CYBERNETICS AND PHYSICS, VOL. 14, NO. 2, 2025, 148–153

CONTROL OF DIRECT CURRENT MOTORS WITH GUARANTEED SPEED MAINTENANCE IN GIVEN BOUNDS

Ba Huy Nguyen
Institute for Problems in Mechanical Engineering
of the Russian Academy of Sciences (IPME RAS)
Russia
leningrat206@gmail.com

The Dong Dang
ITMO University
Russia
dangtd93@gmail.com

Igor Furtat
IPME RAS
St. Petersburg University
Russia
cainenash@mail.ru

Pavel Gushchin
IPME RAS
Russia
guschin.p@mail.ru

Anh Quan Dao
HungVuong University
Vietnam
daoanhquan@hvu.edu.vn

Tung Duong Mai
Saint-Petersburg Electrotechnical
University ETU "LETI"
Russia
maitungduong24@gmail.com

Article history:
Received 07.06.2025, Accepted 01.09.2025

Abstract

This paper addresses the direct current (DC) motor control problem, ensuring that the motor's speed remains within a given bounds. To solve this, a coordinate transformation is applied to convert the original problem into a control problem without constraints on the control variable. A feedback controller is proposed, with the control gain determined by solving an optimization problem. An experimental setup is introduced to validate the proposed control algorithm. Two cases are proposed for testing, and the experimental results show that the algorithm ensures the motor's speed stays within the given set by the designer.

Key words

Direct current (DC) motors, coordinate transformation, nonlinear control, given bounds, experiments.

1 Introduction

DC (direct current) motors are widely used electric motors that operate based on direct current. With their simple design, ease of control, and quick response, DC motors have become the preferred choice in many industrial and household applications. The standout feature of DC motors is their ability to easily adjust speed and torque through variations in the input voltage or current through the windings.

In practice, DC motors are extensively used in speed control systems, autonomous robots, industrial conveyors, and household appliances such as drills and blenders. Additionally, DC motors are crucial components in fields such as electric vehicles, unmanned aerial vehicle (UAV) control, and medical devices, due to their flexibility and high precision. There are several DC motor control methods, such as PID control, feedback control, sliding mode control, etc [Suman and Giri, 2016; Elsrogy et al., 2013; Setiawan et al., 2023; Sharan et al., 2007; Maheswararao et al., 2011; Murtaza and Bhatti, 2012; Sharma and Lather, 2024; Imangazieva, 2024; Konovalov and Matveev, 2025]. An overview of some DC models and methods of control was considered in [Furtat et al., 2022]. Each method has its strengths and limitations. Often, adaptive and robust control methods under conditions of uncertainty and disturbances guarantee a given quality of regulation only in a steady state, that is, after the transients have finished. This paper focuses on obtaining control laws with a guarantee of an output signal in a given bounds at any given time.

To address this issue, papers [Furtat and Gushchin, 2020; Furtat and Gushchin, 2021; Nguyen et al., 2023] present an LMI-based control method that guarantees the output signals always remain within given sets, which are determined by the system designers based on the operational requirements of the overall systems. The method uses a coordinate transformation to convert the original system with output constraints into control problem involving new control variables without these limitations. This control method guarantees the desired signal performance and satisfies the operational requirements of the systems.

In this paper, control method [Furtat and Gushchin, 2020; Furtat and Gushchin, 2021; Nguyen et al., 2023] is applied to adjust the speeds of motors to always stay within predefined sets. This ensures compliance with system constraints and enhances the robustness of the control processes, making them suitable for applications with stringent performance requirements. The effectiveness of the method will be evaluated through experimental testing.

The paper is organized as follows. Section 2 discusses the problem of controlling DC motors that ensuring that the rotational speed signals belong to a given set. Section 3 presents the design of a state feedback controller for linear systems based on the method [Furtat and Gushchin, 2020; Furtat and Gushchin, 2021; Nguyen et al., 2023]. Section 4 validates the theoretical results obtained in the previous section through experimental tests.

