ENERGY TRANSFER BETWEEN HYDRODYNAMICAL SYSTEMS AND EXCITATION MACHINES OF LIMITED POWER

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Abstract
Forced vibrations of fluid free surface in a cylindrical tank under interaction with an excitation machine of a limited power-supply (so-called ”limited excitation” phenomena) are investigated in detail. For a complex system - a tank with fluid and an excitation machine - the regions of parameters for three steady-state regimes: stationary, periodic and chaotic are determined. Attention is concentrated mainly on the properties of chaotic attractors and energy transfer between subsystems. Because the total power for every regime is balanced by the rate of change of the total energy of the whole system it is demonstrated how different is the energy distribution between subsystems for every of steady-state regimes. Changes of the coefficient of the damping force of fluid oscillations may control the chaotic regimes.

Key words
Electromotor, fluid free surface, chaotic attractor.

1 Introduction

In view of its practical importance, the possible modes of vibration of fluid free surface in a rigid container have been studied intensively from different points of view. The revolution in our understanding of the physics of the phenomenon brought by the discovery of chaotic types of motion in deterministic systems has forced reevaluation of previous results, in particular, the details of chaotic types of motion in certain physical systems. In addition, the discovery of chaos has changed the methodology used to study these problems, it has broken down earlier stereotypes, and has led to the rejection of certain unfounded assumptions, such as the method of reduction, which states that the behavior of a complicated system can be determined by the properties of its component sub-systems. The new point of view is that the dynamics of a complicated system depends more on the coupling between the sub-systems than on the sub-systems themselves. For example, in cases where certain normal modes of vibration of a distributed system are coupled and have the same frequency (as in the case of excitation of degenerate modes with equal eigenfrequencies), regular steady-state vibrations of any of the modes will ”deteriorate” into chaotic motion because of the nonlinear interaction between them. Another example of such coupling is the interaction of a vibrating system with an excitation mechanism. This interaction is always present because of the law of conservation of energy. When the vibrating system possesses damping (actually damping is present in all real systems), the dissipation of the energy could introduce essential corrections into the regimes of mechanism functioning. In this way, the vibrating system influences the parameters of the excitation force. This influence is considered significant when the power of the excitation mechanism is comparable to the power dissipated in the vibrating system. In this case the vibrating system has a limited excitation and the mechanism has a limited power-supply. This situation is considered in the present study.

The coupling effect between an excitation machine and vibrational loads was found by Sommerfeld [Sommerfeld, 1902; Timoshenko, 1928] and is a universal phenomenon and a manifestation of the law of conservation of energy. At first equations of motion with explanation the phenomena observed in Sommerfeld’s experiments were obtained by Blekhman [Blekhman, 1953]. However, a rather complete study of the Sommerfeld effect has been given in the works of Kononenko [Kononenko, 1969], so that we call these phenomena as Sommerfeld-Kononenko effect [Krasnopolskaya, 2002]. As shown by Kononenko for a linear oscillator with limited excitation the characteristics of a nonlinear oscillator arise, such as the occurrence of instability regions. In view of this, in the present study, the existence of new possible characteristics is investigated for forced resonant vibrations of the fluid in tanks, which result from the interaction of

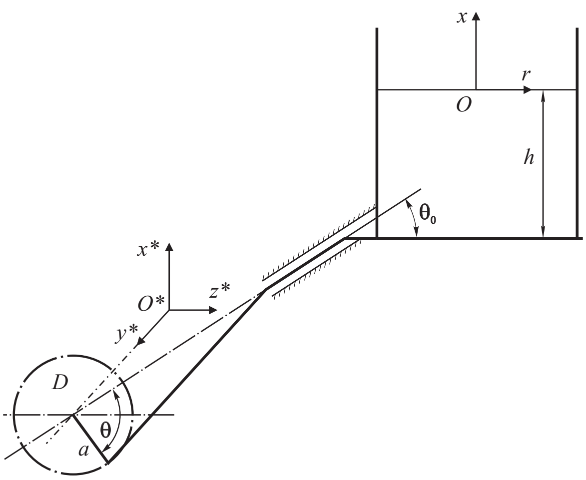


Figure 1. The system scheme

the vibrating system with the energy source - the electromotor (electric motor).

