CYBERNETICS AND PHYSICS, VOL. 13, NO. 3, 2024, 179–186

MATHEMATICAL MODELS OF SYSTEM OF MEASURE OF PRESSURE VARIATION IN COMBUSTION CHAMBERS OF ENGINES

Andrey V. Ankilov
Department of higher mathematics
Ulyanovsk State Technical University
Russia
ankil@ulstu.ru

Petr A. Velmisov
Department of higher mathematics
Ulyanovsk State Technical University
Russia
velmisov@ulstu.ru

Yuliya A. Tamarova
Department of higher mathematics
Ulyanovsk State Technical University
Russia
kazakovaua@mail.ru

Article history:
Received 15.10.2024, Accepted 26.11.2024

Abstract

In this paper the mathematical modelling of a mechanical system designed to control changes in the pressure of the working medium in aircraft engines and consisting of a pipeline and a pressure sensor is carried out. The pipeline is necessary to take the sensor to some distance from the engine in order to mitigate the impact of high temperatures and vibration accelerations on the sensitive element of the sensor, which is an elastic plate. The system takes into account the aerohydrodynamic and thermal effects of the working medium on the plate. Asymptotic equations of aerohydrodynamics in the models of compressible and incompressible medium are used to describe the working medium motion in the pipeline. Both linear and nonlinear models of a deformable solid body are proposed to describe the plate dynamics. When using the compressible medium model, the solution of the problem is reduced to the study of an equation with a deviating argument. To solve the problem using the incompressible medium model, Fourier and Galerkin methods are applied. As a result, for both models the solution of the problem is reduced to the study of ordinary differential equations relating the magnitude of pressure in the motor with the magnitude of deformation of the sensing element, which can be used to control the mode of operation of the motor. The solution of these equations is found with the developed software program using standard functions of Mathematica 12.0. The paper was presented at PhysCon2024.

Key words

Fluid-structure interaction; Pressure sensor; Elastic plate; Dynamics; Partial differential equations.

1 Introduction

In many branches of science and technology the problem of increasing the reliability and durability of structures interacting with the flow of liquid or gas occupies an important place. Such a problem, in particular, arises in the design of pressure sensors for gas-liquid media. In this connection there arises a problem of research of dynamics and stability of oscillations of structural elements, as the impact of the flow can lead to values of amplitude, velocity, accelerations of oscillations, which do not allow to carry out their reliable operation and provide the necessary functional accuracy. The development of rocket-space, aviation and other techniques requires both the development of new types of primary transducers and continuous improvement of existing ones.

In many works the description of sensors of measuring systems, principles of their operation, technical characteristics are presented, for example [Agejkin et al., 1965; Andreeva, 1981; Ash et al., 1992; Etkin, 2004; Kazaryan and Groshev, 2008; Korsunov, 1980]. Each pressure sensor is to a greater or lesser degree critical to the influence of temperatures and vibration accelerations. When pressure sensors are placed directly on the engine, they are exposed to wide temperature ranges and increased vibration accelerations, which leads to additional measurement error and, in some cases, to the destruction of the sensitive element of the sensor. The paper [Pankratov et al., 2003] considers issues related to the construction and optimization of mathematical models of pressure sensors operating in unsteady inhomogeneous fields of the measured and ambient media. The work [Belozubov and Belozubova, 2011] is devoted to the issues of increasing the vibration resistance of thin-film nano- and microsystems and pressure sensors based on them. In the case of an ideal incompressible working medium, mathematical models of the system "pipeline - pressure sensor" were considered in [Ankilov et al., 2024; Velmisov and Pokladova, 2019; Velmisov et al., 2019]. For a compressible working medium, studies of the mechanical system "pipeline - pressure sensor" in a linear model were carried out, for example, in [Velmisov and Tamarova, 2024; Velmisov and Tamarova, 2023]. In works [Chehreghani et al., 2024; Chen et al., 2021; Guo et al., 2022; Kondratov et al., 2023; Mogilevich and Popova, 2023; Reddy et al., 2020; Stetsiuk et al., 2024] the dynamics of elastic pipelines at flowing of a liquid stream is investigated.

In this paper, asymptotic equations of aerohydrodynamics in both compressible and incompressible medium models are used to describe the motion of the working medium in a pipeline. Both linear and nonlinear models of a deformable solid body are proposed to describe the plate dynamics. When using the model of compressible medium, the solution of the problem is based on the introduction of integral characteristics of the mechanical system, as a result of which it was possible to reduce the dimensionality of the problem by one unit and reduce it to the study of an equation with a deviating argument. To solve the problem using the incompressible medium model, the Fourier and Galerkin [Fletcher, 1988] methods are applied. As a result, for both models the solution of the problem is reduced to the study of ordinary differential equations. These equations relate the law of pressure change in the motor with the magnitude of deformation of the sensing element. The solution of these equations is found using standard Mathematica 12.0 functions for specific parameters of the mechanical system and can be used to create a control system of the engine operation mode. A program has been developed, which allows to obtain graphs of deformation of the sensitive element of the sensor at different setting of the law of change of pressure of the working medium. The numerical experiment is carried out and examples of calculation of the dynamics of the sensitive element of the sensor are presented.

