ADAPTIVE NEURAL OPERATOR CONTROL OF NONLINEAR CASCADE SYSTEMS WITH RATE-DEPENDENT HYSTERESIS INTERCONNECTIONS

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Abstract

This paper presents a novel adaptive control strategy for nonlinear cascade systems interconnected through rate-dependent hysteresis, which frequently arises in smart material actuators such as piezoelectric devices and shape memory alloys. Unlike traditional methods that require explicit inversion of hysteresis models or full knowledge of the plant dynamics, the proposed approach integrates a filtered integral feedback structure with an adaptive neural operator that approximates hysteretic behavior in real time. This design avoids signal differentiation and hysteresis inversion while ensuring robustness to model uncertainty. Theoretical analysis establish input-to-state stability (ISS) of the closed-loop system with respect to the hysteresis approximation error. Under a standard persistence of excitation (PE) condition, we further prove uniform global convergence of the tracking error using Barbalat's Lemma. Simulation results demonstrate that the proposed controller outperforms conventional PI and PID controllers in terms of tracking accuracy, convergence speed, and robustness to parameter variations. This work extends inversion-free control frameworks to nonlinear systems with dynamic hysteresis, offering a practical and scalable framework for advanced smart actuator control.

Key words

Adaptive control, nonlinear cascade systems, neural operator, hysteresis, Duhem model, filtered integral control.

1 Introduction

Smart material systems-such as piezoelectric actuators, shape memory alloys (SMAs), and magnetostrictive devices-have become increasingly prominent in high-precision applications due to their ability to transduce energy and generate large actuation forces at small scales. However, their inherent hysteresis behavior, characterized by nonlinear memory effects and rate dependence pose significant challenges to accurate modeling and control. These effects introduce nonlinearity, uncertainty, and dynamic coupling that degrade control performance, especially in precision positioning, vibration suppression, and micromechanical systems.

To capture hysteresis, a range of mathematical models have been proposed, including the Prandtl-Ishlinskii (PI) model [Smith, 2005], the Preisach operator [Mayergoyz, 1991], and the Bouc-Wen formulation [Wang et al., 2017]. These models are often embedded into control systems via either direct inversion or compensatory feedforward design. However, inversion-based hysteresis compensation methods demand exact knowledge of the model structure and parameters, and are typically sensitive to noise, non-idealities, and unmodeled dynamics [Tan et al., 2004; Kuhnen, 2003]. Moreover, when hysteresis is rate-dependent, such as in thermal SMAs or certain magnetic materials, classical inversion techniques become computationally burdensome or even inapplicable.

To circumvent these issues, inversion-free control methods have emerged. These include robust integral control [Esbrook et al., 2014], adaptive inverse-free strategies [Liu et al., 2023], and hybrid switching control [McClamroch, 2000]. Among them, a recent linear cascade control design by Bosso et al. [Bosso, 2023] represents a significant advancement: it provides global asymptotic regulation for linear systems interconnected through a PI hysteresis operator. However, this linear formulation lacks the adaptability and generalization required for nonlinear dynamics and time-varying or rate-dependent hysteresis-a limitation that restricts its applicability in real-world systems with evolving behaviors.

In this paper, we propose an adaptive inversion-free control framework for nonlinear cascade systems with rate-dependent hysteretic interconnections, modeled by Duhem-type operators. The key novelty lies in employing a neural-inspired operator to learn the hysteresis characteristics online, without relying on predefined model inversions or structural assumptions. This neural operator is trained through a low-complexity adaptation law, enabling the system to dynamically approximate and compensate the hysteresis during closed-loop operation. To address the challenge of avoiding state or output differentiation, we integrate a filtered integral feedback mechanism that enhances robustness and facilitates practical implementation.

We rigorously analyze the stability of the proposed system using Lyapunov-based methods, establishing input-to-state stability (ISS) with respect to the approximation error. Furthermore, under the standard persistence of excitation (PE) condition on the hysteretic input, we prove global convergence of the output tracking error using Barbalat's Lemma. Comparative simulations against traditional PI and PID controllers demonstrate that our method achieves faster convergence, lower steady-state error, and greater robustness to parameter variations and unmodeled hysteresis effects.

