CYBERNETICS AND PHYSICS, VOL. 3, NO. 3, 2014 , 147–150

SIMULTANEOUS PARAMETRIC OPTIMIZATION OF PLASMA CONTROLLERS FOR VERTICAL POSITION AND SHAPE

Sergey Zavadskiy
Department of Applied Mathematics and Control Processes
St. Petersburg State University
Russia
sergzav@pochtamt.ru

Anastasiya Kiktenko
Department of Applied Mathematics and Control Processes
St. Petersburg State University
Russia
anakiktenko_89@mail.ru

Abstract
For simultaneous optimization of the plasma shape and vertical position controllers are proposed mathematical model of the structural parameter optimization of the plasma dynamics.Dynamic optimization approach to plasma based on an analysis of the trajectory of the ensemble. This ensemble describes the transition process in a tokamak, expose the raw data and external disturbances. The structure of this approach is given to optimize the dynamics of an ensemble of trajectories in tokamak ITER. The trajectory of the ensemble alarmed in the initial set of points and a set of external disturbances.Earlier, on an example in work [Zavadsky, Ovsyannikov, and Chung, 2009] one regulator for optimization was used. This work is devoted to optimization of two regulators.

Key words
Simultaneous optimization, plasma control, tokamak, ITER, transient process optimization, plasma shape controller.

1 Introduction

Problem analysis and synthesis of stabilizing controller current position and shape of the plasma in a tokamak is very important. Linear systems are widely used in problems of designing control systems for complex objects. Synthesis regulator that stabilizes the plasma in a tokamak form, is made based on the linearization of the distinguishing equations that determine the behavior of the plasma. The procedure for the reduction and LQG-analytical algorithms are applied to the construction of the controller LTI-object, see [McArdle *et al.*, 1998]- [Ovsyannikov, Veremey, and Zhabko, 2006], [Misenov, Ovsyannikov, and Ovsyannikov, 1997] for more information. Object of the control in the state space can be described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + Bu + Gf(t), \\ e &= Lx + Mu, \\ y &= Cx + Du + Ff(t), \end{aligned} \tag{1}$$

where $x \in E^n$ is the state space vector,$u \in E^r$ is the control voltages vector, $y \in E^d$ is the diagnostic signals vector, $e \in E^d$ is the measurement variables vector, all the matrices in the model are known, with constant components, $f(t)$ is the vector function of external disturbances, which is called $l_i, \beta-$*drops* disturbance and is defined in the following form

$$\begin{gathered} f(t) = f_{drop}(t) = \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}, \\ w_1(t) = d_\beta e^{-(t/t_\beta)}, \quad w_2(t) = d_l e^{-(t/t_l)}, \end{gathered} \tag{2}$$

where $d_\beta, d_l, t_\beta, t_l$ are known real numbers. The control object is closed with a set of controllers of a decreased dimension with the LTI-object structure by vectors y and u are the output and input of the object (1) respectively, matrices A_c, B_c, C_c are the constant component matrices of the controller, which can be obtained, for example, using the reduction procedure and the LQG-optimal synthesis [Zavadsky, Ovsyannikov, and Chung, 2009].

By “Synthesis of controllers” we mean a choice of controller parameters component LTI, which gives us a closed object, which is stable. On the trajectory of the closed system, we define a quadratic functional quality and consider the problem of minimizing it.

In addition, we must take into account the nonlinear constraints on the amplitude of the voltage signals control. For example, there are 11 coils control with constraints on the amplitude of the voltage in tokamak

ITER. These constraints have various numerical values u_i where i is index of coil. The numerical value of values u_i depends on the c_i(amplitude maximum for i-th coil). This complicates the design of the controller and allows not only the rise of the value of the control voltage signals.

Note that the controller design model analyzed by linear, but checked at the controller and a nonlinear model must possess the appropriate characteristics [Zavadskiy, 2014; Zavadskiy, 2014(2); Zavadskiy, 2007].

2 Vertical and Shape Simultaneous Optimization of Trajectories Ensemble

Let us describe the mathematical model of ITER plasma control system. Matrices A_c, B_c, C_c are used for describing dynamic shape controller. And matrices A_ν, B_ν, C_ν are used for describing of the vertical position controller, see Fig. 1.

