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HAMILTONIAN CONSTRUCTIONS IN SOLUTIONS OF OPTIMIZATION PROBLEMS IN NAVIGATION

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Abstract

Online dynamic reconstruction problems for controls of navigation systems are under consideration. It is assumed that information about real motions is known with errors. The solution of the inverse problems is suggested with the help of optimization problems for controlled systems. Key elements of the constructions are solutions of corresponding hamiltonian (characteristic) systems.

Key words

navigation, dynamic control reconstruction, hamiltonian systems

1 Introduction

Dynamic reconstruction problems for controls of navigation systems are under consideration. It is assumed that current measurements of the real motion are inaccurate, with known error estimates. We suggest to introduce the pay-off discrepancy functional and solve calculus of variation problems for the navigation systems minimizing the discrepancy with measurements. We consider solutions of the auxiliary calculus of variation problems as approximations of the solution of online control reconstruction problems for the navigation system.

There is a well-known approach solving these inverse problems that was proposed in the studies by Osipov and Kryazhimskii [Kryazhimskii, Osipov, 1983] and [Osipov, Kryazhimskii, 1995]. The proposed method involves a regularized procedure of extremal aiming at the dynamics of a guiding system similar to the navigation one. The construction uses the couple double state variable system. This approach appels to the optimal feedback theory developed in N.N. Krasovskii school [Krasovskii, 1968] and [Krasovskii, Subbotin, 1988].

We introduces and discuss the new method which is close to this approach. In contrast with it, we introduce auxiliary calculus of variation problems with a regularized integral discrepancy functional and the measured fixed initial state and speed. We apply necessary optimality conditions in the terms of hamiltonian system for state and conjugate variables of the navigation system. So, our construction of solution uses coupled state and conjugate variable system. A distinctive feature of the new method is that the negative discrepancies [Subbotina, Tokmantsev and Krupennicov, 2015] can be used.

Both above–mentioned approaches to solving inverse problems of the dynamics of control systems can be regarded as variants of Tikhonov's regularization method [Tikhonov, 1963].

In this paper, we present results of applications of the new method for solving of control reconstruction problems for landing on the Moon [Leitmann, 1962], [Letov, 1969], [Michel, 1977], [Liu et al., 2008].

2 Statement of the control reconstruction problem

We consider the following mathematical model of navigation [Letov, 1969].

We are watching the last stage before landing on the Moon. We assume that:

- the trajectory of the ship is a straight line orthogonal to the surface of the moon at the landing point;
- the Moon is stationary and it is flat in the neighborhood of the landing point;
- there are no aerodynamic forces;
- the ship is powered by gravity mg and traction force (braking) $T = cu$;
- here m is fuel mass, $g = 1,622\,\mathrm{m/sec}^2$ is moon acceleration of gravity, velocity of gas outflow from the nozzle $c = 500\,\mathrm{m/sec}$ is constant, fuel consumption (control) u is limited from above by the constant β.

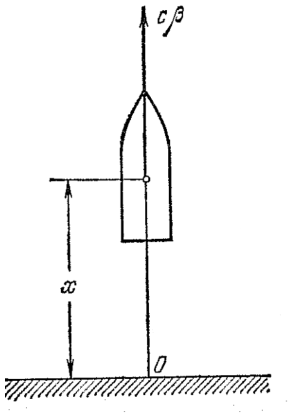

Figure 1. Landing Process on the Moon

So, dynamics of the ship is describe by the equation

$$\ddot{x} = \frac{cu}{m} - g, \qquad 0 \le u \le \beta, \tag{1}$$

or by the system

$$\begin{aligned} \frac{dx_1}{dt} &= x_2; \\ \frac{dx_2}{dt} &= \frac{cu}{x_3} - g; \\ \frac{dx_3}{dt} &= -u; \end{aligned}$$

with the restrictions on controls

$$0 \le u \le \beta. \tag{2}$$

We watch the real landing process $(x_1^*(t), x_2^*(t), x_3^*(t))$ and get online inaccurate state information $(y_1(t), y_2(t), y_3(t)) : \ \|y_1(t) - x_1^*(t)\| \le \delta$, $\|y_3(t) - x_3^*(t)\| \le \delta, \ \delta > 0$. Our aim is to reconstruct the control $u^*(t)$ generated the landing process.

