Functionalization Method in Motion Image Analysis

P.K. Kuznetsov*, B.V.Martemyanov*, V.I. Semavin*, E.Y. Chekotilo*

** Samara State Technical University, Russia*
(Tel: +7 846-2423882; e-mail: kurnesov@mail.ru)

Abstract: A straight method for image motion parameters' calculation by an observed object, which is based on the image sequence analysis, is developed. The main advantage of the method is that it does not involve calculations of the spatial derivatives of the image intensity function.

Keywords: optical flow, image motion, gradient estimation, functionalization method

1. INTRODUCTION

At the present time there exists a class of high performance registration image methods based on optical flow computation. Optical flow (Horn and Schunck, 1981) is called a vector field of apparent local velocity of moving images. The brightness constancy constraint (BCC) equation provides one of the techniques for an optical flow calculation (Black and Anandan, 1993; Lucas and Kanade, 1981; Horn and Schunck, 1981). The BCC equation determines a functional relation of time-spatial image intensity variations and image motion parameters, BCC equation appears as

$$\frac{\partial E(x,y)}{\partial t} + \upsilon_x \frac{\partial E(x,y)}{\partial x} + \upsilon_y \frac{\partial E(x,y)}{\partial y} = 0, \qquad (1)$$

where v_x, v_y – components of an apparent velocity vector $\dot{\mathbf{r}}$, $E(x,y)$ –an image intensity function. The technique of motion estimation methods based upon the equation (1) is referred to as ф gradient-based motion estimation method (GM method). Comprehensive surveys of the GM method are performed in Barron et al. (1981), Irani and Anandan (1994). The GM method is efficient in solving a parametric image registration problem: global translation, rotation, affine, projective motions and some others.

The GM method involves spatio-temporal derivatives of image intensity functions, but these functions are not differentiable and not continuous in a general case. That makes the GM method non rigorous and consequently there arise errors in results of image motion parameters calculations. Considerable efforts were undertaken to eliminate shortcoming of the GM method (Horn and Schunck, 1981; Lucas and Kanade, 1981; Schalkov, 1983; Black and Anandan, 1993; Barron et al., 1994).

Nevertheless the implementations of all the known versions of the method require a preliminary image smoothing through filtration using a convolution. But in general case the convolution is a non-linear operation in respect of velocity vector components. Nonlinearity takes place, for example, when the image rotates. So, smoothing yields additional errors to the results of calculations. In addition, the GM method gives estimations not of the real motion parameters of the observed object but of apparent velocity vectors of its image.

In this paper we develop a straight calculation technique of the motion parameters estimation of the moving object via analysis of its image sequence. The technique is called as functionalization method (Abakumov et al., 1980; Kuznetsov et al., 1990; Kuznetsov et al., 1994; Ageev et al., 1992).

The functionalization method is free from main shortcomings of the GM method as it does not invoke derivatives of the image intensity function. It uses the fundamental functional relation equation (FR equation), that determines a functional relation of measurable characteristics of an observed object image on the object motion parameters. The measurable image characteristic is one that can be calculated as a particular definite integral on any of the image regions of a non-nil square. The spatio-temporal derivatives of the image intensity are not measurable image characteristics in this sense as well as the image intensity function itself. The functionalization method gives a generalization of the class of the GM methods.

2. FUNCTIONALIZATION METHOD

The functionalization method differs from the GM method in operating not upon the image intensity functions but upon the special functionals, determined on a set of the image intensity functions. That brings considerably new features of the universality and accuracy to the technique of the image motion parameters estimation. In this section we develop the method for a case, when an observed object is a flat underlying surface and a platform carrying an imaging device or a video camera undertakes a motion with six degrees of freedom.

2.1. Imaging of the moving underlying surface

We assume that a platform carrying (CP) an optical system executes a translation and a free rotation motion relative to its own mass center, and that the underlying surface is flat, rigid and has a plane albedo and is uniformly illuminated with an off-site light source, the distance between CP and the underlying surface is considerably larger than the focal length of an (OS) in use, as well as OS does not distort an image (feature of an isoplanatism takes place).