2 Mathematical model of the pressure measurement system

The system of control over the change of working medium pressure in the engine combustion chamber presented in Figure 1 is considered. In the system, the sensor is located at some distance from the engine and connected to it by means of a pipeline, which makes it possible to mitigate the effects of temperatures and vibration accelerations. The system contains a pipeline 2 of length l and width H connecting the pressure sensor 3 to the combustion chamber 1. At one end of the pipeline $(x = -l)$, fixed at the outlet of the engine combustion chamber, the pressure of the working medium 4 changes over time. At the other end of the pipeline is a sensing element 5 ($x \in [0, h]$) of a sensor designed to measure this pressure. The sensitive element of the sensor for measuring the pressure of the working medium in the combustion chamber of an aircraft engine is an elastic plate of length H and thickness h.

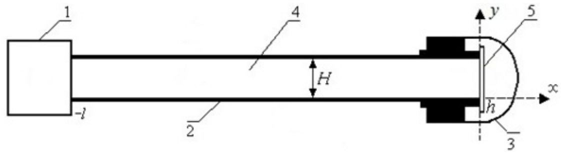

Figure 1. Pipe with sensor

The motion of the working medium in the pipeline is described by differential equations for the velocity potential $\Phi\left(x, y, t\right)$:
– in the incompressible medium model:

$$\Phi_{xx} + \Phi_{yy} = 0, \quad x \in (-l, 0)\,,\ y \in (0, H)\,; \qquad (1)$$

– in the compressible medium model:

$$\Phi_{tt} + 2\Phi_x\Phi_{xt} + 2\Phi_y\Phi_{yt} + \Phi_x^2\Phi_{xx} + \Phi_y^2\Phi_{yy} + 2\Phi_x\times$$
$$\times\Phi_y\Phi_{xy} = \left[a_0^2 - (\gamma_0 - 1)\left(\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_y^2)\right)\right]\times$$
$$\times\left(\Phi_{xx} + \Phi_{yy}\right), \quad x \in (-l, 0)\,,\ y \in (0, H)\,, \qquad (2)$$

where γ_0 – addiabatic exponent, a_0 – velocity of sound in a stationary medium.

Conditions of non-leakage of the pipeline walls $y = 0$, $y = H$ and surface of the elastic element $g\left(x, y, t\right) = 0$, which is a part of the pressure sensor, respectively, have the form:

$$\Phi_y\left(x, 0, t\right) = \Phi_y\left(x, H, t\right) = 0, \quad x \in (-l, 0)\,, \qquad (3)$$

$$\Phi_x g_x + \Phi_y g_y = -g_t,\ g\left(x, y, t\right) = 0,\ y \in (0, H)\,. \qquad (4)$$

At the outlet of the engine combustion chamber there is a pressure change $F(y, t)$ of the working medium:

$$P\left(-l, y, t\right) = F(y, t), \quad y \in (0, H)\,. \qquad (5)$$

The dynamics of the elastic element is described by the equation for the deformation of the elastic element of the sensor $w\left(y, t\right)$:

$$L\left(w\left(y, t\right)\right) = P\left(w\left(y, t\right), y, t\right) - \bar{P},\ y \in (0, H)\,. \qquad (6)$$

The constant $\bar{P}$ is the external pressure on the plate. The pressure $P\,(x,y,t)$ is defined by the formulas:
– for an incompressible medium

$$P = P_0 - \rho_1\left(\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_y^2\right), \qquad (7)$$

– for a compressible medium

$$P = P_0\left[1 - \frac{\gamma_0 - 1}{a_0^2}\left(\Phi_t + \frac{1}{2}\Phi_x^2 + \frac{1}{2}\Phi_y^2\right)\right]^{\frac{\gamma_0}{\gamma_0 - 1}}, \quad (8)$$

where P_0 – pressure in a stationary liquid, ρ_1 – density of the medium.

To determine the thermal field in the system we have the following boundary value problem:

$$\rho_1 c_1 T_{1t} = k_1 T_{1xx} - \beta\,(T_1 - T_0)\,, \qquad (9)$$

$$T_1\,(-l,t) = T_*\,(t)\,, \qquad (10)$$

$$T_{1x}\,(0,t) = 0, \qquad (11)$$

$$\rho_2 c_2 T_{2t} = k_2 T_{2xx}, \qquad (12)$$

$$T_{2x}\,(h,t) = 0, \qquad (13)$$

$$-k_2 T_{2x}\,(0,t) = \alpha(T_1\,(0,t) - T_2\,(0,t)). \qquad (14)$$

Here $T_1\,(x,t)$ is the temperature distribution of the working medium along the length of the pipeline $(x \in (-l,0))$; $T_2\,(x,t)$ is the temperature distribution along the cross-section of the elastic element of the sensor $(x \in (0,h))$; $T_*\,(t)$ is the law of temperature change at the inlet to the pipeline $(x = -l)$; T_0 is the ambient temperature; k_1, k_2, c_1, c_2, ρ_1, ρ_2 – heat conductivity coefficients, heat capacity and density coefficients of the working medium and the material of the sensing element; β – heat transfer coefficient on the lateral surface of the pipeline; α – heat transfer coefficient between the material of the element and the working medium (surface $x = 0$).