2 Problem Statement

Let us consider a DC motor model

$$\dot{\omega}(t) = -\frac{1}{T_d}\omega(t) + \frac{K_d}{T_d}[u(t) + f(t)], \tag{1}$$

where $t \geq 0$, ω is the speed of motor [rad/s], u is input voltage signal [V], f is the unknown input disturbance [V], T_d is the time constant of motor [s], and K_d is dc-gain of motor [rad/(V·s)].

The goal of the paper is to design a controller that ensures the maintenance of the speed motor $\omega(t)$ in the following bounds at any time $t \geq 0$:

$$\mathcal{W} = \left\{\underline{\omega}(t) \leq \omega(t) \leq \overline{\omega}(t)\right\}, \text{ for all } t \geq 0. \tag{2}$$

Here, the functions $\underline{\omega}(t)$ and $\overline{\omega}(t)$ are the differentiable and bounded functions with their first derivatives. These functions are selected by designer according to the requirements of the systems.

If the designer specifies a reference signal $\omega_m(t)$, then this signal can be presented as $\omega_m(t) = 0.5(\underline{\omega}(t) + \overline{\omega}(t))$, where $\underline{\omega}(t)$ and $\overline{\omega}(t)$ are prespecified deviations from reference signal.

It is known that the initial DC motor model is nonlinear [Furtat et al., 2022]. The considered model (1) is a significantly simplified nonlinear model. However, we can assume that nonlinear components can be presented in the disturbance. Also, the applicability of this model for design the control law will be demonstrated in the experiment study.

3 Mathematical preliminaries. Theoretical result

Following [Furtat and Gushchin, 2020; Furtat and Gushchin, 2021; Nguyen et al., 2023], let us introduce the change of coordinate $\Phi : \mathcal{W} \times [0, \infty) \to \mathbb{R}$

$$\varepsilon(t) = \Phi(\omega, t) \triangleq \ln\left(\frac{\omega(t) - \underline{\omega}(t)}{\overline{\omega}(t) - \omega(t)}\right). \tag{3}$$

where $\varepsilon \in \mathbb{R}$ a new auxiliary variable.

As we see, the function $\Phi(\omega, t)$ is differential w.r.t ε and t. Denoting Φ^{-1} as inverse function of the function Φ^{-1} at each time t, one get

$$\omega(t) = \Phi^{-1}(\varepsilon, t) \triangleq \frac{\overline{\omega}(t)\exp(\varepsilon) + \underline{\omega}(t)}{\exp(\varepsilon) + 1}. \tag{4}$$

Taking the time derivative of $\omega(t)$ in (3), we get

$$\dot{\omega} = \frac{\partial \Phi^{-1}}{\partial \varepsilon}\dot{\varepsilon} + \frac{\partial \Phi^{-1}}{\partial t}. \tag{5}$$

Taking into account (1) and $\frac{\partial \Phi^{-1}}{\partial \varepsilon} = \frac{\exp(\varepsilon)(\overline{\omega} - \underline{\omega})}{(\exp(\varepsilon)+1)^2} > 0$, $\forall \varepsilon \in \mathbb{R}$, one can rewrite (5) in the form

$$\dot{\varepsilon} = \left(\frac{\partial \Phi^{-1}}{\partial \varepsilon}\right)^{-1}\left[-\frac{1}{T_d}\omega(t) + \frac{K_d}{T_d}u(t) + \psi\right], \tag{6}$$

where $\psi(\varepsilon, t) \triangleq \frac{K_d}{T_d}f(t) - \frac{\partial \Phi^{-1}(\varepsilon,t)}{\partial t}$. The function $\psi(\varepsilon, t)$ is the bounded with the bound $\bar{\psi} = \frac{K_d}{T_d}\sup_t\{f(t)\} + \sup_{\varepsilon,t}\left\{\frac{\partial \Phi^{-1}(\varepsilon,t)}{\partial t}\right\}$, where $\frac{\partial \Phi^{-1}(\varepsilon,t)}{\partial t} = \frac{\dot{\overline{\omega}}(t)\exp(\varepsilon) + \dot{\underline{\omega}}(t)}{\exp(\varepsilon)+1}$.

Let us choose the controller u in the following form:

$$u = \frac{T_d}{K_d}\left[\frac{1}{T_d}\omega - K\varepsilon\right], \tag{7}$$

where $K > 0$ is a control gain that is defined in the following theorem.