The coordinate systems (Fig. 1) used for the image modeling are the following:

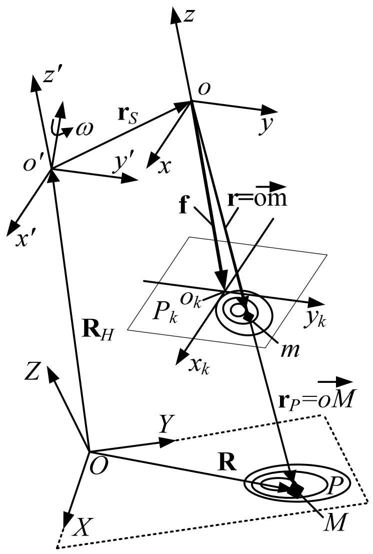


Fig. 1. Coordinate systems used for the image modeling

- *OXYZ,* the axes *OX* and *OY* of it are immovable and belongs to plane *P;*
- *P*- an underlying surface;
- *0′x′y′z′*, the axes of it are fixed to CP, the origin of coordinates *0′* coincides with CP mass center;
- *0xyz*, the axes of it are fixed to CP and are parallel to the similar axes of the coordinate system *0′x′y′z′,* and the origin *0* of the coordinates coincides with the main front focal point of OS, the derection of OS main axis is inverse to the axis *0z*;
- $0_k\, x_k\, y_k$, the axes of it are parallel to the similar axes of the coordinate system *0xy*, the origin 0_k belongs to the main axis of OS and is placed on the image plane (P_k) of OS at the distance of *f* from the origin *0* of the coordinate system *0xyz* $\mathbf{f} = [0\ \ 0\ \ -f\,]^T$ – the rear focal point of OS.

The position of the coordinate system *0xyz* relative to the coordinate system *0′x′y′z′* is identified with vector $\mathbf{r}_S=[x_S\ y_S\ z_S]^T$. The position and orientation of the coordinate system *0′x′y′z′* relative to the coordinate system *OXYZ* is identified with vector $\mathbf{R}_H=\mathbf{R}_H(t)=[X_H(t)\ \ Y_H(t)\ \ Z_H(t)]^T$ and with the transformation matrix (matrix of the directional cosines) $\mathbf{A}=\mathbf{A}(t)=[a_{i,j}(t)]$ (i, j = 1,2,3). It is well known that matrix **A** is orthogonal. Each point, belonging to *P,* maps its image at plane P_k, for example, on Fig. 1 point *M* maps its own image at point *m*. The position of point *M* ($M \in P$) in the *OXYZ* coordinate system is identified in Fig.1 by vector $\mathbf{R}=[X\ Y\ 0]^T$, and in the mobile coordinate system *0xyz* – by vector $\mathbf{r}_P=\mathbf{r}_P(t)=[x_P(t)\ y_P(t)\ z_P(t)]^T$. The position of point *m* ($m \in P_k$) in Fig.1 in the coordinate system *0xyz* is identified with vector $\mathbf{r}=\mathbf{r}(t)=[x(t)\ y(t)\ -f]^T$. From Fig.1 there follows vector equality

$$\mathbf{r} = k_m\mathbf{r}_P = k_m(\mathbf{A}(\mathbf{R} - \mathbf{R}_H) - \mathbf{r}_S), \qquad (2)$$

where

$k_m=k_m(\mathbf{r})=|\mathbf{r}|/|\mathbf{r}_P|= -f/(\mathbf{A}_3(\mathbf{R}-\mathbf{R}_H) - z_S)= -\mathbf{A}^T{}_3\mathbf{r}/(Z_H+\mathbf{A}^T{}_3\,\mathbf{r}_S)$;
$\mathbf{A}_3=[a_{31}\ a_{32}\ a_{33}]$ is a row-matrix that is the third row of matrix **A** and $\mathbf{A}^T{}_3=[a_{13}\ a_{23}\ a_{33}]$ is a row-matrix that is the third row of matrix $\mathbf{A}^T$.

2.2. Image motion equation

To obtain an image motion equation of the underlying surface we derive (2) in respect to time t and then get

$$\dot{\mathbf{r}} = -\left(\frac{1}{f}\mathbf{r}\Omega_3 + \Omega\right)(\mathbf{r} + k_m\mathbf{r}_s) - k_m\left(\frac{1}{f}\mathbf{r}\mathrm{A}_3 + \mathrm{A}_3\right)\mathbf{V}_H\,. \qquad (3)$$

Here $\dot{\mathbf{r}} = [\upsilon_x \upsilon_y 0]^T$ is an image velocity vector;
Ω - the matrix,

$$\mathbf{\Omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix},$$

where ω_1, ω_2, ω_3 are projections of vector $\boldsymbol{\omega}=[\omega_1\ \ \omega_2\ \ \omega_3]$ of CP angular velocity on the axes of *0′x′y′z′* coordinate system; $\mathbf{\Omega}_3=[-\omega_2\ \omega_1\ 0]$ is the third row of matrix **Ω**;
$\mathbf{V}_H = \dot{\mathbf{R}}_H = [V_{XH}\ V_{YH}\ V_{ZH}]^T$ is a velocity vector of the transient motion of CP relative to the underlying surface.
The equation (3) allows getting estimations of an apparent velocity vector of the moving images. We will use this expression below to solve the problem of the image registration and the motion vector velocity estimation of the underlying surface.