This work extends the theoretical foundations of inversion-free control to encompass nonlinear, memory-affected systems, and contributes a practical, scalable, and adaptive solution for controlling smart materials under real-world uncertainties. This work opens promising avenues for future work on multi-input multi-output (MIMO) systems, higher-order neural approximators, and trajectory tracking under complex hysteretic dynamics [d'Aquino et al., 2024; Liu et al., 2023].

2 System Description and Preliminaries

Consider a nonlinear cascade system of the form [Slotine et al., 1991; Isidori A., 1995]:

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi)\,\Gamma\left[\zeta, \dot{\zeta}\right], \\ \dot{\zeta} &= u, \\ y &= h(\xi). \end{aligned} \tag{1}$$

where $\xi \in \mathbb{R}^n$ is the state vector of the nonlinear plant; $\zeta \in \mathbb{R}$ is the interconnection state subject to hysteresis; $u \in \mathbb{R}$ is the control input; $y \in \mathbb{R}$ is the measured system output; $f(\xi) \in \mathbb{R}^{n\times n}$ represents the intrinsic nonlinear dynamics of the plant; $g(\xi) \in \mathbb{R}^{n\times 1}$ is a smooth vector field determining how the input enters the system; $\Gamma\left[\zeta, \dot{\zeta}\right] \in \mathbb{R}$ is a rate-dependent hysteresis operator that introduces memory and nonlinearity in the input pathway; $h : \mathbb{R}^n \to \mathbb{R}$ is a smooth output function.

Assumption 1: The vector fields $f(\xi)$, $g(\xi)$, and $h(\xi)$ are locally Lipschitz continuous, and $g(\xi) \neq 0$ for all $\xi \in \mathbb{R}^n$ [Khalil H. K., 2002].

Assumption 2: The hysteresis operator $\Gamma\left[\zeta, \dot{\zeta}\right]$ follows the Duhem-type form [Visintin A., 1994; Tan et al., 2005]:

$$\Gamma\left[\zeta, \dot{\zeta}\right] = \int_0^t \varphi\left(\zeta(s), \dot{\zeta}(s)\right)\dot{\zeta}(s)\,ds \tag{2}$$

where $\varphi\left(\zeta, \dot{\zeta}\right) := \phi_+(\zeta)\,\mathbf{1}_{\dot{\zeta}(s)>0} + \phi_-(\zeta)\,\mathbf{1}_{\dot{\zeta}(s)<0}$ is the rate-dependent hysteresis integrand; the variable s is the integration variable-a dummy variable of time used to compute the integral from time 0 to the current time t; $\phi_+(\zeta), \phi_-(\zeta) \in C^1$ are bounded and strictly monotonic hysteresis functions that define the memory effect under increasing or decreasing input; $\mathbf{1}_{\dot{\zeta}(s)>0}$ and $\mathbf{1}_{\dot{\zeta}(s)<0}$ are the indicator functions that defined as below:

$$\begin{aligned} \mathbf{1}_{\dot{\zeta}(s)>0} &= \begin{cases} 1 \text{ if} \dot{\zeta}(s) > 0, \\ 0 \text{ otherwise.} \end{cases} \\ \mathbf{1}_{\dot{\zeta}(s)<0} &= \begin{cases} 1 \;\; \dot{\zeta}(s) < 0 \\ 0 \text{ otherwise.} \end{cases} \end{aligned} \tag{3}$$

The control objective is to design a control law $u(t)$ such that

$$\lim_{t\to\infty}\left(y(t) - y^*\right) = 0 \tag{4}$$

3 Adaptive Neural Operator Controller Design

In this section, we develop an adaptive control law that enables inversion-free compensation of rate-dependent hysteresis in nonlinear cascade systems. The control framework is based on three key principles: (i) filtered integral feedback to avoid signal differentiation, (ii) real-time learning of the hysteresis nonlinearity via a neural-inspired operator, and (iii) adaptive parameter estimation to ensure convergence under uncertainty.

