IMPACT DYNAMIC BEHAVIOUR OF MESHING LOADED TEETH IN TRANSMISSION DRIVE RATTLE

M. De la Cruz
Wolfson School of Mechanical & Manufacturing Engineering, Loughborough University, Loughborough, UK
m.delacruz@lboro.ac.uk

S. Theodossiades
Wolfson School of Mechanical & Manufacturing Engineering, Loughborough University, Loughborough, UK
s.theodossiades@lboro.ac.uk

H. Rahnejat
Wolfson School of Mechanical & Manufacturing Engineering, Loughborough University, Loughborough, UK
h.rahnejat@lboro.ac.uk

P. Kelly
Powertrain Engineering, Ford Werke AG, Merkenich, Cologne, Germany
pkelly7@ford.com

Abstract:
The paper describes a model of impact dynamics of meshing gear teeth-pairs under medium to heavy loads and in presence of backlash. The model incorporates classical Hertzian impact, governed by instantaneous geometry of the contact and the prevailing kinematics of contiguous surfaces for pairs of helical teeth. It also includes the effect of flank friction, contributed by a combination of adhesive friction due to asperity-tip interactions, as well as viscous shear of partially lubricated conjunctions. The inclusion of such contact/impact models into gear pair dynamics sheds light on some of the spectral contributions noted from transmission systems under dynamic conditions.

Keywrods
Powertrain, automotive transmissions, NVH, rattle

Notation:

a : Semi-major half-width of the elliptical footprint
A : Apparent Hertzian contact area
A_e : Real contact area of elastic asperity tips
b : Semi-minor half-width of the elliptical footprint
C : Clearance (half the nominal backlash)
$E_{1,2}$: Elastic moduli of bodies in contact
E^* : Reduced elastic modulus of the contact
F : Friction force
h : Lubricant film thickness
h_g : Rigid gap size
K : Contact stiffness non-linearity
m : Equivalent mass of the ellipsoidal solid
M_{res} : Driveline and contact patch resistance
r_p : Contact radius of the pinion in the *xz* plane
r_w : Contact radius of the wheel in the *xz* plane
r_x : Equivalent radius in the *xz* plane
r_y : Equivalent radius in the *yz* plane
R_{bp} : Base radius of pinion
R_{bw} : Base radius of wheel
R_{ow} : Outer radius of wheel
R_{pc} : Pinion radius at point of contact
R_p : Contact radius of the pinion in the *yz* plane
R_w : Contact radius of the wheel in the *yz* plane
R_{wc} : Radius of wheel at the point of contact
u : Speed of entraining motion
v : Impact velocity of equivalent ellipsoidal solid
W : Contact load
α_n : Pressure angle in the normal plane
α_t : Pressure angle in the tangential plane
β : Helix angle
δ : Contact deflection
γ : Average asperity tip radius
φ_p : Pinion rotation angle
φ_w : Wheel rotation angle
λ : Oil film parameter
ρ_p : Curvature of the pinion in the pitch plane
ρ_w : Curvature of the wheel in the pitch plane
σ : Root mean square roughness of contacting surfaces
$\upsilon_{1,2}$: Poisson's ratio for the contacting bodies
τ_s : Shear strength of the contacting solid surfaces

1 Introduction:

With increasing engine power and use of compact transmission systems engine torsional vibration, impact of unselected loose gear pairs and ubiquitous backlash act as sources of excitation [Dogan, Ryborz and Bertzsche, 2004]. The resulting phenomenon is broadly referred to as transmission rattle, but comprises a multitude of interacting events. These include impact of loose gear pair, most audible at low engine speeds, particularly under idle (neutral) condition, referred to as idle rattle [Theodossiades, Tangasawi and Rahnejat, 2007]. At higher speeds and torques the vibration also includes impact of meshing teeth-pairs, which occur due to backlash and transmission error, a condition referred to as drive rattle [Foellinger, 2004]. The system response, as perceived by an observer, includes a broader band, since impact energy is transmitted through the elastic shafts and their supporting bearings into the retaining structures such as the transmission bell-housing, which due to its thin hollow nature has many modal responses within the range of generated impact energy. Therefore, there is a plethora of contributing effects to the overall response both in structure-borne and airborne content.