2 Formulation of the problem

Suppose that the electromotor D is connected by a crank connecting-rod mechanism with a rigid cylindrical tank partly filled with a fluid (Figure 1). The rotation of the electromotor shaft is described by the law of the change of angle $\Theta(t)$. When the crank a turns by the angle Θ , the tank is displaced in the space $u(t) = acos\Theta(t)$, which contains components along the axes O^*x^* and O^*z^* of the absolute coordinate system. These components are equal to $u_z = acos\Theta_0^*cos\Theta(t)$ and $u_x = asin\Theta_0^*cos\Theta(t)$, respectively, where Θ_0^* is the three-dimensional angle formed by the plane of tank platform motion and the horizontal plane $y^*O^*z^*$. The axis of the electromotor shaft is assumed to be parallel to the axis O^*y^* . For the description of the fluid free surface vibrations in the tank, we introduce the cylindrical coordinate system $Oxr\theta$, with origin on the undisturbed surface of the fluid. Then the equation of the fluid free surface may be written in the form

$$x = \eta(r, \theta, t) \tag{1}$$

3 The Lagrangian

For description of the fluid surface we use a representation in the form of the sum of eigenmodes,

$$\eta(r, \theta, t) = \eta_n(t)\psi_n(r, \theta), \tag{2}$$

where the summation is carried out for values with identical indices, and η_n are the generalized coordinates;

$$\psi_n = \psi_n^{c,s} = N_{ij}^{-1}J_i(k_{ij}r)(cosi\theta, sini\theta)$$

$$J_i'(k_{ij}R) = 0; N_{ij}^2 = \frac{1}{2}(1+\delta_{0i})[1-(i/k_{ij}R)^2]J_i^2(k_{ij}R), \tag{3}$$

where J_i are Bessel functions. Then Lagrangian L can be written in the form

$$L = \frac{1}{2}I\dot{\Theta}^2 + \frac{1}{2}m_0a^2\dot{\Theta}^2sin\Theta(t)+$$

$$\rho S[\frac{1}{2}a_{mn}\dot{\eta}_m\dot{\eta}_n - \frac{1}{2}(g + \ddot{u}_y)\eta_n\eta_n - \ddot{u}_xQ_n\eta_n^c], \tag{4}$$

where I is the moment of inertia of the electromotor shaft, m_0 is the mass of the tank filled with fluid, ρ is the density of the fluid, S is the cross-sectional area of the circular cylindrical tank, a_{mn} are nonlinear functions of η_n , $Q_n = \frac{1}{SN_{ij}}\int\int rcos\theta\psi_n^c rdrd\theta$, g is the acceleration of gravity, $\ddot{u}_x$ and $\ddot{u}_z$ are the vertical and horizontal acceleration, respectively, of the tank. The first two terms in (4) denote the kinetic energy of the electromotor shaft and of the tank with fluid as a whole.

Since the power of the electromotor which excites three-dimensional vibrations of the tank is comparable to the power dissipated in the fluid at vibrations with internal damping, the change of fluid vibratory regime has a reverse influence on the process of formation of the excitation force. For these reasons, the rotation speed of the electromotor shaft $\dot{\Theta}(t)$ should not be considered as a prescribed value, it depends on the characteristics of the electromotor, but also on the vibration of the fluid.