3 Solution of the reconstruction problem

To solve the control reconstruction problem online we apply the new method similar to method suggested for solving control reconstruction problems a posteriori [5] where all incorrect information about real motion was known after the end of the observation.

3.1 Auxiliary calculus of variations problem

We introduce the following controlled system

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 + u_1; \\ \frac{dx_2}{dt} &= \frac{cu_3}{x_3} - g + u_2; \\ \frac{dx_3}{dt} &= -u_3; \end{aligned} \tag{3}$$

where (u_1, u_2, u_3) are controls, $t \in [0, T]$. We introduce the pay-off functional of the form $I(u(\cdot), x(\cdot)) =$

$$\begin{aligned} = \int_0^T &-\frac{(x_1(t) - y_1(t))^2}{2} - \frac{(x_2(t) - y_2(t))^2}{2} \\ &-\frac{(x_3(t) - y_3(t))^2}{2} + \frac{\alpha^2}{2} u_1(t)^2 + \\ &+\frac{\alpha^2}{2}[u_2(t)^2 + u_3(t)^2] dt. \end{aligned} \tag{4}$$

Here $\alpha > 0$ is a small regularizing parameter.

We consider the following calculus of variations problem: we need to minimize the functional (4) over the set of continuously differential functions $x(\cdot) = (x_1(\cdot), x_2(\cdot), x_3(\cdot)) \ : \ [0, T] \to R^3; \ \ u(\cdot) = (u_1(\cdot), u_2(\cdot), u_3(\cdot)) \ : \ [0, T] \to R^3$, which satisfy the fixed initial conditions

$$x_i(0) = y_i(0), \quad \dot{x_i}(0) = \dot{y_i}(0), \quad i = 1, 2, 3, \tag{5}$$

and satisfy the differential relations (3), and the control restrictions

$$0 \le u_3 \le \beta. \tag{6}$$

3.2 Solution of the calculus of variations problem

According to the necessary optimality conditions of the calculus of variations problem we get the following hamiltonian (characteristic) system

$$\begin{aligned} \frac{dx_1}{dt} &= x_2(t) + u_1^0(t); \\ \frac{dx_2}{dt} &= -g + \frac{c}{x_3(t)} u_3^0(t) + u_2^0(t); \\ \frac{dx_3}{dt} &= -u_3^0(t); \\ \frac{ds_1}{dt} &= x_1(t) - y_1(t); \\ \frac{ds_2}{dt} &= x_2(t) - y_2(t) - s_1(t); \\ \frac{ds_3}{dt} &= x_3(t) - y_3(t) + \frac{cs_2(t)}{(x_3(t))^2} u_3^0(t). \end{aligned} \tag{7}$$

Here

$$\begin{aligned} u_i^0(t) &= -\frac{s_i(t)}{\alpha^2}, \quad i = 1, 2; \\ u_3^0(t) &= \begin{cases} u_3[t], \text{ as } u_3[t] \in (0, \beta), \\ 0, \quad\ \ \text{as } u_3[t] \le 0, \\ \beta, \quad\ \ \text{as } u_3[t] \ge \beta, \end{cases} \end{aligned} \tag{8}$$

where

$$u_3[t] = -\frac{1}{\alpha^2}\Big[\frac{cs_2(t)}{x_3(t)} - s_3(t).\Big]$$

State variables $x_1(\cdot), x_2(\cdot), x_3(\cdot)$ of the solution of the system with initial conditions (5) are called state characteristics. Conjugate variables $s_1(\cdot), s_2(\cdot), s_3(\cdot)$ of the solution of the system with initial conditions (5) are called impulse characteristics.