2.3 Functional relation equation (FR equation)

Let it a singly connected regular region *D* of analysis (an analyzing window) with boundary *B(D)* be allocated at plane P_k. Then we will define functional $\Phi(\mathbf{r}, t)$ on a set of the image intensity functions $E(\mathbf{r}, t)$ in window *D*:

$$\Phi = \iint_D K(\mathbf{r})E(\mathbf{r}(t), t)\, ds \qquad (4)$$

where $K(\mathbf{r})$ – a weight function that is continuous, uniformly bounded and differentiable one time almost everywhere in respect to all arguments. We assume that $K(\mathbf{r}) = 0$, if $\mathbf{r} \in B(D)$;
$E(\mathbf{r}(t), t)$ – an image intensity function, uniformly bounded.

Let it be calculated a total derivative of the functional (4) with respect to time *t* under the assumption that image motion equation (3) is fulfilled. Then we get

$$\dot{\Phi}(t) = \Phi'_t(t) + \iint_D K(\mathbf{r})E(\mathbf{r}(t),t)'_{\mathbf{r}}\,\dot{\mathbf{r}}\,ds, \qquad (5)$$

With the Green formula connecting double and curvilinear integrals the transformations of (5) yields

$$\dot{\Phi}(t) = \Phi'_t(t) + \Phi_1(t) + \Phi_2(t), \qquad (6)$$

where

$$\Phi_1(t) = \iint_D [K(\mathbf{r})]'_{\mathbf{r}}\, E(\mathbf{r}(t),t)\big((\mathbf{r}\mathbf{\Omega}_3 / f + \mathbf{\Omega})(\mathbf{r} + k_m\mathbf{r}_s)$$
$$+ k_m(\mathbf{r}\mathbf{A}_3 / f + \mathbf{A})\mathbf{V}_H\big)\, ds;$$

$$Ц_2(t) = -\iint_D K(\mathrm{r})E(\mathrm{r}(t), t)V_{ZH} / \big((\mathrm{A}_3^T\mathrm{r}_s + Z_H) - 3k_m\mathrm{A}_3\mathrm{V}_H / f$$
$$-3\mathbf{\Omega}_3(\mathbf{r} + k_m\mathbf{r}_s)/ f + \mathrm{A}_3^T\mathbf{\Omega}\mathbf{r}_s / (\mathrm{A}_3^T\mathbf{r}_s + Z_H)\big)ds;$$

$K(\mathbf{r})'_{\mathbf{r}} = [K(\mathbf{r})'_x \; K(\mathbf{r})'_y \; 0]$ is a vector of partial derivatives of weight function K with respect to spatial variables x and y correspondingly. Consequently, we admit that $\Phi'_t(t) = 0$, i.e. the off-site light source has a constant intensity in time.

The equation (6) is the desired FR equation. It functionally relates the object motion parameters $\boldsymbol{V_H}$, $\boldsymbol{\Omega}$ with the object image measurable characteristics; the characteristics are the double integrals defined on analyzing window D.

The example of a suitable weight function K gives a pyramid like function presented in Fig. 2.

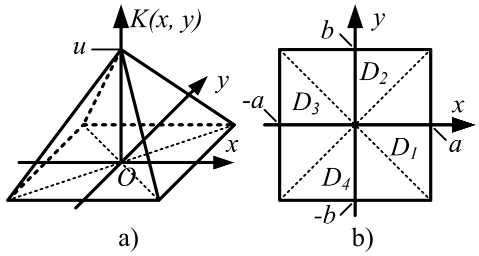


Fig. 2. Pyramid like weight function (a) and location of subregions $D_1 ... D_4$ on the analyzing window (b)