Introducing expressions for accelerations $\ddot{u}_y$ and $\ddot{u}_x$ into L, we obtain

$$L = \frac{1}{2}I\dot{\Theta}^2 + \frac{1}{2}m_0a^2\dot{\Theta}^2sin^2\Theta(t) + \frac{\rho S}{2}a_{mn}\dot{\eta}_m\dot{\eta}_n+$$

$$a\rho Scos\Theta_0^*(\dot{\Theta}^2cos\Theta + \ddot{\Theta}sin\Theta)Q_n\eta_n^c+$$

$$\frac{a}{2}\rho Ssin\Theta_0^*(\dot{\Theta}^2cos\Theta+\ddot{\Theta}sin\Theta)\eta_n\eta_n-\frac{\rho S}{2}g\eta_n\eta_n. \tag{5}$$

On the basis of (5) we can easily construct the equations of Lagrange for the generalized coordinates of the electromotor, i.e., for the angle $\Theta(t)$

$$I\ddot{\Theta} = -m_0a^2\dot{\Theta}^2sin\Theta cos\Theta - m_0a^2\ddot{\Theta}sin^2\Theta+$$

$$a\rho Scos\Theta_0^*(\dot{\Theta}^2sin\Theta - \ddot{\Theta}cos\Theta)Q_n\eta_n^c-$$

$$2a\rho Scos\Theta_0^*(\dot{\Theta}cos\Theta)Q_n\dot{\eta}_n^c$$

$$+a\rho S sin\Theta_0^*(\dot{\Theta}^2 sin\Theta - \ddot{\Theta} cos\Theta)\eta_n\eta_n -$$

$$2a\rho S sin\Theta_0^*(\dot{\Theta} cos\Theta) + \Phi(\dot{\Theta}) - H(\dot{\Theta}) \qquad (6)$$

Last two summations in the right-hand side of (6) are the driving torque and the torque of resistive forces of the electromotor [Kononenko, 1969]. The remaining terms in the right-hand side are torque of the reverse influence forces of vibrations of the fluid-filled tank and of the fluid free surface. The equation of Lagrange for η_n may be obtained also from (5). In this case the problem is reduced to the analysis of infinite number of nonlinear mutually related equations relative to η_n. This system has to be completed with the equation of energy source (6).

In the following the resonant forcing of the electromotor on free surface vibrations will be considered. We analyze the forced resonant vibrations of the fluid free surface. We assume $\Theta_0^* = 0$, which means that tank vibrations occur in the horizontal plane along the axis o^*z^* . Moreover we assume that, the angular speed of the electromotor shaft $\dot{\Theta}(t)$ in the stationary regime is close to the eigenfrequency ω_1 of the vibrations of the free surface of the first antisymmetric modes $\eta_n(t)\psi_n(r,\theta)$ $(n = 1, 2)$.

We introduce a small positive parameter

$$\epsilon = (aQ_1k_{11})^{(1/3)} \qquad (7)$$

In this case the detuning of frequencies $\dot{\Theta}$ and ω_1 will be taken as a small value, proportional to ϵ^2, in the form

$$\dot{\Theta}(t) - \omega_1 = \frac{1}{2}\epsilon^2\omega_1\beta(t), \qquad (8)$$

here $\omega_1 = (gk_{11}tanhk_{11}d)^{1/2}$, d is the depth of the fluid in the tank; β is tuning parameter, which measures the offset of frequency $\dot{\Theta}$ and ω_1. The vibrations of the free surface are approximated by dominant forms of vibrations $\eta_1\psi_1^c$ and $\eta_2\psi_2^s$ as well as by secondary modes containing the harmonics $cos2\theta, sin2\theta, cos0 \equiv 1$ [Miles, 1984]. We assume

$$\eta_n = \epsilon\lambda[p_n(\tau)cos\Theta(t) + q_n(\tau)sin\Theta(t)], \qquad n = 1, 2 \qquad (9)$$

for dominant modes and

$$\eta_n = \epsilon\lambda[A_n(\tau)cos2\Theta(t) + B_n(\tau)sin2\Theta(t) + C_n(\tau)] \qquad (10)$$

$$n \neq 1, 2$$

for secondary modes, where $\lambda = k_{11}^{-1}tanh(k_{11}d)$,

$$\tau = \frac{1}{2}\epsilon^2\Theta(t) \qquad (11)$$

is slow time; and the variables $p_n(\tau), q_n(\tau), A_n(\tau), B_n(\tau), C_n(\tau)$ are slowly varying dimensionless amplitudes of the dominant and the secondary modes.