To describe the dynamics of an elastic element (deformable plate), linear and nonlinear mathematical models of a solid deformable body are used, for example

$$L\,(w\,(y,t)) \equiv Mw_{tt} + Dw_{yyyy} + N\,(t)\,w_{yy} +$$
$$+\gamma w + \beta_1 w_t + \beta_2 w_{yyyyt}; \qquad (15)$$

$$L\,(w\,(y,t)) \equiv Mw_{tt} + Dw_{yyyy} + N\,(t)\,w_{yy} +$$
$$+\gamma w + \beta_1 w_t + \beta_2 w_{yyyyt} -$$
$$-w_{yy}\left(\mu\int\limits_0^H w_y^2 dy + \eta\left(\int\limits_0^H w_y^2 dy\right)_t\right); \qquad (16)$$

$$L\,(w\,(y,t)) \equiv Mw_{tt} + \left[Dw_{yy}\left(1 - \frac{3}{2}w_y^2\right)\right]_{yy} +$$
$$+N\,(t)\,w_{yy} + \gamma w + \beta_1 w_t + \beta_2 w_{yyyyt}; \qquad (17)$$

$$L\,(w\,(y,t)) \equiv Mw_{tt} + \left[Dw_{yy}\left(1 - \frac{3}{2}w_y^2\right)\right]_{yy} +$$
$$+N\,(t)\,w_{yy} + \gamma w + \beta_1 w_t +$$
$$+\beta_2\left[w_{yy}\left(1 - \frac{3}{2}w_y^2\right)\right]_{yyt}. \qquad (18)$$

Assume that the ends of the plate are rigidly fixed and the temperature of the plate $T_2\,(x,t)$ is variable, then

$$w\,(0,t) = w_y\,(0,t) = w\,(H,t) = w_y\,(H,t) = 0, \quad (19)$$

$$N\,(t) = N_0 + \frac{E\alpha_T}{1-\nu}\int\limits_0^h T_2\,(x,t)\,dx. \qquad (20)$$

Here the coefficients M, D, E, ν, $N\,(t)$, N_0, γ, β_1, β_2, η, μ, α_T are the parameters of the mechanical system. The operator (16) takes into account the nonlinearity of the longitudinal force resulting from the elongation of the plate due to its deformation; the operator (17) takes into account the nonlinearity of the bending moment; the operator (18) refines the operator (17) in the case of taking into account the nonlinearity of the plate damping.

3 Solving of the thermal problem

The solution to the thermal problem (9) - (14) is divided into two parts: first, the temperature distribution along the length of the pipeline is found ((9) - (11)), then the temperature distribution along the thickness of the plate (problem (12) - (14)).

The solution to problem (9) - (11), obtained by the method of separation of variables, has the form

$$T_1(x,t) = T_*(t) - \sum_{n=0}^{\infty}\chi_n\cdot e^{-\gamma_n t}\sin\nu_n(x+l)\times$$
$$\times\left[\frac{\beta^0 T_0}{\gamma_n} - T_1^0 + e^{\gamma_n t}\left(T_*(t) - \frac{\beta^0 T_0}{\gamma_n}\right) - \right. \qquad (21)$$
$$\left. -a_1^2\nu_n^2\int\limits_0^t e^{\gamma_n\tau}T_*(\tau)d\tau\right],$$

where

$$\gamma_n = a_1^2 \nu_n^2 + \beta^0,\ \nu_n = \frac{\pi(2n+1)}{2l},\ \chi_n = \frac{4}{\pi(2n+1)},$$

$$T_1^0 = T_1(x,0) = const,\ a_1^2 = \frac{k_1}{\rho_1 c_1},\ \beta^0 = \frac{\beta}{\rho_1 c_1}.$$

The solution to problem (12) - (14), which makes it possible to find the temperature distribution over the thickness of the plate at an arbitrary point in time, has the form

$$\begin{gathered} T_2(x,t) = \tilde{T}(t) + \sum_{n=0}^{\infty} A_n e^{-\delta_n t} \cos \mu_n (x-h) \times \\ \times \left[T_2^0 - T_1^0 - \int_0^t e^{\delta_n t} \tilde{T}'(t) dt \right] = \tilde{T}(t) + \\ + \sum_{n=0}^{\infty} A_n e^{-\delta_n t} \cos \mu_n (x-h) \left[T_2^0 - T_1^0 - \right. \\ - \sum_{k=0}^{\infty} \frac{\chi_k \cdot \gamma_k \sin \nu_k l}{\delta_n - \gamma_k} \left(\frac{\beta^0 T_0}{\gamma_k} - T_1^0 + \frac{a_1^2 \nu_k^2 T_*}{\gamma_k} \right) \times \\ \left. \times \left(e^{(\delta_n - \gamma_k)t} - 1 \right) \right], \end{gathered} \tag{22}$$

where

$$T_2^0 = T_2(x,0) = const,\ a_2^2 = \frac{k_2}{\rho_2 c_2},\ \delta_n = a_2^2 \mu_n^2,$$

$$A_n = \frac{(-1)^n 2\alpha \sqrt{\alpha^2 + k_2^2 \mu_n^2}}{\mu_n \left[h(\alpha^2 + k_2^2 \mu_n^2) + k_2 \alpha \right]},$$

the function $\tilde{T}(t) = T_1(0,t)$ is determined by formula (21), the values μ_n $(n = 0 \div \infty)$ are the positive roots of equation $tg \mu_n h = \alpha/(k_2 \mu_n)$. Substituting (22), we find the coefficient (20).