In idle gear rattle the impact energy is fairly low and is transmitted to the structural elements through lightly loaded lubricated impact zones, as described by Gnanakumarr *et al* [Gnanakumarr, Theodossiades and Rahnejat, 2002] and [Tangasawi, Theodossiades and Rahnejat, 2007].The lubricant behaviour in conjunction of meshing teeth pair for lightly loaded impacts can be considered as hydrodynamic. Under such conditions, the crushing stiffness of a pair of teeth in contact/impact is governed by the low stiffness of the lubricant film, rather than the Hertzian effective stiffness of the contact, which is in series with it and considerably larger in magnitude. The situation alters under medium to heavy loaded impacts, such as under conditions pertaining to drive rattle in the conjunctions created by selected meshing pairs. In such conjunctions the regime of lubrication pertains to elastohydrodynamics, where the lubricant film becomes incompressible, often with very thin thickness, and at high shear rates behaves in a non-Newtonian manner [Gohar, 2001]. Therefore, its stiffness usually exceeds that of the contiguous surfaces and a Hertzian type analysis would suffice. In fact, the localised deflection of solid surfaces far exceeds the fluctuations on the surface of the oil film, as shown by Mehdigoli *et al* [Mehdigoli, Rahnejat and Gohar, 1990]. The main contribution of the elastohydrodynamic film would be due to viscous shear, providing a coherent film is formed in parts of the generally accepted mixed regime of lubrication in the conjunction. This paper provides such an approach to transmission drive rattle, contributed by the selected meshing pair of gears.

2 Theory

2.1 Impact dynamics of meshing teeth

The Hertzian contact force is given by [Hertz, 1896]:

$$W = K\delta^{3/2} \tag{1}$$

where clearly: $$K = \frac{4}{3}\left(r_x r_y\right)^{1/4} E^* \tag{2}$$

K is the contact stiffness non-linearity and r_x and r_y are the effective radii of contact in the normal plane *xz* and the tangential plane *yz*. These are obtained by consideration of principal radii of bodies in contact at any instant of time. Each body has two such principal radii; one in the direction of the normal plane and the other along the conforming flanks for helical gear teeth (tangential plane *yz* (see figure 1).
For the *xz* plane, the equivalent radius is:

$$r_x = \frac{r_p r_w}{r_p + r_w} \tag{3}$$

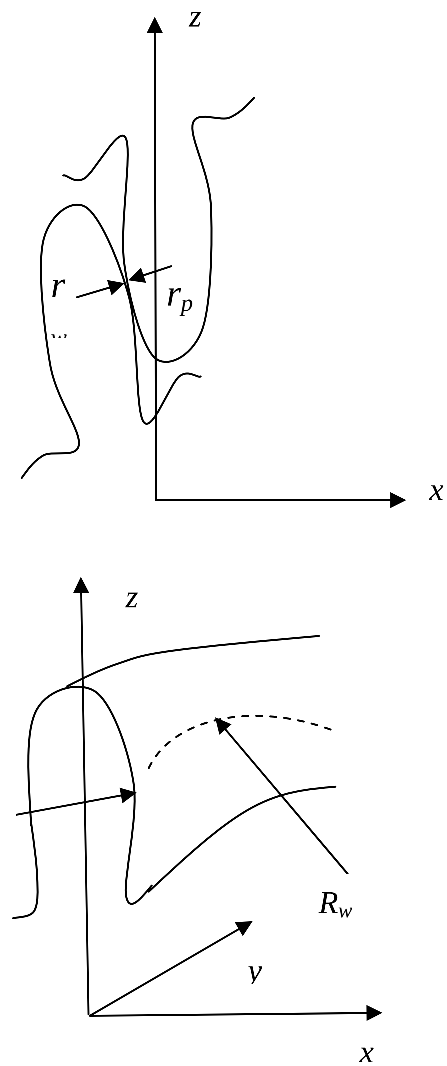

Figure 1: Principal radii of bodies in contact

Noting that:

$$\left.\begin{aligned} r_p &= \sqrt{\left(R_{bp}^2 + \rho_p^2\right)} \\ r_w &= \sqrt{\left(R_{bw}^2 + \rho_w^2\right)} \end{aligned}\right\} \tag{4}$$

and:

$$\left.\begin{aligned} \rho_w &= \frac{\sqrt{R_{ow}^2 - R_{bw}^2} - \varphi_p R_{bp}}{\cos\beta} \\ \rho_p &= \frac{c_c \sin\alpha_t}{\cos\beta} - \rho_w \end{aligned}\right\} \tag{5}$$

For the *yz* plane a partially conforming contact exists, in which the radius of the wheel is considered as negative due to its concavity, thus:

$$r_y = \frac{R_w R_p}{R_p - R_w} \tag{6}$$

Therefore, as shown in figure 2, there is an equivalent ellipsoidal solid contacting/impacting a semi-infinite elastic half-space at any instant of time.