Upon introducing (8)-(10) into (5) and averaging L over the fast time $\Theta(t)$, an expression may be obtained for the averaged Lagrangian $< L >$. After determination of A_n , B_n and C_n and their introducing into $< L >$ we finally find

$$< L > =$$

$$\frac{1}{2}I\dot{\Theta}^2 + \frac{1}{4}m_0a^2\dot{\Theta}^2 + \frac{1}{2}\epsilon^4 g\lambda^2\rho S[\frac{1}{2}(\frac{dp_n}{d\tau}q_n - p_n\frac{dq_n}{d\tau}) +$$

$$p_1 + \frac{\ddot{\Theta}}{\omega_1^2}q_1 + \beta(\tau)E + \frac{1}{2}AE^2 + \frac{1}{2}BM^2], \qquad (12)$$

where A, B - constant coefficients used in [Miles, 1984];

$$E = E_1 + E_2; \quad E_n = \frac{1}{2}(p_n^2 + q_n^2); \quad M = p_1q_2 - p_2q_1.$$

E and M are the energy and the angular momentum respectively of the vibrations of the fluid in the fundamental modes.

4 EVOLUTION EQUATIONS

We write the equations of Hamilton which follow from (12). We take into account of the forces of viscous damping $\epsilon^2\delta\dot{\eta}_n$

$$\frac{dp_1}{d\tau} = -\alpha p_1 - (\beta + AE)q_1 + BMp_2;$$

$$\frac{dq_1}{d\tau} = -\alpha q_1 + (\beta + AE)p_1 + BMq_2 + 1;$$

$$\frac{dp_2}{d\tau} = -\alpha p_2 - (\beta + AE)q_2 - BMp_1; \qquad (13)$$

$$\frac{dq_2}{d\tau} = -\alpha q_2 + (\beta + AE)p_2 - BMq_1,$$

where $\alpha = \delta/\omega_1$.

In problems of an ideal excitation of vibrations of the fluid free surface (when the power of excitation mechanism is infinite and the feedback of the vibrating system on this mechanism may be neglected) the system of equations (13) would have been a four -parametric one. However in the formulation of the problem considered in this paper, when the excitation unit - electromotor is "sensitive" to the level of energy dissipation by the vibrating system, we must consider $\beta(\tau)$ (8) not as a constant coefficient, but as an additional unknown. Since the value $\Theta(t)$ depends on vibrations of the liquid, the value of frequencies difference $\beta(\tau)$ will be determined by the whole history of interaction between the rotation of an electromotor shaft and the vibrations of the fluid free surface.

In order to close the system (13) we need an equation for β. We proceed in the following manner: we introduce a change of variables, as usual in problems of limited excitation

$$\dot{\Theta}(t) = \Omega(\tau) \qquad (14)$$

Then from (9) and (10) by averaging over the fast time $\Theta(t)$ we can write the equation (6) in the following form ($\Theta_0^* = 0$)

$$\frac{d\Omega}{dt} = \epsilon^4[M_1(\Omega) - \alpha_1\lambda\Omega^2 q_1 - \alpha_1\lambda\frac{d\Omega}{dt}p_1] + \epsilon^5 ... \qquad (15)$$

Here

$$\epsilon^4 M_1(\Omega) = \frac{\Phi(\Omega) - H(\Omega)}{I + 0.5m_0a^2}; \qquad \epsilon^3\alpha_1 = \frac{aQ_1\rho S}{2I + m_0a^2}.$$

In the slow time we have

$$\frac{d\Omega}{d\tau} = \epsilon^2 M_2(\Omega) - \epsilon^2\mu q_1, \qquad (16)$$

when

$$\epsilon^2 M_2(\Omega) = \frac{2\epsilon^2}{\omega_1}M_1; \qquad \mu = 2\lambda\alpha_1\omega_1.$$

As we are interesting in the steady-state response, the static characteristic of $\Phi(\Omega)$ of the electromotor [Kononenko, 1969] will be used. Accordingly, we assume $\epsilon^2 M_2(\Omega) = \epsilon^2(N_0 - N_1\Omega); N_0, N_1$ are constants.