4 Compressible medium

Let's explore the system (2), (3), (4), (5), (6), (8), (15). Let us represent the solution of this problem as an expansion in terms of a small parameter ε. Such a parameter can be the ratio of the thickness of the elastic element h to its length H $\left(\varepsilon = \frac{h}{H}\right)$, or the ratio of the width of the pipeline H to its length l $\left(\varepsilon = \frac{H}{l}\right)$.

$$\begin{gathered} \Phi(x,y,t) = \varepsilon \psi_1(x,y,t) + ...,\ F(y,t) = P_0 + \\ + \varepsilon P_*(y,t) + ...,\ w(y,t) = \varepsilon w_1(y,t) + ... \end{gathered} \tag{23}$$

According to the Lagrange-Cauchy integral (8), we have an asymptotic formula for the pressure

$$P = P_0 - \varepsilon \rho_1 \psi_{1t} + ... \tag{24}$$

From equation (2) in the first approximation, leaving the senior terms (of order ε), we obtain the equation for potential ψ_1 :

$$\psi_{1tt} = a_0^2 \left(\psi_{1xx} + \psi_{1yy} \right). \tag{25}$$

Here $a_0^2 = \frac{\gamma_0 P_0}{\rho_1}$ is the square of the speed of sound in a medium at rest.

The boundary conditions (3), (5) take the form

$$\psi_{1y}(x,0,t) = \psi_{1y}(x,H,t) = 0, x \in (-l,0), \tag{26}$$

$$-\rho_0 \psi_{1t}(-l,y,t) = P_*(y,t), y \in (0,H). \tag{27}$$

The boundary condition (4) at $\varepsilon \to 0$ in first approximation takes the form

$$\psi_{1x}(0,y,t) = w_{1t}(y,t). \tag{28}$$

Setting the value of the external load $\bar{P} = P_0$ and substituting (23) into (15), we obtain

$$\begin{gathered} M w_{1tt} + D w_{1yyyy} + N(t) w_{1yy} + \gamma w_1 + \\ + \beta_1 w_{1t} + \beta_2 w_{1yyyyt} = -\rho_1 \psi_{1t}(0,y,t). \end{gathered} \tag{29}$$

Let us consider one of the ways to solve the problem (25)-(29). Let us introduce the averaged characteristics of the main quantities of the dynamic system

$$\begin{gathered} \varphi(x,t) = \int_0^H \psi_1(x,y,t) dy,\ \xi(t) = \int_0^H w_1(y,t) dy, \\ G(t) = \int_0^H P_*(y,t) dy. \end{gathered} \tag{30}$$

Let us assume $w(y,t) = g(y)\theta(t)$, where the function $g(y)$ satisfies the boundary conditions corresponding to the type of fastening of the elastic element. According to the rigid fixation of the ends of the element (19), we take the function $g(y)$ as $g(y) = \xi_1(y)$, where

$$\begin{gathered} \xi_n(y) = ch(\mu_n y) - \cos(\mu_n y) - \\ - \frac{ch(\mu_n H) - \cos(\mu_n H)}{sh(\mu_n H) - \sin(\mu_n H)} \left(sh(\mu_n y) - \sin(\mu_n y) \right), \end{gathered} \tag{31}$$

and μ_n are found from the equation

$$ch(\mu_n H) \cos(\mu_n H) = 1.$$

Performing integration in (25), (27)-(29) over y within the limits from 0 to H, taking into account the boundary conditions (26), we obtain

$$\varphi_{tt} - a_0^2 \varphi_{xx} = 0, \tag{32}$$

$$\varphi_x(0,t) = w_0 \theta_t(t), \tag{33}$$

$$-\rho_1 \varphi_t(-l,t) = G(t), \tag{34}$$

$$\begin{gathered} -\rho_1 \varphi_t(0,t) = \\ = M_0 \theta_{tt}(t) + \alpha_0 \theta_t(t) + \gamma_0(t)\theta(t), \end{gathered} \tag{35}$$

where

$$w_0 = \int_0^H g(y)dy, \quad M_0 = M\int_0^H g(y)dy,$$

$$\alpha_0 = \beta_1 \int_0^H g(y)dy + \beta_2 \int_0^H g_{yyyy}(y)dy,\ \gamma_0(t) =$$

$$= D\int_0^H g_{yyyy}(y)dy + N(t)\int_0^H g_{yy}(y)dy + \gamma \int_0^H g(y)dy.$$

Thus, the solution of problem (25)-(29) is reduced to the study of the one-dimensional system (32)-(35) for functions $\varphi(x,t)$, $\theta(t)$, for the study of which several methods have been proposed and implemented.