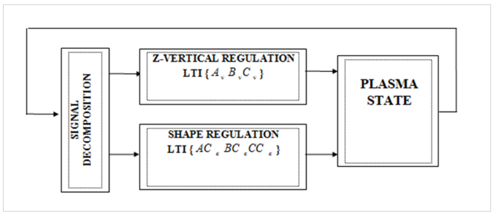

Figure 1. Simultaneous optimization for vertical position and shape.

The elements of these matrices will be taken as parameters that are to be optimized and combined into a vector of parameters

$$p = \{p_k\} \longleftrightarrow \{A_c, B_c, C_c, A_\nu, B_\nu, C_\nu\}.$$

A mathematical model of the structural dynamics of parametric optimization of a number of paths that alarmed in the initial set of points and a set of external disturbances. The structure of this approach is the optimization of transients full-size control object that is closed regulator of reduced dimension.

$$\dot{x} = \begin{pmatrix} A_p & 0 & 0 & 0 & B_pC_p \\ W_{wp}C_p & A_{wp} & B_{wp}C_v & 0 & 0 \\ 0 & B_vC_{wp} & A_v & 0 & 0 \\ 0 & B_fC_{wp} & 0 & A_f & 0 \\ 0 & 0 & 0 & B_cC_f & A_c \end{pmatrix} x,$$
$$y = \begin{pmatrix} 0 \; C_{wp} \; 0 \; 0 \; 0 \end{pmatrix} x. \quad (3)$$

Where matrixes of the structure are defined in [Zavadsky, Ovsyannikov, and Chung, 2009]. Proposed to use a composite criterion as executive function, which allows you to optimize the transition process, alarmed by the initial set of points and a set of external disturbances.

Let's examine the object of the control equation (1) with constant agitation applied (2), which is closed vertical position and the shape controllers. We combine a system of linear differential equations using a mathematical model of ITER plasma control system.To do that, let us introduce the following vectors and matrices: extended state-space vector $x = \{x_{st}, x_\nu, x_p, x_f, x_c\}$ that includes plasma states, vertical controller states, power system states, filter system states and shape controller states; matrices P and N with constant components such what P is a matrix of the linear part of the system mentioned above, and N is the coefficient of the non-linear part; the matrices L and K for linear combinations with extended state-space vector. So, by using the newly introduced variables, we represent the control system in the following form:

$$\begin{gathered} \dot{x} = P(p)x + N(p)f(t), \\ x(0) = x_0, \\ f(t) = f(d_\beta, t_\beta, d_l, t_l, t), \\ e = Lx(t, x_0, p), \\ u = K(p)x(t, x_0, p), \end{gathered} \quad (4)$$

where $x \in E^{112}$ is the extended state space vector,$e \in E^d$ is the measurement variables vector,$u \in E^{11}$ is the control voltages vector,P, N, L, K are the above introduced constant component matrices,$f(t)$ is the $l_i, \beta - drops$ disturbance, $p = \{p_k\}$ is a vector of parameters. Note, that the matrices $A_c, B_c, C_c, A_\nu, B_\nu, C_\nu$ of a designed regulators (1) will be taken as parameters that are to be optimized. We combine the elements of these matrices into a vector of parameters $p = \{p_k\}$, where each parameter has it own index. By labeling it $P(p)$and $N(p)$ we emphasize that it depends on the parameters that are being optimized. Based on this differential system (4) we have measurement variables vector **e** and control voltages vector **u**.

The transition process is described in the ensemble of trajectories [Zavadsky, Ovsyannikov, and Sakamoto, 2010]. In this paper we just mentioned that the transient component used executive criterion $I(p)$. This criterion must be minimized, it is performed by gradient.

3 Numerical Simultaneous Optimization Results

The results of the optimization is considered based on the transient authenticated documents - gaps that are members of the vector of variables $e \in E^d$ measuring object contr ol (4). Simulation of transient object has agreed with optimized control and primary controller is shown. This initial controller obtained using the approach described in [Zavadsky, Ovsyannikov, and Chung, 2009; Misenov, Ovsyannikov, and Ovsyannikov, 1997], and already has the appropriate features.