So, the state characteristics and controls $u_1^0(\cdot), u_2^0(\cdot), u_3^0(\cdot)$ of the form (8) are the unique solution of the considering calculus of variations problem.

3.3 Solution of the reconstruction problem

We consider the control $u^0(t) = u_3^0(t)$ (8) as the approximation of the solution of online control reconstruction problem:

$$\frac{dx_1}{dt} = x_2;$$
$$\frac{dx_2}{dt} = \frac{cu}{x_3} - g;$$
$$\frac{dx_3}{dt} = -u^0(t);$$

$$x_i(0) = y_i(0), \quad i = 1, 2, 3.$$

4 Discussion

We have applied in construction $u^0(t)$ ideas and results of solving control reconstruction problems a posteriori [Subbotina, Tokmantsev and Krupennicov, 2015] where all incorrect information about real motion was known after the end of the observation. It was proven that the solutions of inverse problems with perturbed (inaccurate) sampling of trajectory converge in L_2 to the normal control, while the accuracy of sampling becomes more precise. The normal control is an admissible control that has the least norm in L_2 on the set of admissible controls generating the real motion. The key role here plays the sight minus before discrepancies

$$\frac{(x_1(t) - y_1(t))^2}{2} + \frac{(x_2(t) - y_2(t))^2}{2} + \frac{(x_3(t) - y_3(t))^2}{2}$$

in the pay-off functional (4). It imply that the matrix of linearized part of the characteristic system has imaginary eigenvalues. So, solutions of the system are oscillators. State characteristics are oscillating around measurements of the trajectories of the real system. Conjugate characteristics are oscillating around zero with amplitude α. We suggest a new numerical method to approximate the precise normal solution of the reconstruction control problem. To provide convergence of the controls (8) to the normal solution of the reconstruction control problem, a concordance condition for parameters α, β and the steps of numerical integration h is required [5].

5 Numerical example

There are results of numerical experiments in this section.

We watch the real landing process $(x_1^*(t), x_2^*(t), x_3^*(t))$ and get online inaccurate discrete state information $(y_1(t_j), y_2(t_j), y_3(t_j))$:

$$\|y_1(t_j) - x_1^*(t_j)\| \le \delta, \quad \|y_3(t_j) - x_3^*(t_j)\| \le \delta,$$

$t_0 = 0 < t_1 < t_2, \ldots, < t_N = T$, $\delta > 0$. We construct smooth continuous approximations $(y_1(t), y_2(t), y_3(t))$ of the measurements and apply the above presented method to get the approximation $u^0(t)$ of the reconstructing control $u^*(t)$ with a small delay.

We put $\alpha = 0.1$, $\beta = 50$,

$$u^*(t) = \begin{cases} 15 + t^2, 0 \le t < 5, \\ 0, 5 \le t \le 10. \end{cases}$$

Red lines on the pictures 2–5 are real control and trajectory, blue lines are the reconstructed control and the reconstructed trajectories.

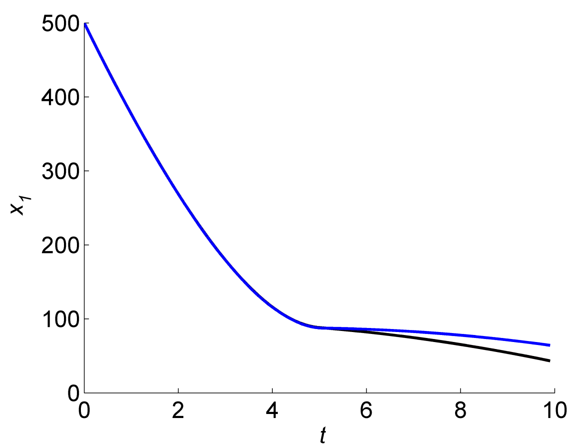

Figure 2. State information $y_1(\cdot)$ (red line) and reconstructed trajectory $x_1(\cdot)$ (blue line)