A) The analytical solution of the problem leads to the study of an equation with a deviating argument. In this case, the general solution of equation (32) is written as:

$$\varphi(x,t) = A\left(t - \frac{x}{a_0}\right) + B\left(t + \frac{x}{a_0}\right), \tag{36}$$

where A, B are arbitrary functions of their arguments. Substituting (36) into (33)-(35) and performing a number of simple mathematical operations, we obtain a differential equation with a deviating argument, connecting the function $\theta(t)$, characterizing the deformation of the sensitive element of the sensor, with the function $G(t)$, characterizing the law of change in the pressure of the working medium in the engine

$$\begin{gathered} M_0\left[\theta_{tt}\left(t - \frac{l}{a_0}\right) + \theta_{tt}\left(t + \frac{l}{a_0}\right)\right] + \\ +\alpha_0\left[\theta_t\left(t - \frac{l}{a_0}\right) + \theta_t\left(t + \frac{l}{a_0}\right)\right] + \\ +\gamma_0(t)\left[\theta\left(t - \frac{l}{a_0}\right) + \theta\left(t + \frac{l}{a_0}\right)\right] - \\ -\rho_1 a_0 w_0\left[\theta_t\left(t - \frac{l}{a_0}\right) - \theta_t\left(t + \frac{l}{a_0}\right)\right] = 2G(t). \end{gathered} \tag{37}$$

If $l/a_0 = \varepsilon$ is a small parameter (for example, for water $a_0 \approx 1403$ m/sec at a temperature of 0^0C, and the length l does not exceed several meters), then, by carrying out an expansion by degrees of ε in (37) and setting aside the highest order terms, we can obtain an approximate equation (without deviating the argument t), connecting $\theta(t)$ and $G(t)$

$$\begin{gathered} (M_0 + \rho_1 w_0 l)\theta_{tt}(t) + \alpha_0\theta_t(t) + \gamma_0(t)\theta(t) + \\ +\frac{1}{2}\varepsilon^2\left[\left(M_0 + \frac{1}{3}\rho_1 w_0 l\right)\theta_{tttt}(t) + \alpha_0\theta_{ttt}(t) + \right. \\ \left. +\gamma_0(t)\theta_{tt}(t)\right] + O\left(\varepsilon^4\right) = G(t). \end{gathered} \tag{38}$$

Solutions of the linear differential equation (38) with constant coefficients are constructed both numerically and analytically; in particular, a study was conducted of resonance phenomena in the case of pulsating pressure in the combustion chamber.

B) Numerical and analytical study of problem (32)-(35) was also carried out using the Galerkin method. In this case, is represented in the form of segments of series in complete on the interval $(-l, 0)$ of systems of the functions $\{z_m(x)\}$, which satisfy homogeneous boundary conditions corresponding to conditions (33), (34) or (34), (35). As a result, the study is reduced to solving the Cauchy problem for a linear system of ordinary differential equations, on the basis of which a numerical experiment was carried out.

A software package for mathematical modeling of the mechanical system "pipeline - pressure sensor" has been developed. It is designed to study the joint dynamics of the sensitive element of the pressure sensor and the working environment in the pipeline connecting the combustion chamber of the engine with the sensor, and allows obtaining graphs of the function $\theta(t)$, characterizing the deformation of the elastic element of the sensor, with various assignments of the mechanical parameters of the system, including when assigning the law of change in the pressure of the working environment in the engine (i.e., the function $G(t)$).

Let the working medium be water ($\rho_1 = 1000$, $c_1 = 4182$, $k_1 = 0.683$), the plate be made of steel ($E = 2 \cdot 10^{11}$, $\rho_2 = 7.8 \cdot 10^3$, $\alpha_T = 7.3 \cdot 10^{-6}$, $c_2 = 460$, $k2 = 45.4$). The system parameters are: $P_0 - \bar{P} = 0$, $P_*(t) = 10^5(50 + \cos 5t)$, $a_0 = 1481$, $T_* = 1800$, $T_0 = T_1^0 = T_2^0 = 293.15$, $\alpha = 21$, $\beta = 0.15$, $l = 0.5$, $H = 0.01$, $h = 0.0005$, $N_0 = -3.11 \cdot 10^5$, $M = \rho_2 h = 3.9$, $D = \dfrac{Eh^3}{12(1-\nu^2)} = 2.29$, $\nu = 0,3$, $\gamma = 0.2$, $\beta_1 = 0.3$, $\beta_2 = 0.1$ (all values are given in the SI system).

Using the Mathematica mathematical system, solutions of equation (38) are numerically obtained in the case of rigidly fastening. The initial conditions are specified as: $\theta(0) = \theta_t(0) = \theta_{tt}(0) = \theta_{ttt}(0) = 0$, and the function $g(y) = \xi_1(y)$ is also specified. Figures 2 and 3 show the deformations of the plate at the midpoint of the plate $y_0 = 0.005$ at $t \in [0, 1]$ and $t \in [0, 20]$ respectively.