Theorem 1. *Given numbers $c > 0, \alpha > 0$. Let $\hat{K}$ is solution of the following optimization problem:*

$$\begin{gathered} \lambda \to min, \ \ s.t. \\ 0 \leq K \leq \lambda, \\ \begin{bmatrix} -K + \alpha + \tau_1 & 0.5 \\ \star & -\tau_2 \end{bmatrix} \leq 0, \\ -c^2\tau_1 + \bar{\psi}^2\tau_2 \leq 0. \end{gathered} \tag{8}$$

with respect to the scalar variables $K \geq 0$ and $\tau_1, \tau_2 > 0$.

Hence, the controller (7) with the gain $K = \hat{K}$ ensures the goal (2). The symbol "$\star$" denotes a symmetric element in a symmetric matrix.

Proof. Applying the controller (7) to (6), we get

$$\dot{\varepsilon} = \left(\frac{\partial \Phi^{-1}}{\partial \varepsilon}\right)^{-1}\left[-K\varepsilon + \psi\right]. \tag{9}$$

Let us choose Lyapunov function in the form

$$V(\varepsilon) = 0.5\varepsilon^2. \tag{10}$$

Taking the time derivative of Lyapunov function (10) along the solution of (9), we get

$$\dot{V} = \varepsilon\dot{\varepsilon} = \varepsilon\left(\frac{\partial \Phi^{-1}}{\partial \varepsilon}\right)^{-1}\left[-K\varepsilon + \psi\right]. \tag{11}$$

Let us define the set Ω as follows:

$$\Omega := \{\varepsilon \in \mathbb{R}^m : |\varepsilon| < c,\ c > 0\}, \tag{12}$$

where c is a positive number.

In order to stabilize the trajectory ε into the set Ω, we require for any $\alpha > 0$: $\dot{V} \leq -2\alpha V$, $\forall \varepsilon \notin \Omega$, i.e., $\dot{V} < 0$, $\forall \varepsilon : |\varepsilon| \geq c$. Since $\frac{\partial \Phi^{-1}}{\partial \varepsilon} > 0$ does not affect the sign of the expression (11), these conditions can be rewritten as

$$\begin{aligned} -(K-\alpha)\varepsilon^2 + \varepsilon\psi \leq 0, \forall\, (\varepsilon, \psi) : \\ \varepsilon^2 \geq c^2,\ \psi^2 \leq \bar{\psi}^2. \end{aligned} \tag{13}$$

Denoting $z = [\varepsilon, \psi]^T$, rewrite (13) as follows:

$$\begin{aligned} z^T \begin{bmatrix} -K+\alpha & 0.5 \\ \star & 0 \end{bmatrix} z \leq 0, \\ z^T \begin{bmatrix} -1 & 0 \\ \star & 0 \end{bmatrix} z \leq -c^2,\ z^T \begin{bmatrix} 0 & 0 \\ \star & 1 \end{bmatrix} z \leq \bar{\psi}^2. \end{aligned} \tag{14}$$

Applying the S-procedure [Boyd et al., 1994; Poznyak et al., 2014; Polyak et al., 2021], one can conclude that inequalities (14) are satisfied if two last inequalities in (8) hold. Therefore, the controller (7) with $K = \hat{K}$ ensures the boundedness of ε, and ε is stabilized into the set Ω. According to the properties of the change of coordinate (3), the goal (2) is satisfied.

Theorem 1 is proved.

Remark 1. *The LMI technique and S-procedure allow us to design the controller for a system under the influence of unknown bounded disturbances based on analysis the input to state stability of the closed-loop system. Moreover, the problem of finding the control gain of (7) can be reduced to the problem of finding the solutions to the minimizing problem (8), which can be easily solved using popular solvers for semidefinite programming (such as SEDUMI [Sturm, 1999], SDPT3 [Toh et al., 1999], CSDP [Borchers, 1999] and others.)*

4 Experimental result

4.1 Experimental Setup

In this paper, we create an education board for experiments (Fig. 1). The board includes the following main components:

1. Lego NXT DC Motor: with a built-in incremental encoder with a resolution of 180 (pulses/rev) for measuring the rotor's angular position, enabling the implementation of position or velocity feedback control;
2. L298N H-Bridge Driver: used for motor control, providing direction reversal and speed adjustment via pulse-width modulation (PWM);
3. STM32F4-Discovery Microcontroller Board: serves as the central control unit of the prototype. The microcontroller handles encoder signal processing, PWM signal generation for the driver, and execution of control algorithms. Since the STM32F4 board includes an internal x4 (Quadrature) circuit, for the encoder one revolution of the motor produces $180 \times 4 = 720$ (pulses/rev).

The control process utilizes the Waijung Blockset library [Hmidet and Hasnaoui, 2018], which facilitates the integration between Simulink/MATLAB and the STM32 microcontroller via the UART interface. The configuration of the experimental setup is illustrated in Fig. 2.

Development and implementation are divided into two main components: the "Target program" and the "Host program". Both are created in MATLAB/Simulink. The Target program is responsible for configuring the microcontroller to interact with the motor's H-driver, encoder, and computer. This program is compiled to generate C code, which is then loaded onto the microcontroller module. The Host program is used to generate disturbances to simulate perturbations, synthesize the controller for the studied control system, and visualize the resulting data. Together, these components enable the experimental setup to function in HIL (Hardware-In-the-Loop) mode [Kelemen et al., 2014]. This mode provides real-time closed-loop control of the physical system, facilitating comprehensive testing and validation of the control algorithms.

For the experiments, the PWM period is set to 2 kHz and the sample time is 0.02 seconds. The UART baud rate is 460800 bits/s.

4.2 Evaluation of Model Parameters and Controller Testing

Let us rewrite the mathematical model (1) in the form of a transfer function $W(s)$ relating the voltage u to the speed ω, expressed as

$$W(s) = \frac{\omega(s)}{u(s)} = \frac{K_d}{T_d s + 1}. \tag{15}$$

Since the transfer function W(s) (15) represents a first-order linear system, the coefficient K_d is the motor static gain to ω.

For the time constant T_d, it can be determined as the time at which the speed equals 63.2% of its steady-state value.

Table 1 presents the steady-state values $\omega(\infty)$ obtained for different constant input voltages. Based on these values, the coefficients can be approximated as $K_d \approx 2.05$, $T_d \approx 0.2$. Fig. 3. compares the measured speed ω (the blue line) with the simulated speed (the dash red line) for an input voltage $u = 9[V]$.

Now, let us consider two cases for testing the proposed controller.

Case 1.

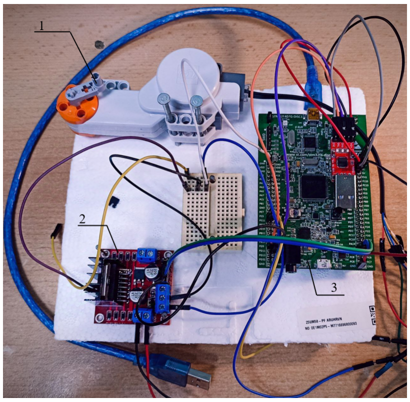

Figure 1. The experimental board: 1-Lego DC motor, 2-H-Bridge Driver, 3-STM32F4-Discovery board.