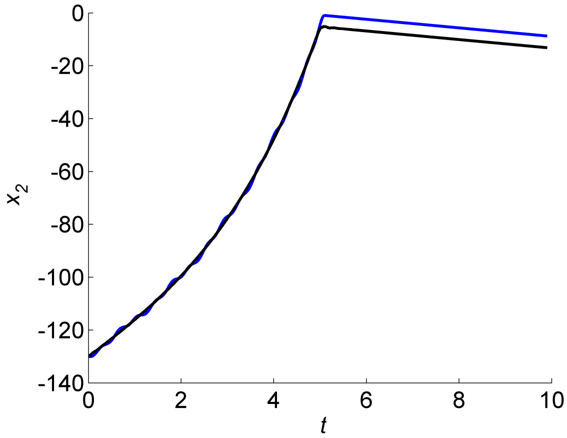

Figure 3. State information $y_2(\cdot)$ (red line) and reconstructed trajectory $x_2(\cdot)$ (blue line)

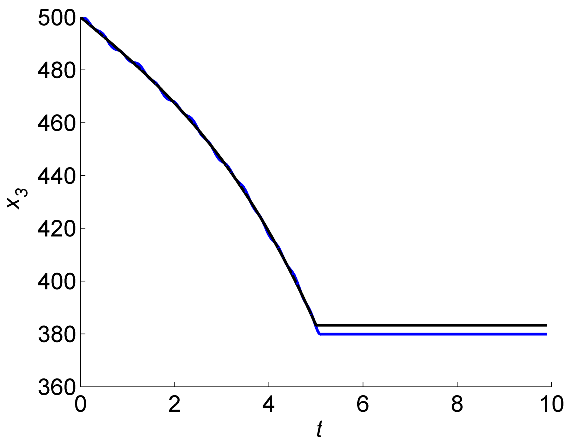

Figure 4. State information $y_3(\cdot)$ (red line) and reconstructed trajectory $x_3(\cdot)$ (blue line)

Further we put $\alpha = 0.01$, $\beta = 2$,

$$u^*(t) = \begin{cases} \beta\frac{(e^t-1)}{e^T-1}, & \text{as } t \in [0,5], \\ \beta, & \text{as } t \in (5,10]. \end{cases}$$

Red lines on the pictures 6–9 below are real control and trajectory, blue lines are the reconstructed control and the reconstructed trajectories.

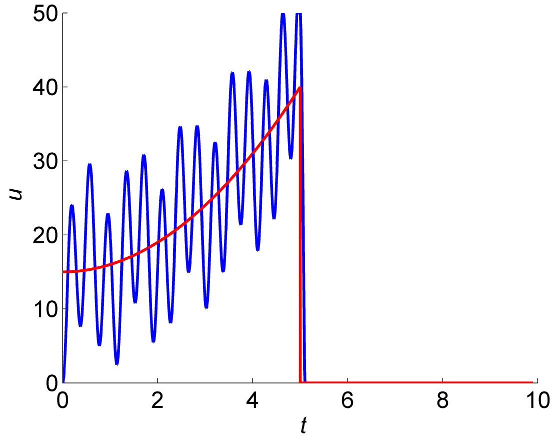

Figure 5. Unknown control $u_3^*(\cdot)$ (red line) and reconstructed control $u^0(\cdot)$ (blue line)