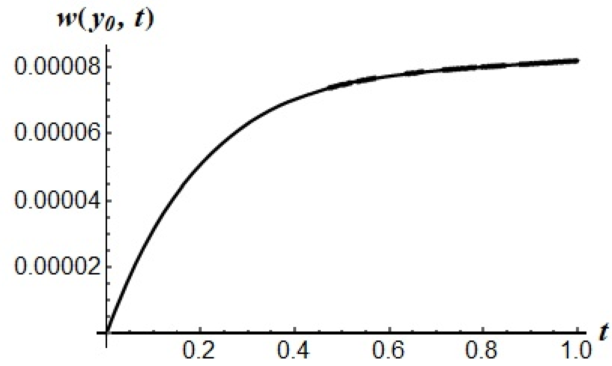

Figure 2. Graph of the deformation function of the elastic element of the sensor at $t \in [0, 1]$

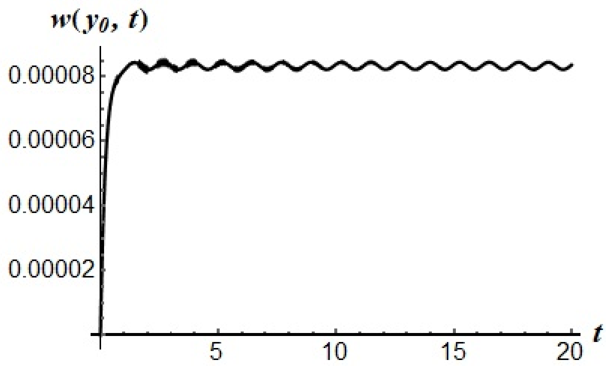

Figure 3. Graph of the deformation function of the elastic element of the sensor at $t \in [0, 20]$

5 Incompressible medium

Let's explore the system (1)–(7), (9)–(14), (19). Substituting (23) into system (1)–(7) at $\psi_1(x, y, t) = \varphi(x, y, t)$ and limiting ourselves to terms of order ε, we obtain an asymptotic model of the problem in the first approximation:

$$\varphi_{xx} + \varphi_{yy} = 0, \tag{39}$$

$$\varphi_y(x, 0, t) = \varphi_y(x, H, t) = 0,\ x \in (-l, 0)\,, \tag{40}$$

$$\varphi_x\,(0, y, t) = w_{1t}(y, t), \tag{41}$$

$$\begin{gathered} M w_{1tt} + D w_{1yyyy} + N(t) w_{1yy} + \gamma w_1 + \\ + \beta_1 w_{1t} + \beta_2 w_{1yyyyt} = -\rho_1 \varphi_t\,(0, y, t)\,. \end{gathered} \tag{42}$$

$$-\rho_1 \varphi_t\,(-l, y, t) = P_*\,(y, t)\,. \tag{43}$$

Let us assume that the excess pressure does not depend on the coordinate y, i.e. $P_*\,(y, t) = P_*\,(t)$. Then we will look for the potential $\varphi(x, y, t)$ in the form

$$\begin{gathered} \varphi\,(x, y, t) = -\frac{1}{\rho_1}\int\limits_0^t P_*\,(z)\,dz + (x + l)\,\alpha\,(t) + \\ + \sum_{n=1}^{\infty} \varphi_n\,(t) \cos\lambda_n y \cdot sh\lambda_n\,(x + l)\,,\ \lambda_n = \frac{n\pi}{H}. \end{gathered} \tag{44}$$

Function (44) satisfies the Laplace equation (39) and conditions (40), (43).

We will look for the function $w_1(y, t)$ in the form of a series expansion in a complete on the segment $[0, H]$ system of functions $\{\xi_n\,(y)\}_{n=1}^{\infty}$ that satisfy the boundary conditions corresponding to the conditions for rigidly fixing the ends of the plate (19).

According to (19), we will look for the function $w_1(y, t)$ in the form

$$w_1(y, t) = \sum_{n=1}^{\infty} w_n\,(t)\,\xi_n\,(y)\,, \tag{45}$$

where $\xi_n\,(y)$ identified in (31).

Let's substitute (44), (45) into condition (41)

$$\begin{gathered} \alpha\,(t) + \sum_{n=1}^{\infty} \varphi_n\,(t) \cos\lambda_n y \cdot \lambda_n \mathrm{ch}\lambda_n l = \\ = \sum_{n=1}^{\infty} w_{nt}\,(t)\,\xi_n\,(y)\,. \end{gathered} \tag{46}$$