We can also say that with the help of the envelope estimate, since we can select a maximum voltage control according to their limitations, and then raise them and get a better job for the variables measuring e_i, see Fig. 2, Fig. 3.

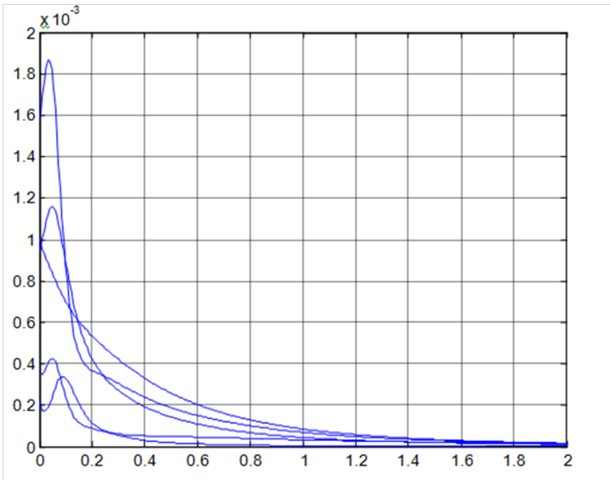

Figure 2. Trajectories ensemble with initial parameters values. In vertical axis ensemble bound of squared gaps, in horizontal axis time, sec.

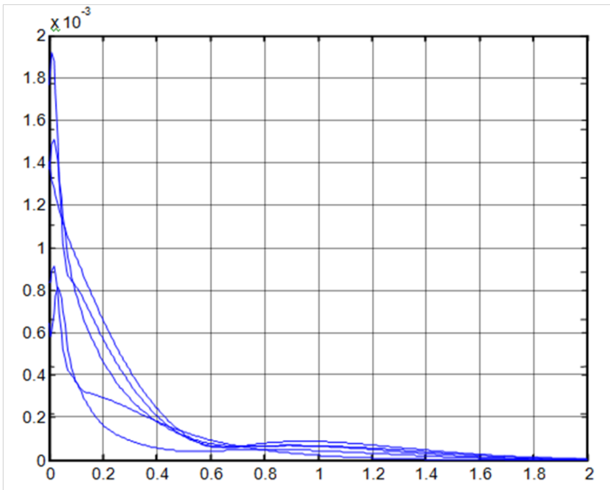

Figure 3. Trajectories ensemble after vertical and shape simultaneous optimization. In vertical axis ensemble bound of squared gaps, in horizontal axis time, sec.

In addition, we consider the basis for performance optimization of numerical features such as the integral squared gaps

$$I_{gaps} = \int_0^T \sum_{i=1,\dots,6} g_i^2(t)dt$$

and the settling time

$$t_{settling} = \min_{\tilde{t}}\{\tilde{t} : \max_{i=1,\dots,6} g_i(t)^2 \leq 0.2, \forall t \geq \tilde{t}\},$$

Table 1. Settling time and integral of squared gaps for control object.

previous results	new results
$I_{gaps} = 0.024$	$I_{gaps} = 0.0197$
$t_{settling} = 0.61 sec.$	$t_{settling} = 0.415 sec.$

which are presented in Table 1.

4 Conclusion

This work is dedicated to questions concerning synthesis and optimization of tokamak plasma control system. The model of ITER plasma control system is discussed and the simultaneous parametric optimization is suggested. The model of control system includes controllers of vertical position and shape of plasma. It is optimized both of these controllers simultaneously. We take the structural diagram of control system as differential equations system 4. The gradient method of simultaneous optimization is implemented for C++ and Matlab environments. Results of the computations are obtained and discussed. Numerical characteristics such as the integral of squared gaps and the settling time are presented for the both initial and the optimized controllers. For the optimized controller the squared gaps and settling time are lower, correspondingly.

References

McArdle, G.J., Belyakov, V.A., Ovsyannikov, D.A., qnd Veremey, E.I. (1998) The MAST plasma control system, *Proc. 20th Intern. Symp. on Fusion Tecnology SOFT'98.* Marseille, France, pp. 541–544.