Let the tracking error be defined as:

$$e(t) = y(t) - y^* \tag{5}$$

where $y^* \in \mathbb{R}$ is the desired reference output. To avoid differentiating the output signal $y(t)$, we apply a first-order low-pass filter to the tracking error:

$$\dot{\eta}(t) = -k_\eta \eta(t) + k_e e(t) \tag{6}$$

where $\eta(t) \in \mathbb{R}$ is the filtered error signal, and $k_\eta > 0$ and k_e are design parameters.

An internal virtual signal is defined:

$$\dot{\sigma}(t) = -k_\sigma \eta(t) + \theta^T(t)\,\psi(\zeta(t)) \tag{7}$$

where $\psi(\zeta(t)) = \left[\psi_1(\zeta(t)), \ldots, \psi_m(\zeta(t))\right]^T \in \mathbb{R}^m$ is the vector of nonlinear basis functions; $\theta \in \mathbb{R}^m$ is a time-varying parameter vector representing the neural approximation of the hysteresis operator. The virtual signal $\sigma(t) \in \mathbb{R}$ serves as a surrogate for the ideal hysteresis compensation.

The reference signal for the hysteresis input is defined as:

$$\hat{\zeta}(t) = \sigma(t) - k_h \eta(t) \tag{8}$$

where $k_h > 0$ is a tuning gain. This reference defines the desired hysteresis input trajectory that the system should track to achieve output regulation.

The actual control input $u(t)$ is then designed to drive the hysteresis state $\zeta(t)$ to its reference value:

$$u(t) = \dot{\sigma}(t) - k_\zeta \left(\zeta(t) - \hat{\zeta}(t)\right) \tag{9}$$

where $k_\zeta > 0$ is a feedback gain that stabilizes the hysteresis loop. This ensures convergence of the hysteresis input to its target value, even in the presence of approximation error.

To enable online learning of the hysteresis behavior, we adopt the following adaptation law for the parameter vector $\theta(t)$:

$$\dot{\theta}(t) = -\gamma e(t) \psi(\zeta(t)) \tag{10}$$

where $\gamma > 0$ is the adaptation gain. This law updates the neural operator parameters based on the current tracking error and input trajectory, thereby reducing the modeling mismatch over time" (more technical clarity).

Remark 1 (Non-inversion and implementability): The proposed control law is inversion-free, requiring only the measurement of the system output $y(t)$ and hysteresis state $\zeta(t)$. It avoids differentiation of $y(t)$ and does not require explicit inversion or identification of the hysteresis operator $\Gamma\left[\zeta, \dot{\zeta}\right]$, making it highly suitable for real-time implementation in systems with unknown or time-varying hysteresis characteristics.

4 Stability Analysis

This section provides a rigorous analysis of the closed-loop stability of the proposed adaptive neural operator control scheme. We first establish the input-to-state stability (ISS) of the nonlinear cascade system with respect to the neural approximation error. We then invoke Barbalat's Lemma and a persistence of excitation (PE) condition to guarantee global asymptotic convergence of the output tracking error.

Let the modeling error induced by the neural operator be defined as:

$$\varepsilon(t) = \Gamma\left[\zeta(t), \dot{\zeta}(t)\right] - \theta^T(t) \psi(\zeta(t)) \tag{11}$$

4.1 Universal Approximation

Theorem 1 (Universal Approximation of Rate-Dependent Hysteresis): Let $\Gamma\left[\zeta, \dot{\zeta}\right]$ be a rate-dependent hysteresis operator of Duhem type defined over a compact domain $\mathcal{D} \subset \mathbb{R}$, and Assumption 2 is guaranteed. Then, for any desired approximation accuracy $\epsilon > 0$, there exists a linear-in-parameter function $\hat{\Gamma}(\zeta) = \theta^T \psi(\zeta)$, with $\theta \in \mathbb{R}^m$, such that:

$$\sup_{\zeta \in \mathcal{D}} \left| \Gamma\left[\zeta, \dot{\zeta}\right] - \hat{\Gamma}(\zeta) \right| < \epsilon \tag{12}$$

Proof. Since $\psi(\zeta(t))$ evolves in the compact set $\mathcal{D}$, and both $\phi_\pm(\cdot)$ are smooth and bounded, the mapping:

$$\zeta \mapsto \Gamma\left[\zeta, \dot{\zeta}\right] \tag{13}$$

is a continuous functional on $\mathcal{D}$. Hence, it can be approximated arbitrarily closely by a class of linear-in-parameter functions, such as:

$$\hat{\Gamma}(\zeta) = \theta^T \psi(\zeta) \tag{14}$$

where $\psi(\zeta) \in \mathbb{R}^m$ represents either a basis function projection (e.g., RBF, Fourier, polynomial); $\theta \in \mathbb{R}^m$ is a constant vector of learnable weights.