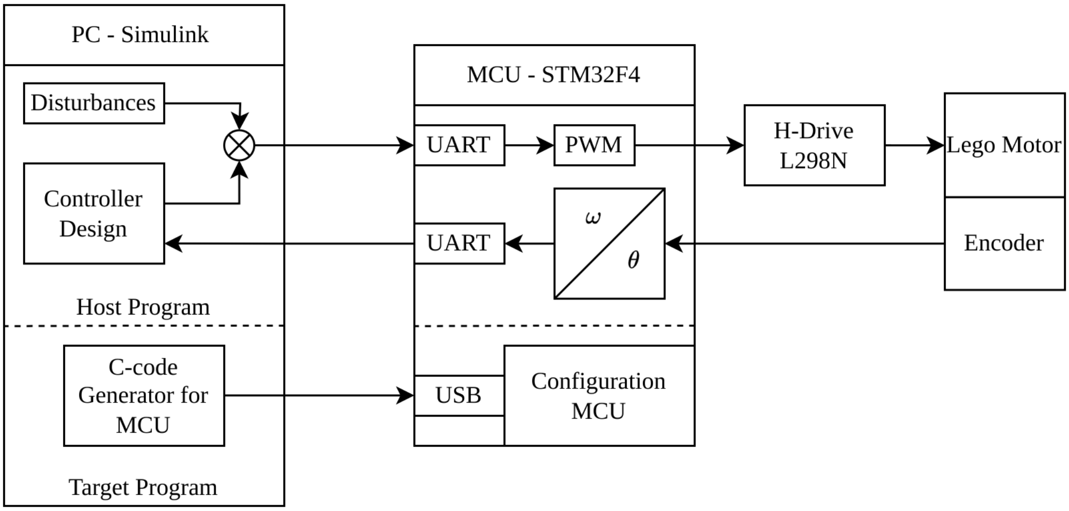

Figure 2. The configuration of experimental setup.

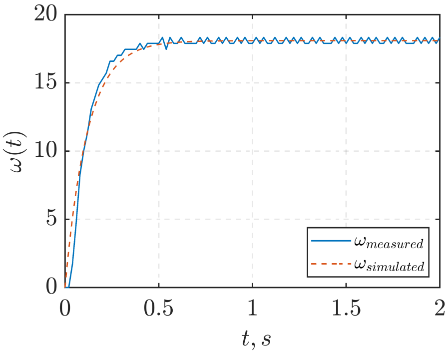

Figure 3. Speed response ω for input $u = 9[V]$: the blue line represents the measured speed, and the dashed red line represents the simulated speed.

u	$\omega(\infty)$	K_d
5	10.25	2.05
6.5	13.3	2.05
7	14.35	2.05
8.5	17.34	2.05
9	18.33	2.04

Table 1. Values $\omega(\infty)$, and K_d for different inputs u.

Let us choose $\underline{\omega}$ and $\overline{\omega}$ in the form

$$\overline{\omega}(t) = 11 + \sin(\pi t + \tfrac{\pi}{2}),$$
$$\underline{\omega}(t) = \begin{cases} -10 + 16\sin(\frac{\pi}{4}t), & t < 2, \\ 4 + 2\cos(\pi t), & t \geq 2. \end{cases} \quad (16)$$

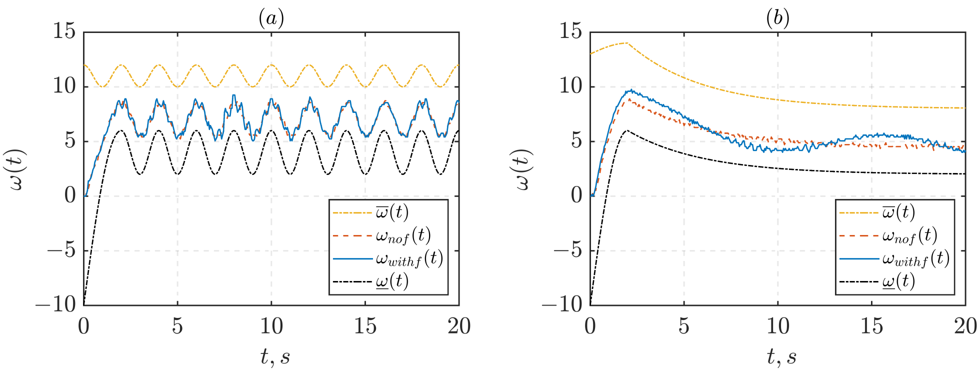

Figure 4. Controlled speed ω: (a) Case 1, (b) Case 2. The yellow and black dashed lines denote the performance bounds $\overline{\omega}(t)$ and $\underline{\omega}(t)$, respectively. The dashed red lines show the speed $\omega(t)$ without disturbance f, while the blue lines depict $\omega(t)$ with disturbance f applied.