Ovsyannikov, D.A., Veremey, E.I., Zhabko, A.P. et al. (2006) Mathematical methods of plasma vertical stabilization in modern tokamaks *PNuclear Fusion.* Vol 46, pp. 652–657.

Zavadsky, S., Ovsyannikov, A., Sakamoto, N. (2010) Parametric optimization for tokamak plasma control system *World Scientific Series on Nonlinear Science, Series B.* Singapore. Vol. 15, pp. 353–358.

Zavadsky, S.V., Ovsyannikov, D.A., and Chung, S.-L. (2009) Parametric optimization methods for the tokamak plasma control problem *International Journal of Modern Physics A.* Vol. 24, pp. 1040–1047.

Ovsyannikov, D.A., Veremey, E.I., Zhabko, A.P. et al. (2006) Mathematical methods of plasma vertical stabilization in modern tokamaks *Nuclear Fusion.* Vol 46, pp. 652–657.

McArdle, G.J., Belyakov, V.A., Ovsyannikov, D.A., and Veremey, E.I. (1998) The MAST plasma control system. In: *Proc. 20th Intern. Symp. on Fusion Technology SOFT'98.* Marseille, France, pp. 541–544.

Ovsyannikov, D.A., Zhabko, A.P., Veremey, E.I. et al. (1998) Plasma current and shape stabilization with control power reducing. In: *Proc. of the 5th Intern. workshop: "Beam Dynamics & Optimization".* St. Petersburg, pp. 103–111.

Ovsyannikov, D.A., Veremey, E.I., Zhabko, A.P. et al. (1996) Mathematical methods of tokamak plasma shape control *Proc. of the 3th Intern. workshop: "Beam Dynamics & Optimization"*. St.-Petersburg, pp. 218–229.

Misenov, B.A., Ovsyannikov, D.A., Ovsyannikov, A.D. et al. (2000) Analysis and Synthesis of Plasma Stabilization Systems in Tokamaks. In: *Control Application of Optimization: Preprints of the Eleventh IFAC Intern. Workshop.* St. Petersburg, pp. 249–254.

Misenov, B.A., Ovsyannikov, A.D., Ovsyannikov, D.A. et al. (1997) Non-linear model of tokamak plasma shape stabilization. In: *Intern. conf. on Informatics and Control (ICI&C'97).* St.-Petersburg, pp. 382-388.

Misenov, B. (1995) Computational aspects of plasma shape control synthesis problem *Proc. of 2nd Intern. Workshop "Beam Dynamics and Optimization"*. St. Petersburg, pp. 138–145.

Kirichenko, N.F. (1978) *Introduction to the Motion Stabilization Theory.* Kiev (in Russian).

Ovsaynnikov, D.A. (1990) *Modeling and Optimization of Charged Particle Beam Dynamics.* St. Petersburg, (in Russian).

Veremey, E.I., and Zhabko, N.A. (2003) Plasma current and shape controllers design for ITER-FEAT tokamak. *Book of abstracts of Workshop on Computational Physics Dedicated to the Memory of Stanislav Merkuriev.* St. Petersburg, 52 p.

Zavadskiy, S.V. (2014) Concurrent optimization of plasma shape and vertical position controllers for ITER tokamak. In: *Beam Dynamics and Optimization (BDO), 2014 20th International Workshop on.* St. Petersburg, DOI: 10.1109/BDO.2014.6890102.

Zavadskiy, S.V. (2014) Joint optimization of composition regulators for control systems in tokamaks. In: *Proc. of XII All-Russian Conference on Control Problems (VSPU 2014). V. A. Trapeznikov Institute of Control Sciences of Russian academy of sciences*, Moscow, pp. 4297–4302, (in Russian).

Zavadskiy, S.V. (2007) Structural parametric optimization in a plasma stabilization problem. *Bulletin of St. Petersburg State University. Series 10: Applied Mathematics. Informatics. Control processes.* Vol. 3, St. Petersburg, pp. 22–29, (in Russian).