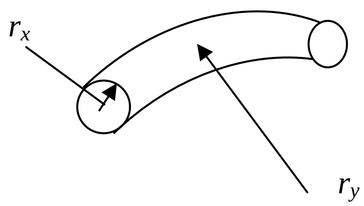

Figure 2: The equivalent curvilinear ellipsoidal solid

The equivalent radius is that of an ellipsoidal solid impacting/contacting a semi-infinite elastic half-space of reduced elastic modulus:

$$E^* = \frac{2}{\left(\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}\right)}$$

Under rattle condition a pair of teeth impact each other and rebound/separate, when the maximum Hertzian penetration is reached. For each pair of impacting teeth, represented by an ellipsoidal solid mass, *m*, in localised Hertzian impact (below their modal behaviour), principle of conservation of energy is upheld, when:

$$\frac{1}{2}mv^2 = \frac{2}{5}K\delta_{\max}^{5/2}, \text{ thus: } \delta_{\max} = \left(\frac{5mv^2}{4K}\right)^{2/5} = \left(\frac{15mv^2}{16\left(r_x r_y\right)^{1/4} E^*}\right)^{2/5} \tag{7}$$

where *m* is the mass of the equivalent ellipsoidal solid: $m = \rho V = \pi\rho r_x^2 l$

l is the length of the equivalent ellipsoidal solid, which can be approximated by the length of contact footprint. For an elliptical footprint this is the long diameter, given by 2*a*. Rolling of the bodies occur along the minor axis of the elliptical footprint, with the minor diameter, 2*b*. Knowing the contact load, *W*, from Hertzian theory [Hertz, 1896]:

$$\sqrt{ab} = \left(\frac{3W\sqrt{r_x r_y}}{4E^*}\right)^{1/3}$$

Also:

$$a/b \approx \left(r_x/r_y\right)^{2/3}$$

Thus:
$$l = 2a = 2\left(\frac{9W^2 r_y^2}{16E^*}\right)^{1/6} \tag{8}$$

v is the orthogonal velocity of approach of the pair of teeth. In other words is the velocity of the equivalent ellipsoidal solid described by r_x, r_y, l and m, impacting a semi-infinite elastic half-space of the reduced elastic modulus, E^*. v is obtained as:

$$v = v_{pitch} = \left|R_{pc}\dot{\varphi}_p - R_{wc}\dot{\varphi}_w\right|\cos\beta \tag{9}$$

2.2 Gear-pair dynamics

Note that the motion of the pinion is prescribed according to the transmission input shaft (i.e. it is kinematic). This is measured for $\ddot{\varphi}_p$, $\dot{\varphi}_p$ and φ_p. For the wheel, the equation of motion is:

$$I_w\ddot{\varphi}_w = \left(W - F\right)R_{wc} - M_{res} \tag{10}$$

where *W* is the Hertzian contact force and the flank friction force is given in section 2.3. M_{res} is the resistance representing typical driveline and contact patch contributions. A step-by-step integration algorithm is used to determine $\dot{\varphi}_w$ and φ_w.

2.3 Determination of flank friction

In reality the flank friction force is due to a combination of asperity interactions and viscous action due to the presence of a lubricant film. The gap between any meshing pair of teeth can be obtained as:

$$h = h_g + \delta_{\max} = C - \frac{\left|\left(R_{cw}\varphi_w - R_{cp}\varphi_p\right)\right|}{\cos\alpha_n \cos\beta} + \delta_{\max} \tag{11}$$

where h_g is the rigid gap, and C is half the backlash. Note that initial conditions correspond to a gap of: $h = C,\ \delta_{\max} = 0,\ \varphi_w = \varphi_p = 0$.

Flank friction is given as [Gohar and Rahnejat, 2008]:

$$F = A_e \left\{ \psi \tau_s + (1-\psi) \frac{\eta u}{h} \right\} \tag{12}$$

where ψ is the proportion of dry contact: $0 \le \psi \le 1$. This depends on the thickness of film *h*. The regime of lubrication is determined by the oil film parameter: $\lambda = \frac{h}{\sigma}$, where σ is the root mean square value of roughness of the contiguous solids (the meshing teeth surfaces). For typical gear flank surfaces: $\sigma \approx 0.4\ \mu\text{m}$. If, $\lambda \le 1$, then a boundary regime of lubrication results, for which it is safe to assume that: $\psi = 1$, and thus any viscous friction is ignored. On the contrary, if $\lambda \ge 3$, then fluid film lubrication is prevalent, and thus: $\psi = 0$. A procedure is required to establish the value of ψ, when $1 < \lambda < 3$, which is termed as mixed regime of lubrication. For this purpose the actual area of contact for asperity tip interactions without the presence of a film of lubricant is first considered [Gohar and Rahnejat, 2008]:

$$A_e \approx 3.2 \frac{W}{E^*} \sqrt{\frac{\gamma}{\sigma}} \tag{13}$$

where γ is the average asperity tip radius. Now the value of ψ is proportioned according to the ratio: $\frac{A_e}{A}$, where *A* is the apparent contact area, obtained through Hertzian theory as:

$$A = \pi \left(\frac{3W\sqrt{r_x r_y}}{4E^*} \right)^{2/3} \tag{14}$$

Thus:
$$\psi \approx \frac{A_e}{A} \tag{15}$$

As this ratio increases the contribution due to boundary friction becomes larger. When the ratio is unity, no viscous friction exists. u in equation (12) is the speed of entraining motion of the lubricant into the contact conjunction. This is the average velocity of the contacting surfaces. Thus:

$$u = \frac{v_p + v_w}{2} \tag{16}$$

where:

$$\left. \begin{aligned} v_p &= v_{pitch} \left(\sin \alpha_t + \frac{l_j}{R_{pc}} \right) \cos \beta \\ v_w &= v_{pitch} \left(\sin \alpha_t - \frac{l_j}{R_{wc}} \right) \cos \beta \end{aligned} \right\} \tag{17}$$

and: $l_j = R_{wc} \sin \alpha_t - \rho_w$

3 Results and Discussion

The conditions simulated correspond to the nominal engine speed of 830 rpm with a 4-cylinder 4-stroke diesel engine, and with the transmission engaged in the second gear with a ratio of 1.95 The simulated condition corresponds to part loading at low speed, a form of drive rattle termed creep rattle. The measured kinematics of the input shaft are dominated by 2nd engine order at 26 Hz and its multiples at 53 Hz and 79 Hz as expected for such an engine. With a gear ratio of 1.95 one would expect a steady state speed of 44.5 rad/s for the wheel and the transmission output shaft, with superimposed fluctuations. Figure 3 shows the simulation results for this case. The fluctuations, however, are not simply due to the previously mentioned engine orders, but also include 3-4 teeth pairs in simultaneous impact under different conditions, with a repetitive pattern corresponding to the meshing cycle.

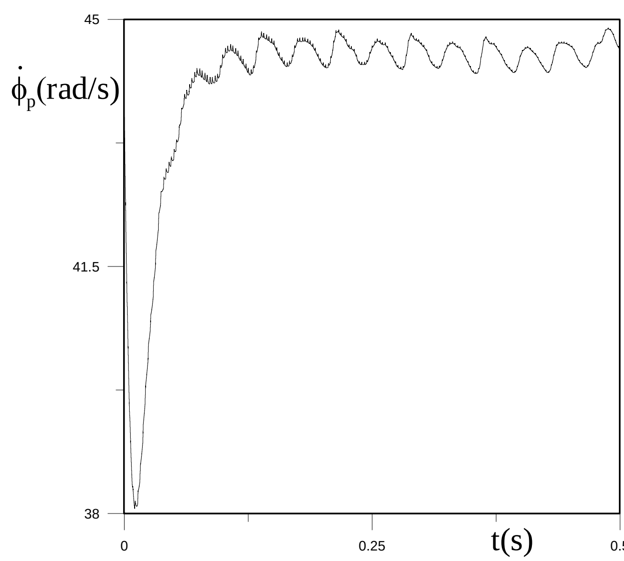

Figure 3: Time history of gear wheel velocity

The overall impact force transmitted onto the retaining structures is responsible for perceived vibration and noise, for which refinement is usually sought via palliation. Engine orders are function of combustion process and piston-connecting rod-crankshaft sub-system, and are a given input for transmission engineers. Therefore, refinement is directed to attenuation of impact-induced spectrum. Figure 4 shows the impact force variation, corresponding to a pair of teeth.

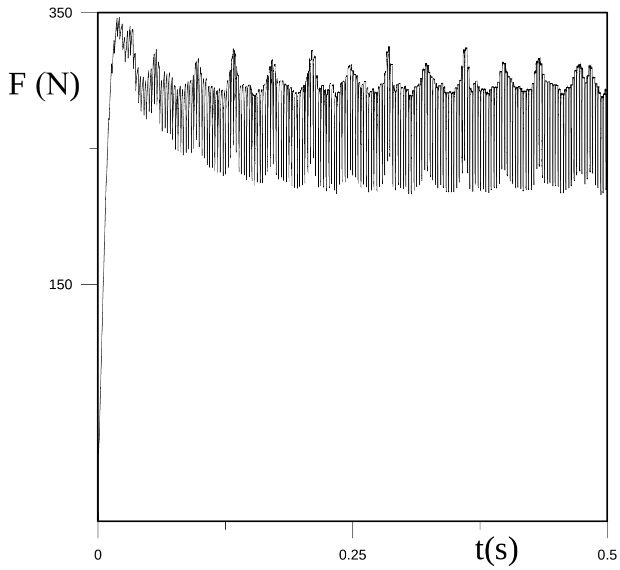

Figure 4: Time history of impact force within a tooth-pair

A complex variation in the impact force can be observed. This is expected as the phenomenon is highly non-linear and broad-band in nature. Note that the stiffness of the contact is given as:

$$k = \frac{\partial W}{\partial \delta} = 2\left(r_x r_y\right)^{1/4} E^* \sqrt{\delta} \propto v^{5/4}$$

Since the impact velocity changes due to fluctuations in gear speed (as shown in figure 3), then the stiffness of the contact alters through impact. This means that the response characteristics are transitory in nature, within each impact pair as well as for a number of simultaneous pairs. This can be seen by the power spectrum of impact force in figure 5.

The maximum deflection is typically of the order of 1μm, with the impact velocity being of the order of 4 mm/s. The dominant period of impact action corresponds to a frequency of 380 Hz, with a sub-harmonic response at 190 Hz. Other half and full order harmonics of the 380 Hz response also occur in the spectrum, which contains all the contributions up to 1500 Hz. The lower band of contributions is due to engine orders, dominated by the 2nd engine order. Thus, the spectrum shows the significant role of the impact velocity, rather than the extent of backlash, and explains the underlying reason for the relative success of dual mass flywheels in attenuation of rattle.

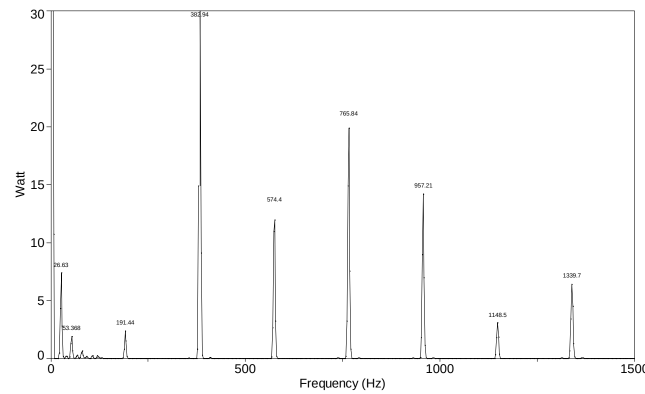

Figure 5: Typical spectral response of the impacting force of a gear teeth pair

Acknowledgements
The authors would like to express their gratitude to the EPSRC and Ford Motor Company for their financial support extended to this research project.

References:

Dogan, S.N., Ryborz, J. and Bertsche, B. "Low-noise automotive transmissions – investigation of rattling and clattering", in Rahnejat and Rothberg (eds.), Multi-body Dynamics: Monitoring & Simulation Techniques III, pp.323-338, Professional Engineering Publishing, IMechE, 2004

Theodossiades, S., Tangasawi, O. and Rahnejat, H. "Gear teeth impacts in hydrodynamic conjunctions promoting idle gear rattle", Journal of Sound and Vibration, Vol. 303, 2007, pp. 632-658.

Foellinger, H. "Advanced CAE simulations and prediction of drivetrain attributes", in Rahnejat and Rothberg (eds.), Multi-body Dynamics: Monitoring & Simulation Techniques III, pp.207-220, Professional Engineering Publishing, IMechE, 2004

Gnanakumarr, M., Theodossiades, S. and Rahnejat, H. "Tribo-contact dynamics phenomenon in torsional impact of loose Gears, promoting gear rattle", *Proc. SAE-IBEC & ATT Congress, Paris, Pap. No. 02-ATT-138, July 2002.*

Tangasawi, O., Theodossiades, S. and Rahnejat, H. "Lightly loaded lubricated impacts: idle gear rattle", Journal of Sound and Vibration, Vol. 308, 2007, pp. 418-430.

Gohar, R. Elastohydrodynamics, 2nd edition, Imperial College Press, London, 2001.

Mehdigoli, H., Rahnejat H. and Gohar R. "Vibration response of wavy surfaced disc in elastohydrodynamic rolling contact", Wear, Vol.139, 1990, pp.1-15.

Hertz, H. Miscellaneous papers by H. Hertz, Jones and Schort (eds.), Macmillan, London, 1896.

Gohar, R. and Rahnejat, H. Fundamentals of tribology, Imperial College Press, in print queue, expected 2008.