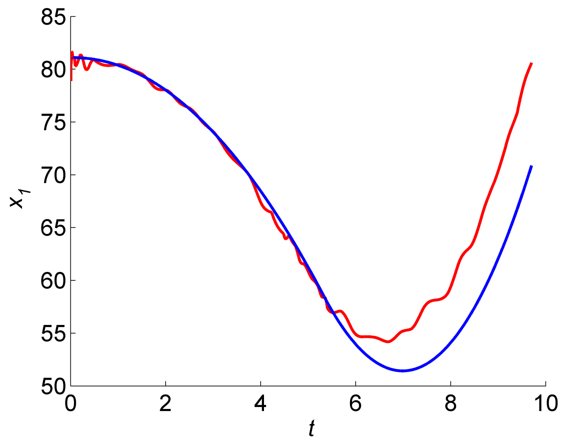

Figure 6. State information $y_1(\cdot)$ (red line) and reconstructed trajectory $x_1(\cdot)$ (blue line)

6 Conclusion

In the paper dynamic reconstruction problems for controls of navigation systems are considered in assumption that online information about real motions is known with errors. A new method for solving the inverse problem is suggested on the base of solutions of auxiliary calculus of variations problems. A corresponding numerical method is created. Results of simulations are exposed. The effective method will be developed for the navigation deterministic systems of general form and greater dimensions in the future papers.

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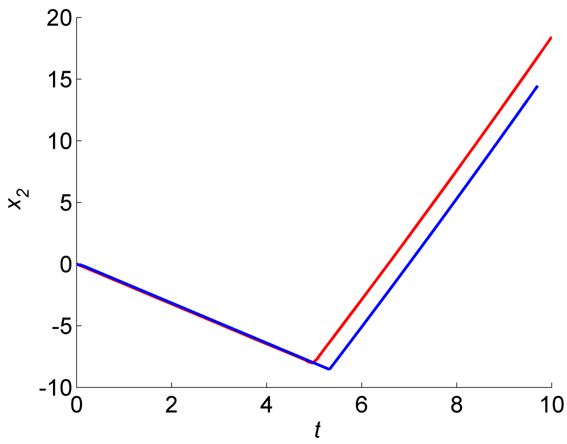

Figure 7. State information $y_2(\cdot)$ (red line) and reconstructed trajectory $x_2(\cdot)$ (blue line)

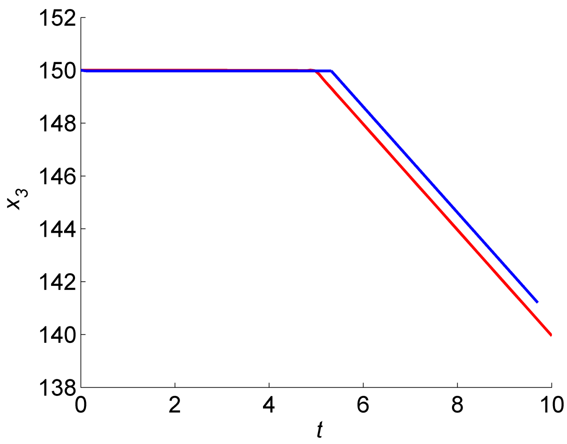

Figure 8. State information $y_3(\cdot)$ (red line) and reconstructed trajectory $x_3(\cdot)$ (blue line)

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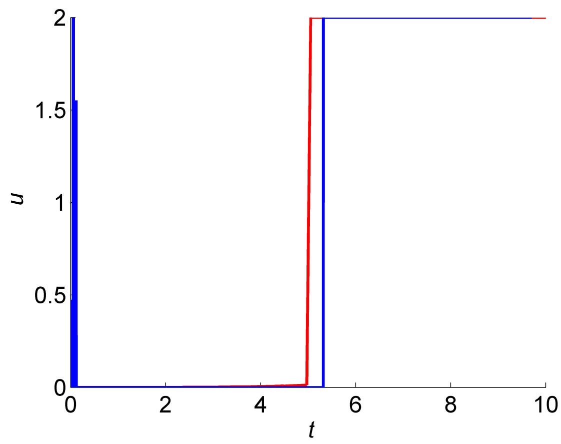

Figure 9. Unknown control $u_3^*(\cdot)$ (red line) and reconstructed control $u^0(\cdot)$ (blue line)

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