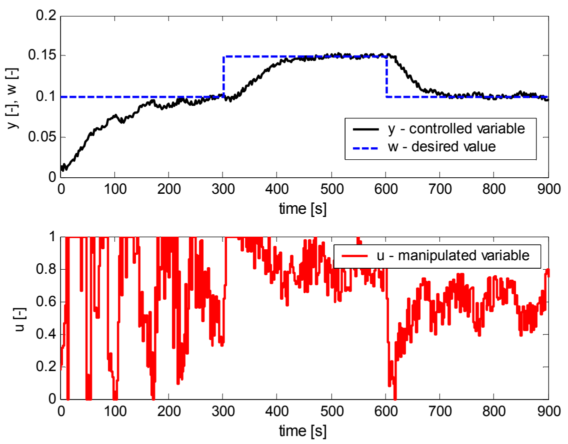

Fig. 7. Experimental results: the adaptive control with STC

Fig. 7 shows the experimental results of adaptive control with simple self-tuning controller. It can be seen that the basic requirement to ensure permanent zero control error were satisfied. Controlled variable achieved the desired value slower than in the previous case. The manipulated variable contains smaller changes in its course compare to multiestimation scheme.

The performance of adaptive control with multiestimation scheme and simple self-tuning controller were evaluated from quality control point of view. The results can be seen in Table 1.

Table 1 Comparison by quadratic criterions

Method	S_u	S_y
MEScheme	0.110663	0.000548
Simple STC	0.039119	0.000716

The quadratic criteria were defined

$$S_u = \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \Delta u^2(k) \tag{9}$$

$$S_y = \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} e^2(k) \tag{10}$$

$e(k)$ denotes control error, $u(k)$ is controller output and $k_1 = 1, \quad k_2 = 300$.

From Table 1 can be concluded that the multiestimation scheme provides better results compare to simple self-tuning controller from controlled variable point of view.

5. CONCLUSION

One possibility of improvement of a self-tuning controller reliability and performance were introduced. A simple estimation scheme is replaced by so-called a multiestimation scheme and the self-tuning controller is than synthesized from this scheme. A higher level switching structure between various estimation schemes is used to supervise the reparametrization of the self-tuning controller in real time. From experimental results can be concluded the proposed scheme improved the accuracy of estimated parameters of the controlled system and then better transient response is obtained.

The main drawback of the proposed scheme is that in order to improve transient response the free parameters in the performance index must be properly chosen.

6. ACKNOWLEDGEMENTS

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