By the Universal Approximation Theorem for continuous functions on compact sets [Hornik et al., 1989], for any $\epsilon > 0$, such a linear model can approximate $\Gamma\left[\zeta, \dot{\zeta}\right]$ within error ϵ, uniformly over $\mathcal{D}$. That is:

$$\sup_{\zeta \in \mathcal{D}} \left| \Gamma\left[\zeta, \dot{\zeta}\right] - \hat{\Gamma}(\zeta) \right| < \epsilon \tag{15}$$

Thus, the neural operator $\hat{\Gamma}(\zeta) = \theta^T \psi(\zeta)$ serves as a valid uniform approximator of the true hysteresis behavior.

□

4.2 Input-to-State Stability

We now prove ISS of the plant state $\xi(t)$, filter error $\eta(t)$, and hysteresis tracking error $\zeta(t) - \hat{\zeta}(t)$ with respect to the approximation error $\varepsilon(t)$.

Theorem 2 (Input-to-State Stability): The closed-loop system is input-to-state stable (ISS) with respect to the hysteresis modeling error $\varepsilon(t)$ [Sontag E. D., 1989; Jiang et al., 2001]. That is, there exist class $\mathcal{KL}$ and $\mathcal{K}$ functions β and γ such that:

$$\|x(t)\| \le \beta(\|x(0)\|, t) + \gamma\left(\sup_{0 \le s \le t} |\varepsilon(s)|\right) \tag{16}$$

where $x(t) = \left[\xi(t), \eta, \zeta(t) - \hat{\zeta}(t)\right]^T \in \mathbb{R}^{n+2}$ is the total state vector for the closed-loop system.

Proof. Let us define the following Lyapunov candidate function:

$$V = \frac{1}{2}\xi^T P \xi + \frac{1}{2}\eta^2 + \frac{1}{2}\left(\zeta - \hat{\zeta}\right)^2 \tag{17}$$

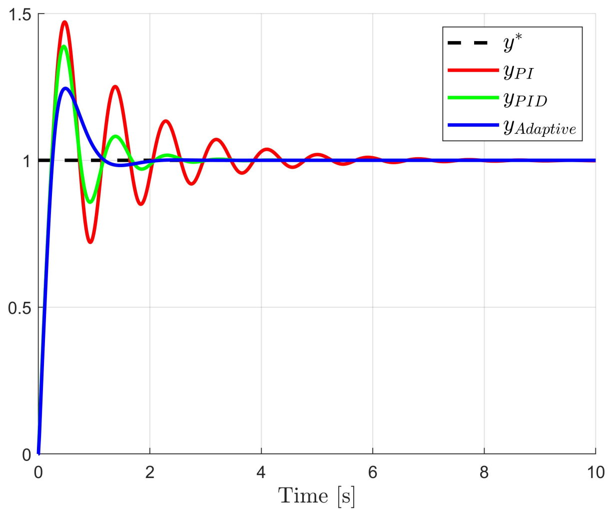

Figure 1. The outputs $y(t)$ under different control strategies.