Moreover, we transform (16) into equation for $\beta(\tau)$

$$\frac{d\beta}{d\tau} = N_3 - N_1\beta - \mu_1 q_1, \qquad (17)$$

where

$$N_3 = \frac{2}{\omega_1}(N_0 - N_1\omega_1); \qquad \mu_1 = \frac{2\mu}{\omega_1}.$$

Consequently, we conclude that the process of interaction between vibrations of the fluid free surface in dominant resonant modes and the shaft rotation of the electromotor with limited power-supply is described by a system of five evolution equations

$$\frac{dp_1}{d\tau} = -\alpha p_1 - (\beta + AE)q_1 + BMp_2;$$

$$\frac{dq_1}{d\tau} = -\alpha q_1 + (\beta + AE)p_1 + BMq_2 + 1;$$

$$\frac{dp_2}{d\tau} = -\alpha p_2 - (\beta + AE)q_2 - BMp_1; \qquad (18)$$

$$\frac{dq_2}{d\tau} = -\alpha q_2 + (\beta + AE)p_2 - BMq_1;$$

$$\frac{d\beta}{d\tau} = N_3 - N_1\beta - \mu_1 q_1.$$

In the following we analyze the steady solutions of the system of equations (18), which may represent equilibrium states, periodic and almost-periodic and also chaotic solutions. In the five-dimensional phase-space $(p_1, q_1, p_2, q_2, \beta)$, these solutions correspond asymptotically to a point, a limit cycle, a limit torus and a chaotic attractor respectively. The condition for the occurrence of a chaotic attractor is the combination of total compression with local instability.

The system of equations (18) is nonlinear, and closed-form solutions are not possible, and numerical solutions were obtained. In the space of parameters $(\alpha, A, B, N_1, N_3, \mu_1)$ of the equations system (18) extensive numerical experiments were carried out in order to find the regions of existence of chaotic solutions, and to investigate the transition from regular to chaotic regimes. The structural reorganization of phase portraits of the chaotic attractors was also investigated. The main computational method of the numerical integration of the equations (18) was a fourth-order Runge-Kutta method with the correction of the variable computational interval according to Dormand-Prince. A local numerical error of $O(10^{-8})$ or less was ensured. In the case of chaotic vibrations the number of points in those cross-sections was about 10^4 . The Lyapunov exponents were computed using Bennettin's method [Benettin, Galgani and Strelcyn, 1976]. In order to minimize the effect of transients, all the temporal realizations of the dynamic processes were analyzed after a prolonged time interval. The system of equations (18) has six parameters $(\alpha, A, B, N_1, N_3, \mu_1)$ which together with the initial conditions determine its behavior in the steady regimes.

We assume that the tank is filled by fluid to the depth $d > 3a$, so, we may use [Miles, 1984; Krasnopolskaya & Shvets, 1994]: $A = 1.112; \quad B = -1.531; \quad \alpha = 0.1; \quad N_3 = -0.1; \quad \mu_1 = 0.5$. The following initial conditions were chosen:

$$p_1(0) = q_1(0) = \beta(0) = 0; p_2(0) = q_2(0) = 0.01.$$

The parameter N_1 is the bifurcation parameter. That parameter characterizes the angle of the static characteristic slope of the source of vibrations excitations. As known, one of the most reliable features that confirm an existence of the chaotic attractor in the system is the presence of at least one positive Lyapunov exponent. On the basis of calculations of the highest Lyapunov exponent on the value N_1 the regions of chaotic attractors were defined. For values N_1 in the interval $0.05 \leq N_1 < 0.01$, a stable equilibrium state exists. The coordinates of the equilibrium positions of the system are:

$$p_1 = const; q_1 = const; \beta = const; \quad p_2 = q_2 = 0.$$

In other words all the equilibrium positions (for the chosen initial values) have zero coordinates p_2, q_2. At the point $N = 0.1$ the equilibrium position loses its stability. The system then undergoes a Hopf bifurcation, and a stable limit cycle appears. When $N_1 = 0.10153$ the coordinates of the equations system are periodic functions that correspond to a two - turn cycle in phase - space (fig. 2(a)). Results for the dimensionless power of the motor $P_o = N_3 - N_1\beta$, the power consumed by the damping force under fluid free surface oscillations $P_c = -\alpha\mu_1(p_1^2 + q_1^2 + p_2^2 + q_2^2)$ [Miles, 1984], the total power $P = P_o + P_c$ are shown in fig. 2(b). For the considered case the powers show typical periodic behaviour. In order to obtain this dimensionless powers the procedure which was used by Kononenko [Kononenko, 1969] was applied (for more details see [Krasnopolskaya & Shvets, 1993]). The total power is balanced by the rate of change of the total dimensionless energy of the whole system $E_t = \mu_1(p_1^2+q_1^2+p_2^2+q_2^2)/2 + \beta$. Further increases of the value of N_1 result in a cascade of period-doubling bifurcations. The infinite succession of bifurcation of period doubling ends, when $N_1 = 0.101632$, at the appearance of the chaotic attractor. The attractor which appeared in the system is a quasichaotic attractors and it has a spiral structure. At the value $N_1 = 0.10164$ an attractor shift occurs, and as a result, the two-cycled spiral turns into a single-cycled one. It is worth noting that all the foregoing regimes (the regular and the chaotic states), have one common important property: they are planar regimes since in all of them $p_2 = q_2 = 0$. It can be concluded that the vibrations of fluid free surface occur in the first mode only. The chaotic trajectory for $N_1 = 0.10164$ is shown in fig. 3(a). Power curves for this case are shown in fig. 3(b). The total power also oscillates around zero