According to the Galerkin method, we project (46) onto the complete system of functions $\{\cos\lambda_k y\}_{k=0}^{\infty}$. Projecting onto the first verification function $\cos\lambda_0 y = 1$, we obtain

$$\alpha\,(t) = \frac{1}{H}\sum_{n=1}^{\infty} A_n w_{nt}\,(t)\,, \tag{47}$$

where

$$A_n = \int\limits_0^H \xi_n\,(y)\,dy.$$

Projecting (46) onto the remaining functions $\{\cos\lambda_k y\}_{k=1}^{\infty}$, according to Galerkin's method we obtain:

$$\sum_{n=1}^{\infty} \left(C_{nk} w_{nt}\,(t) - B_{nk}\varphi_n\,(t)\right) = 0,\ k = 1, 2, ... \tag{48}$$

where

$$B_{nk} = \lambda_n ch\lambda_n l \int\limits_0^H \cos\lambda_n y \cdot \cos\lambda_k y dy,$$

$$C_{nk} = \int\limits_0^H \xi_n(y) \cos\lambda_k y dy.$$

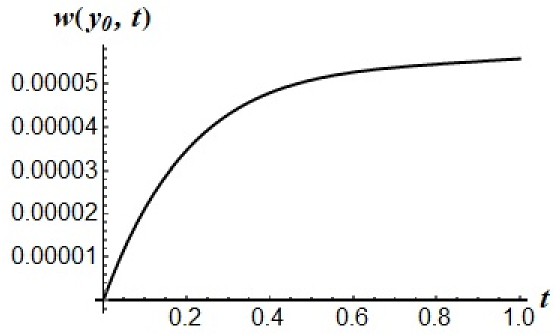

Figure 4. Graph of the deformation function of the elastic element of the sensor at $t \in [0, 1]$

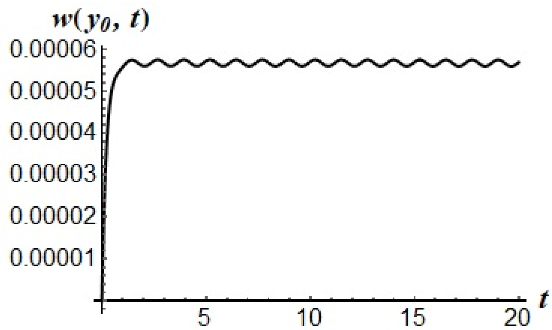

Figure 5. Graph of the deformation function of the elastic element of the sensor at $t \in [0, 20]$

Let's substitute (44), (45) into equation (42)

$$M\sum_{n=1}^{\infty} w_{ntt}(t)\,\xi_n(y) + D\sum_{n=1}^{\infty} w_n(t)\,\xi_{nyyyy}(y) +$$
$$+N(t)\sum_{n=1}^{\infty} w_n(t)\,\xi_{nyy}(y) + \gamma\sum_{n=1}^{\infty} w_n(t)\,\xi_n(y) +$$
$$+\beta_1\sum_{n=1}^{\infty} w_{nt}(t)\,\xi_n(y) + \beta_2\sum_{n=1}^{\infty} w_{nt}(t)\,\xi_{nyyyy}(y) =$$
$$= P_*(t) - \rho_1 l\alpha_t(t) -$$
$$-\rho_1\sum_{n=1}^{\infty} \varphi_{nt}(t)\cos\lambda_n y \cdot sh\lambda_n l. \quad (49)$$

Projecting (49) onto the system of orthogonal functions $\{\xi_k(y)\}_{k=1}^{\infty}$, according to Galerkin's method we obtain:

$$\begin{gathered}\sum_{n=1}^{\infty}\left[\left(MH\delta_{nk} + \frac{\rho_1 l A_k A_n}{H}\right) w_{ntt}(t) +\right.\\ +\left(\beta_1 H\delta_{nk} + \beta_2 F_{nk}\right) w_{nt}(t) +\\ +\left(DF_{nk} + N(t)G_{nk} + \gamma H\delta_{nk}\right) w_n(t) +\\ \left.+E_{nk}\varphi_{nt}(t)\right] = A_k P_*(t),\ k = 1, 2, ...\end{gathered} \quad (50)$$

where δ_{nk} – Kronecker delta,

$$F_{nk} = \int_0^H \xi_{nyyyy}(y)\,\xi_k(y)\,dy,$$

$$G_{nk} = \int_0^H \xi_{nyy}(y)\,\xi_k(y)\,dy,$$

$$E_{nk} = \rho_1 \cdot sh\lambda_n l \int_0^H \cos\lambda_n y \cdot \xi_k(y)\,dy.$$

As a result, we obtain a system of ordinary differential equations (48), (50) for determining unknown functions $\varphi_n(t)$, $w_n(t)$. Let us carry out a numerical experiment, limiting the number of terms in expansions (44), (45) by the number $m = 4$, at the previously introduced parameters of the mechanical system. Let's take the initial conditions $w_k(0) = 0$, $w_{kt}(0) = 0$, $k = 1, ..., m$. Figures 4 and 5 show the deformations of the plate at the midpoint of the plate $y_0 = 0.005$ at $t \in [0, 1]$ and $t \in [0, 20]$ respectively.

From the graphs presented in Figures 2-5, we can see that for different models (compressible and incompressible medium) the same qualitative behaviour of the pressure sensor sensing element takes place, while the quantitative difference should be explained both by the difference of physical properties of the working medium (compressibility and incompressibility) and some peculiarities of approximate methods of solving problems for different mathematical models.