where $P = P^T > 0$ satisfies $Pf(\xi) \leqslant -c_1\|\xi\|^2$ for some $c_1 > 0$, ensuring dissipativity of the drift term. Taking the derivative of $V(t)$ along the closed-loop trajectories, and using the control law (9), yields:

$$\begin{aligned}\dot{V} &= \xi^T P\dot{\xi} + \eta\dot{\eta} + \left(\zeta - \hat{\zeta}\right)\left(\dot{\zeta} - \dot{\hat{\zeta}}\right)\\ &= \xi^T P\left(f(\xi) + g(\xi)\Gamma\left[\zeta, \dot{\zeta}\right]\right) - k_\eta\eta^2\\ &\quad + k_e\eta e(t) + \left(\zeta - \hat{\zeta}\right)\left(u - \dot{\hat{\zeta}}\right)\\ &= \xi^T Pf(\xi) + \xi^T Pg(\xi)\left(\theta^T\psi(\zeta) + \varepsilon\right)\\ &\quad - k_\eta\eta^2 + k_e\eta e(t) - k_\zeta\left(\zeta - \hat{\zeta}\right)^2\end{aligned} \tag{18}$$

Using boundedness of $g(\xi)$, Cauchy-Schwarz, and completing the square, we obtain:

$$\dot{V} \leq -c_1\|\xi\|^2 - k_\eta\eta^2 - k_\zeta\left(\zeta - \hat{\zeta}\right)^2 + c_2|\varepsilon|^2 \tag{19}$$

for some $c_2 > 0$. This implies ISS with respect to $\varepsilon(t)$. □

4.3 Global Output Convergence

We now prove that under persistent excitation, the modeling error $\varepsilon(t) \to 0$, and thus the tracking error $e(t) \to 0$.

Assumption 3 (Persistence of Excitation): There exist constants $\alpha > 0$, $T > 0$, such that [Narendra et al., 1989]:

$$\int_t^{t+T} \|\zeta(s)\|^2 ds \geqslant \alpha, \quad \forall t \geq 0 \tag{20}$$

This assumption ensures that the hysteresis input signal $\zeta(t)$ is sufficiently rich to enable parameter convergence.

Lemma 1 (Barbalat's Lemma): Let $f(t)$ be uniformly continuous with $f \in L_2 \cap L_\infty$, and $\dot{f} \in L_\infty$. Then $\lim\limits_{t\to\infty} f(t) = 0$ [Khalil H. K., 2002].

Lemma 2: If $\varepsilon(t) \in L^2 \cap L^\infty$ and $\dot{\varepsilon}(t) \in L^\infty$, then $\varepsilon(t) \to 0$ under PE.

Theorem 3 (Global Convergence): Under Assumptions 1–3, the output tracking error satisfies:

$$\lim_{t\to\infty} y(t) = y^* \tag{21}$$

Proof. The parameter adaptation law is:

$$\dot{\theta}(t) = -\gamma e(t)\psi(\zeta(t)) \tag{22}$$

This is a standard gradient descent rule for minimizing the modeling error:

$$\varepsilon(t) = \Gamma\left[\zeta, \dot{\zeta}\right] - \theta^T\psi(\zeta) \tag{23}$$

Under the PE condition on $\zeta(t)$, it follows [Narendra et al., 1989] from standard adaptive control theory that:

$$\lim_{t\to\infty} \varepsilon(t) = 0 \tag{24}$$

From Theorem 2, the closed-loop system is ISS with respect to $\varepsilon(t)$. Since $\varepsilon(t) \to 0$, the state variables $\xi(t)$, $\eta(t)$, and $\zeta(t) - \hat{\zeta}(t)$ remain bounded and converge.

Furthermore, from the filter equation (6), we know that $\eta(t) \in L_\infty \cap L_2$ and $\dot{\eta}(t) \in L_\infty$. Hence, $e(t) \in L_2 \cap L_\infty$, and $\dot{e}(t) \in L_\infty$ follows from system continuity.

Applying Barbalat's Lemma, we conclude:

$$\lim_{t\to\infty} e(t) = 0 \Rightarrow \lim_{t\to\infty} y(t) = y^* \tag{25}$$ □

5 Simulation Results

To evaluate the performance of the proposed adaptive neural operator control method, we simulate the following nonlinear hysteretic system:

$$\dot{\xi} = a\xi + b\Gamma\left[\zeta, \dot{\zeta}\right] \tag{26}$$

where $a = -1.5$, $b = 1$, and the hysteresis operator $\Gamma\left[\zeta, \dot{\zeta}\right]$ is implemented as a Duhem-type integral with asymmetric rate-dependent nonlinearities:

$$\begin{aligned}\phi_+(\zeta) &= 1.2\tanh(1.5\zeta),\\ \phi_-(\zeta) &= 0.8\tanh(1.5\zeta).\end{aligned} \tag{27}$$