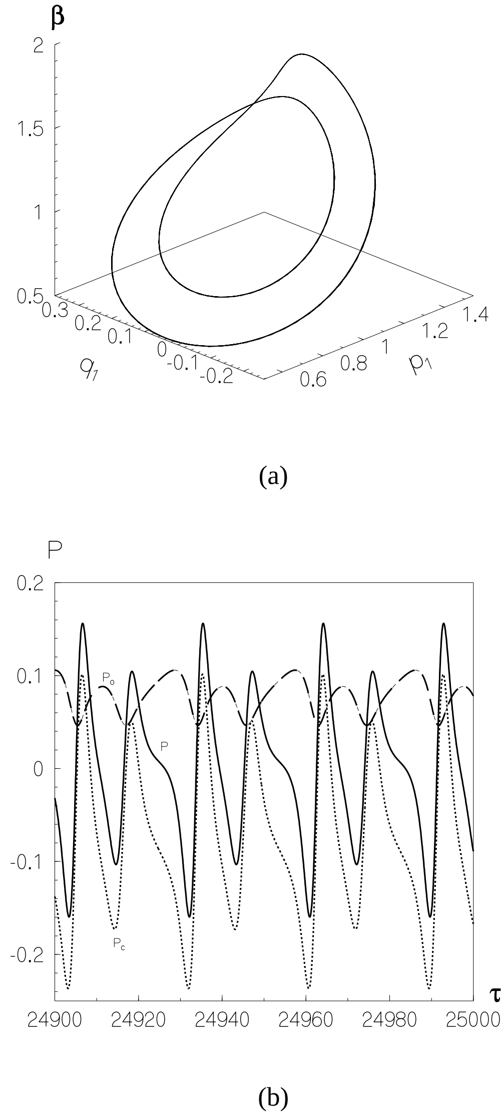
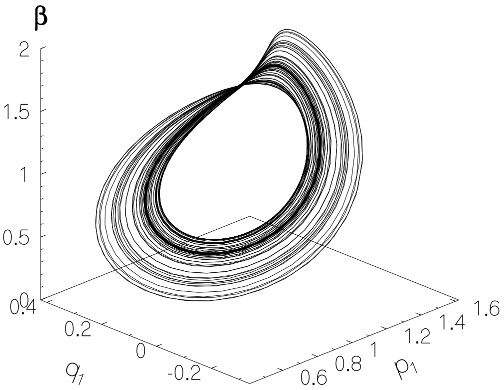
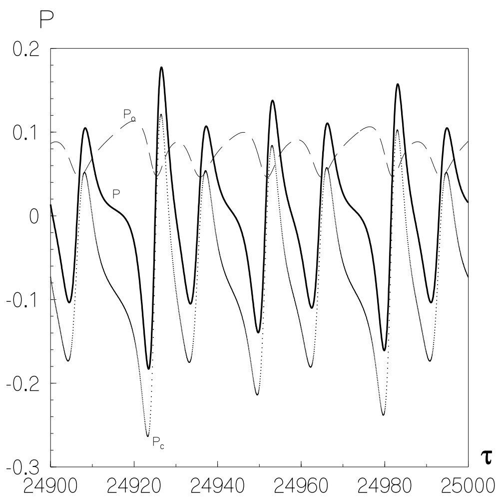


Figure 2. Graphs of (a) the trajectory and (b) the powers P_o, P_c and the total power P at $N_1 = 0.10153$ for the periodic regime

(as in the periodic regimes), but no constant period exists in slow time, for which the average power will be zero. And this total power is balanced by the rate of change of the total energy of the whole system, which is changing in chaotic way. Further increases of the slope angle for the electric drive static characteristic at the point $N_1 = 0.10165$ an additional qualitative shift causes rearrangement of the structure of the phase portrait. In physical space the second mode appears. This results in substitution of the "planar" chaotic attractor by the chaotic attractor of a completely different type which exists when the values of the parameter N_1 change within limits $0.10165 \leq N_1 \leq 0.373$.



(a)



(b)

Figure 3. Phase portrait of (a) the chaotic attractor and (b) graphs of the powers P_o, P_c and the total rower P for $N_1 = 0.10164$

5 Conclusion

Summarizing, in the system "electromotor- fluid free surface in a tank" three classes of steady state regimes are determined. The first class (I) consists of the stationary regimes, when vibrations of the fluid free surface occur with constant amplitude and frequency and the electromotor shaft rotates with a constant speed. For this case the power of the motor, the power, consumed by the damping force under fluid free surface oscillations and the total power are constant. Moreover, the total power as well as the rate of change of the total energy are equal to zero.

The second class (II) contains regimes with periodically changing amplitude and frequency of fluid free surface vibrations and the shaft speed periodically changes with time. For that class all considered powers have periodic behaviour and the total power oscillates around zero line. The total power is balanced by the rate of change of the total energy of the whole complex system.

Finally, the third class (III) corresponds to chaotic regimes when amplitude and frequency of vibrations and the electromotor speed change in time chaotically. For this case all powers have oscillating behaviour. The total power oscillates around zero (as in the II regimes), but no constant period exists in slow time, for which the average power will be zero. The total power is balanced by the rate of change of the total energy of the system, which is the chaotic function of time. The last regime is asymptotically established in the system. The system cannot leave this regime without assistance or be approximated by regimes of the first two classes.

It was shown that changes of the damping force coefficient of fluid free surface oscillations may control chaotic regimes: for very small and big coefficients chaotic regimes disappeared. They exist for intermediate values of the coefficient.

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