Similar studies have been carried out for nonlinear models of deformable solids (16)-(18). The plate deformation graphs presented in Figures 2-5 qualitatively and quantitatively coincide with sufficient accuracy.

6 Conclusion

Mathematical models of the mechanical system "pipeline-pressure sensor", which serves to control pressure changes in combustion chambers of aircraft engines, are proposed. The models of both compressible and incompressible working medium in the engine are considered. The solution of the corresponding initial boundary value problems is reduced to the study of ordinary differential equations relating the value of pressure in the engine to the value of deformation of the sensing element, which can be used to create a control system for the engine operation mode. On the basis of the developed computer program in Matematica 12.0 the calculations for specific parameters of the mechanical system are made.

Acknowledgements

The work was supported by a grant № 23-21-00517 from the Russian Science Foundation.

References

Agejkin, D. I., Kostina, E. N. and Kuznecova, N. N. (1965). *Sensors of control and regulation.* 928 p. Moscow. Mashinostroyeniye Publ.

Andreeva, L. E. (1981). *Elastic elements of devices.* 392 p. Moscow. Mashinostroenie.

Ankilov, A. V., Velmisov, P. A. and Ankilov, G. A. (2024). Mathematical modeling of the pressure measurement system in aircraft engines. *Middle Volga mathematical society journal*, **26(3)**, pp. 294-312.

Ash, J., et al. (1992). *Sensors of measuring systems: in 2 books.* Moscow. Mir Publ.

Belozubov, E. M. and Belozubova, N. E. (2011). Increasing the resistance of thin-film nano- and microsystems and pressure sensors based on them to the effects of increased vibration accelerations. *Proceedings of the International Symposium "Reliability and Quality"*, **2**, pp. 426–429.

Chehreghani, M., Misra, A. K. and Paidoussis, M. P. (2024). Dynamics of a cantilevered pipe conveying fluid and partially subjected to a confined countercurrent external axial flow of a different fluid. *Journal of Sound and Vibration*, **590**.

Chen, W., Wang, L. and Peng Z. A. (2021). A magnetic control method for large-deformation vibration of cantilevered pipe conveying fluid. *Nonlinear Dynamics*, **105(2)**, pp. 1459–1481.

Etkin, L. G. (2004). *Vibration sensors. Theory and practice.* 407 p. Moscow. MGTU im. N. E. Baumana Publ.

Fletcher, K. (1988). *Numerical methods based on the Galerkin method.* 353 p. Moscow. Mir Publ.

Guo, Y., Zhu, B. and Li, Y. (2022). Nonlinear dynamics of fluid-conveying composite pipes subjected to time-varying axial tension in sub- and super-critical regimes. *Applied Mathematical Modelling*, **101**, pp. 632–653.

Kazaryan A. A. and Groshev G. P. (2008). Universal pressure sensor. *Izmeritel'naya tekhnika*, **3**, pp. 26–30.

Kondratov, D. V., Kondratova, T. S., Popov, V. S. and Popova M. V. (2023). Modeling hydroelastic response of the channel wall resting on a nonlinear elastic foundation. *Lecture Notes in Mechanical Engineering*, pp. 261-270.

Korsunov, V. P. (1980). *Elastic sensitive elements.* 264 p. Saratov. Saratov State Univ. Publ.

Mogilevich, L. I. and Popova, E. V. (2023). Longitudinal waves in the walls of an annular channel filled with liquid and made of a material with fractional nonlinearity. *Izvestiya VUZ. Applied Nonlinear Dynamics*, **31(3)**, pp. 365–376.

Pankratov V. M., et al. (2003). Mathematical models of the functioning of a pressure sensor for spacecraft at non-stationary temperatures of the measured and environmental media. *Microsystems engineering*, **6**, pp. 20–29.

Reddy, R. S., Panda, S. and Natarajan, G. (2020). Nonlinear dynamics of functionally graded pipes conveying hot fluid. *Nonlinear Dynamics*, **99(3)**, pp. 1989–2010.

Stetsiuk, S., Doroshenko, Y., Bondarenko, R., Filipchuk, O. and Volovetskyi, V. (2024). Investigation on the dynamics of movement of cylindrical cleaning pigs through the bends of pipeline systems for fluid transportation. *Scientific Journal of Silesian University of Technology. Series Transport*, **123**, pp. 303-317.

Velmisov, P. A. and Pokladova, Yu. V. (2019). Mathematical modelling of the "pipeline – pressure sensor" system. *Journal of Physics: Conference Series*, **1353**.

Velmisov, P. A., Pokladova, Yu. V. and Mizher, U. J. (2019). Mathematical modelling of the mechanical system "pipeline – pressure sensor". *AIP Conference Proceedings*, **2172**.

Velmisov, P. A. and Tamarova, Y. A. (2024). Mathematical modeling of the dynamics of the Aeroelastic pipeline-pressure sensor system. *PNRPU Mechanics Bulletin*, **2024(2)**, pp. 69–78.

Velmisov, P. A. and Tamarova, Y. A. (2023). Nonlinear mathematical model of pressure measurement systems in gas-liquid media. *Middle Volga mathematical society journal*, **25(4)**, pp. 313–325.