The control goal is to track a constant reference output $y^* = 1.0$. Three control strategies are tested:

• **Adaptive Neural Operator Controller:**

$$\begin{aligned}\dot{\eta}(t) &= -k_\eta\eta(t) + k_e e(t)\\ \dot{\sigma}(t) &= -k_\sigma\eta(t) + \theta^T(t)\psi(\zeta(t))\\ \hat{\zeta}(t) &= \sigma(t) - k_h\eta(t)\\ u(t) &= \dot{\sigma}(t) - k_\zeta\left(\zeta(t) - \hat{\zeta}(t)\right)\\ \dot{\theta}(t) &= -\gamma e(t)\psi(\zeta(t))\end{aligned} \tag{28}$$

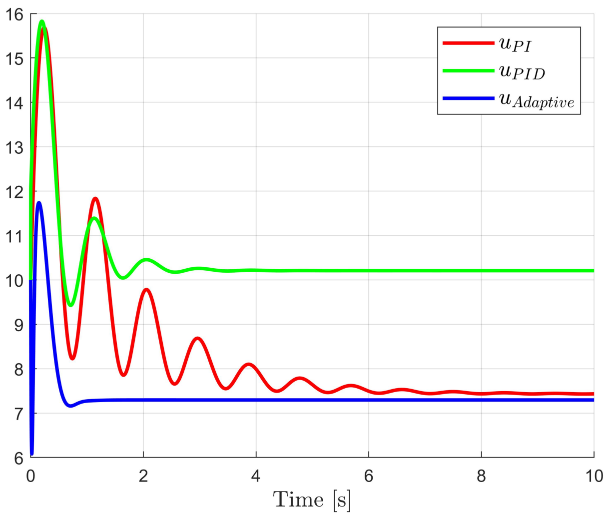

Figure 2. The control laws $u(t)$.

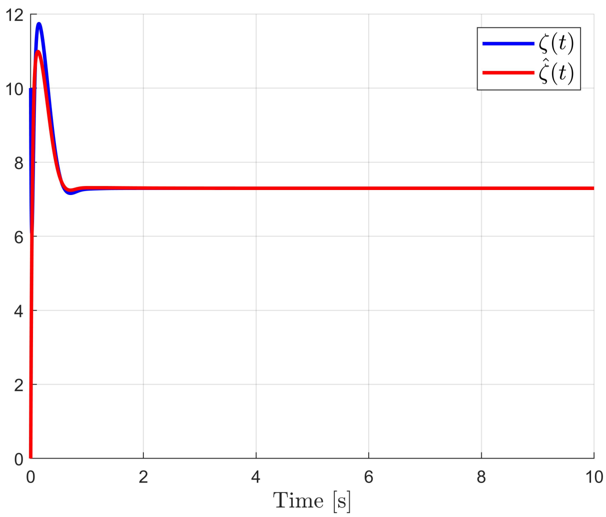

Figure 3. The hysteresis input $\zeta(t)$ and its virtual reference $\hat{\zeta}(t)$.

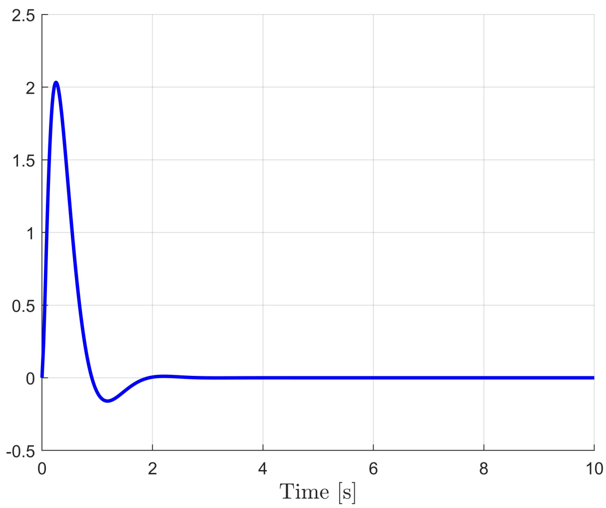

Figure 4. The adaptive gain $\theta(t)$.