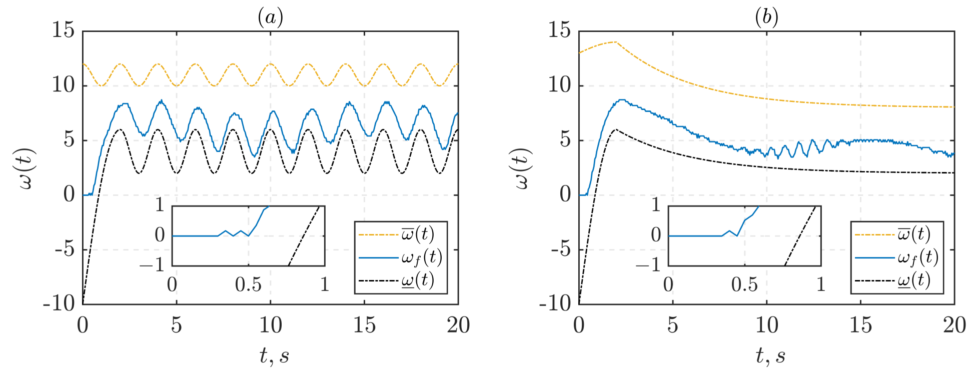

Figure 5. Controlled speed ω when time-delay $h = 0.3$ seconds: (a) Case 1, (b) Case 2. The yellow and black dashed lines denote the performance bounds $\overline{\omega}(t)$ and $\underline{\omega}(t)$, respectively. The blue lines depict $\omega(t)$ with disturbance f applied.

Assume that $f(t) = 5\sin(0.5t)$, hence $\bar{\psi} = 63.82$. By selecting $c = 0.5$, $\alpha = 1$, and solving the problem in equation (8), we obtain $K = 127.73$. Fig. 4 shows the graphs of the controlled speed ω when no disturbance f affects the system (represented by the dashed red line) and when f is present in the system (represented by the blue line). As illustrated in Figure 4, the signal ω remains within the given set at any time.

Case 2.

Let us choose $\underline{\omega}$ and $\overline{\omega}$ in the form

$$
\begin{aligned}
\overline{\omega}(t) &= \begin{cases} 13 + \sin(\frac{\pi}{4}t), & t < 2, \\ (4 + 2\cos(\frac{\pi}{2}t)\exp(-0.25(t-2)) + 8, & t \geq 2, \end{cases} \\
\underline{\omega}(t) &= \begin{cases} -10 + 16\sin(\frac{\pi}{4}t) - 0.2, & t < 2, \\ (2 + 2\cos(\frac{\pi}{2}t)\exp(-0.25(t-2)) + 2, & t \geq 2. \end{cases}
\end{aligned} \tag{17}
$$

Assume that $f(t) = 5\sin(0.5t) + 2$, hence $\bar{\psi} = 84.31$. By selecting $c = 0.5$, $\alpha = 1$, and solving the problem in equation (8), we obtain $K = 168.73$. Fig. 5 shows the graphs of the controlled speed ω when no disturbance f affects the system (represented by the dashed red line) and when f is present in the system (represented by the blue line). As in Case 1, the signal ω remains within the given set at any time.

Let us now consider the performance of the proposed control algorithm in the presence of time-delay in the input channel and parameters as in Case 1 and Case 2. This time-delay can occur due to remote control or when using digital devices. The experimental results have shown that with a time-delay of less than 0.3 seconds, the proposed control law guarantees the fulfilment of the goal. Thus, in the present paper we propose the control law developed without taking into account the time-delay in the input channel, but which can be robust to small values of the time-delay. The transients in the presence of a time-delay are presented in the Fig. 5.

5 Conclusion

This paper presents a control method for DC motors, ensuring that the motor's speed remains within a given set. The method applies a coordinate transformation to convert the control problem into one without constraints on the controlled variable. The control gain is determined by solving an optimization problem.

Experimental results show that the proposed algorithm is effective in maintaining the motor's speed within the given set by the designer. The experiments were conducted in two specific cases, and the results demonstrate that the algorithm ensures stability and adherence to the design requirements while improving the robustness of the control system.

Acknowledgements

The study was supported by the grant of the Russian Science Foundation No. 25-19-20075, https://rscf.ru/en/project/25-19-20075/ at the IPME RAS.