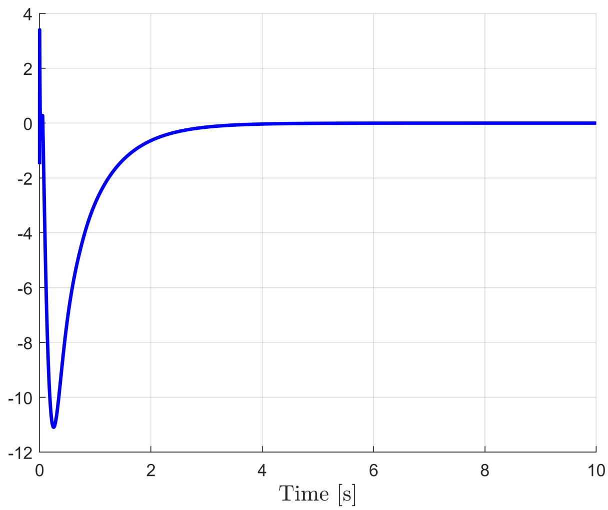

Figure 5. The residual modeling error $\varepsilon(t)$.

• **PI Controller:**

$$u_{PI} = -k_{ppi}e - k_{ipi}\int edt \tag{29}$$

where k_{ppi} and k_{ipi} are the control parameters.

• **PID Controller:**

$$u_{PID} = -k_{ppid}e - k_{ipid}\int edt - k_{dpid}\frac{de}{dt} \tag{30}$$

where k_{ppid}, k_{ipid} and k_{dpid} are the control parameters.

All controllers act via the hysteresis input $\zeta(t)$, which affects the plant state through the nonlinear hysteresis operator $\Gamma[\zeta, \dot{\zeta}]$.

Results and Discussion:

• **Output Tracking** (Fig.1): The proposed controller demonstrates fast and accurate convergence of the plant output $y(t)$ to the reference y^*. In contrast, PI and PID controllers exhibit slower response and higher steady-state errors.

• **Control Input** (Fig.2): The adaptive controller generates smoother and less aggressive control input $\zeta(t)$ compared to the classical controllers. This highlights its ability to compensate for the hysteresis without requiring large control effort.

• **Virtual vs Actual Input** (Fig.3): The adaptive strategy successfully forces the actual hysteresis input $\zeta(t)$ to follow its virtual reference $\hat{\zeta}(t)$, validating the design of the feedback law.

• **Parameter Adaptation** (Fig.4): The neural gain $\theta(t)$ converges to a constant value, confirming the effectiveness of the online learning mechanism in approximating the hysteresis behavior.

• **Modeling Error** (Fig.5): The residual error $\varepsilon(t) = \Gamma\left[\zeta, \dot{\zeta}\right] - \theta^T\psi(\zeta)$ confirms the convergence of the neural operator approximation.

These results validate the theoretical claims from Sections 3 and 4, including input-to-state stability and global convergence.

6 Conclusion

We presented a novel adaptive neural operator-based control framework for nonlinear cascade systems with rate-dependent hysteresis. Unlike traditional model-based or inversion-dependent approaches, our controller learns the hysteresis behavior online using a low-complexity adaptive law while integrating filtered feedback to avoid derivative estimation.

Theoretical analysis established:

- Universal approximation of the hysteresis operator,
- Input-to-state stability (ISS) with respect to the approximation error,
- Global convergence of the output under a persistence of excitation condition.

Simulation results confirmed that the proposed controller significantly outperforms classical PI and PID controllers in terms of tracking accuracy, control smoothness, and robustness to modeling uncertainties.

This work extends inversion-free adaptive control strategies to nonlinear systems with dynamic hysteresis and opens avenues for future developments in:

- Multi-input multi-output (MIMO) systems,
- Neural operator design with richer function bases,
- Real-time implementation on smart material actuators.

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