References

Borchers, B. (1999). CSDP, a C library for semidefinite programming. *Optimization Methods & Software*, **11**, pp. 613–623.

Boyd, S., Ghaoui, L. E., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. SIAM.

Elsrogy, W. M., Fkirin, M. A., and Hassan, M. A. M. (2013). Speed control of DC motor using PID controller based on artificial intelligence techniques. In *2013 International Conference on Control, Decision and Information Technologies (CoDIT)*, pp. 196–201.

Furtat, I. and Gushchin, P. (2020). Control of dynamical systems with given restrictions on output signal with application to linear systems. *IFAC-PapersOnLine*, **53**, pp. 6384–6389.

Furtat, I. and Gushchin, P. (2021). Control of dynamical plants with a guarantee for the controlled signal to stay in a given set. *Automation and Remote Control*, **82**, pp. 654–669.

Furtat, I., Slobodzyan, N., and Pryanichnikov, R. (2022). Overview of models and control methods for step motors and permanent magnet motors. *Cybernetics and Physics*, **11** (4), pp. 190–197.

Hmidet, A. and Hasnaoui, O. (2018). Waijung blockset-STM32F4 environment for real time induction motor speed control. In *2018 IEEE 5th International Congress on Information Science and Technology (CiSt)*, pp. 600–605.

Imangazieva, A. (2024). Robust control of one class of nonlinear systems with a distributed delay. *Cybernetics and Physics*, **13** (1), pp. 42–48.

Kelemen, M., Kelemenová, T., Virgala, I., Mikova, L., and Liptak, T. (2014). Rapid control prototyping of embedded systems based on microcontroller. *Procedia Engineering*, **96**.

Konovalov, P. and Matveev, A. (2025). Distributed algorithms for self-spreading of robotic networks over unknown complex areas in GPS-denied environments. *Cybernetics and Physics*, **14** (1), pp. 52–61.

Maheswararao, C., Yadlapati, S., and Amaresh, K. (2011). Sliding mode speed control of a DC motor.

Murtaza, G. and Bhatti, A. I. (2012). Control of DC motors using sliding mode. In *Proceedings of 2012 9th International Bhurban Conference on Applied Sciences & Technology (IBCAST)*, pp. 37–42.

Nguyen, B. H., Furtat, I., and Vrazhevsky, S. (2023). Control of linear systems with guarantee of outputs in given sets at any time. In *2023 62nd IEEE Conference on Decision and Control (CDC)*, pp. 1667–1672.

Polyak, B. T., Khlebnikov, M. V., and Shcherbakov, P. S. (2021). Linear matrix inequalities in control systems with uncertainty. *Automation and Remote Control*, **82** (1), pp. 1–40.

Poznyak, A., Polyakov, A., and Azhmyakov, V. (2014). *Attractive Ellipsoids in Robust Control*. Springer International Publishing.

Setiawan, N. R., Ma'arif, A., and Widodo, N. (2023). DC motor controller using full state feedback. *Control Systems and Optimization Letters*, **1**, pp. 7–11.

Sharan, R. V., Onwubolu, G. C., Singh, R., Reddy, H., and Kumar, S. (2007). A simple speed feedback system for low speed DC motor control in robotic applications. In Sobh, T., Elleithy, K., Mahmood, A., and Karim, M., editors, *Innovative Algorithms and Techniques in Automation, Industrial Electronics and Telecommunications*, Dordrecht, Springer Netherlands, pp. 397–399.

Sharma, A. and Lather, J. (2024). Synchronization of coupled quadcopters using contraction theory. *Cybernetics and Physics*, **13** (2), pp. 142–151.

Sturm, J. F. (1999). Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones. *Optimization Methods and Software*, **11**, pp. 625–653.

Suman, S. K. and Giri, V. K. (2016). Speed control of DC motor using optimization techniques based PID controller. In *2016 IEEE International Conference on Engineering and Technology (ICETECH)*, pp. 581–587.

Toh, K.-C., Todd, M. J., and Tütüncü, R. H. (1999). SDPT3 – a MATLAB software package for semidefinite programming, version 1.3. *Optimization Methods & Software*, **11**, pp. 545–581.