Analysis of the FR equation (6) shows, that it can be transformed to an algebraic equation that is linear with respect to the components of the full motion vector

$$b_1 V_{XH} + b_2 V_{YH} + b_3 V_{ZH} + b_4 \omega_1 + b_5 \omega_2 + b_6 \omega_3 = \dot{\Phi}, \quad (7)$$

where V_{XH}, V_{YH}, V_{ZH}, и ω_1, ω_2, ω_3 – components of the full velocity vector of the CP; $\dot{\Phi} = d\Phi / dt$ –a total derivative of functional Φ with respect to time t;
b_1 - b_6 are coefficients, values of which are determined by the magnitude of the integrals in (6).
The set of the equations needed to calculate the components of the full velocity vector $\Lambda = [V_{XH} \; V_{YH} \; V_{ZH} \; \omega_1 \; \omega_2 \; \omega_3]^T$ is composed by a parameterization of functional Φ. There is a vast option for the sort of functional Φ parameterization. As parameters may be set forth a time t, a position, a number, a shape of the analyzing window as well as the weight function $K(\mathbf{r})$ itself *etc*. The parameterization results in a system of equations

$$\mathbf{B}\Lambda = \dot{\Phi}, \quad (8)$$

where $\mathbf{B} = [b_{i,j}]$; $\dot{\Phi} = [\dot{\Phi}_i]$ – a column-vector; $i = 1 \ldots N$ ($N \geq M$), $j = 1 \ldots 6$ – a number of the CP motion degrees of freedom.

To decrease errors in the velocity estimations we over-constrain the system (7) ($N >> M$) and the generalized inverse method brings the following solution

$$\Lambda = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \dot{\Phi}. \quad (9)$$

In a case, when the main axis of the OS is directed to nadir the FR equation (7) takes the form of

$$\dot{\Phi} = \omega_3 \begin{pmatrix} \frac{1}{a}\left(\Phi_0^1(y) - \Phi_0^3(y)\right) - \frac{1}{b}\left(\Phi_0^2(x) - \Phi_0^4(x)\right) \\ + \frac{f}{z_S + Z_H}\left(\frac{y_S}{a}\left(\Phi_0^1 - \Phi_0^3\right) - \frac{x_S}{b}\left(\Phi_0^2 - \Phi_0^4\right)\right) \end{pmatrix}$$

$$- V_{XH} \frac{f}{z_S + Z_H}\left(\frac{a_{11}}{a}(\Phi_0^1 - \Phi_0^3) + \frac{a_{21}}{b}(\Phi_0^2 - \Phi_0^4)\right) \quad (10)$$

$$- V_{YH} \frac{f}{z_S + Z_H}\left(\frac{a_{12}}{a}(\Phi_0^1 - \Phi_0^3) + \frac{a_{22}}{b}(\Phi_0^2 - \Phi_0^4)\right)$$

$$- V_{ZH} \frac{1}{z_S + Z_H}\left(\frac{3}{a}\left(\Phi_0^1(x) - \Phi_0^3(x)\right) + \frac{3}{b}\left(\Phi_0^2(y) - \Phi_0^4(y)\right) + 2\Phi_0\right),$$

where

$$\Phi_0 = \iint_D E(\mathrm{r}, t)\, ds; \; \Phi_0^i = \iint_{D_i} E(\mathrm{r}, t)\, ds;$$

$$\Phi_0^i(\xi) = \iint_{D_i} \xi E(\mathbf{r}, t)\, ds, \; \xi \in \{x, y\};$$

$i = 1 \ldots 4$ – the number of subregions D_i in the analyzing window (Fig. 2). In addition, when $z_s = 0$, $\omega_3 = 0$, $V_{ZH} = 0$ the equation (10) takes the simplest form of

$$\dot{\Phi} = -V_{XH} \frac{f}{Z_H}\left(\frac{a_{11}}{a}(\Phi_0^1 - \Phi_0^3) + \frac{a_{21}}{b}(\Phi_0^2 - \Phi_0^4)\right)$$

$$-V_{YH} \frac{f}{Z_H}\left(\frac{a_{12}}{a}(\Phi_0^1 - \Phi_0^3) + \frac{a_{22}}{b}(\Phi_0^2 - \Phi_0^4)\right).$$

While comparing the last equation with BCCE (1) we see that the right hand sight of the FR equation contains measurable image characteristics only.

The functionalization method affords to estimate not only velocity vector $\boldsymbol{\Lambda}$. It provides a way to calculate an attitude of an optical axis of OS at any time $t > t_0$. To realize this way it is necessary to add to FR equation (9) a cinematic equation of the angular motion of CP

$$\dot{\mathbf{A}}(t) = -\boldsymbol{\Omega}\,\mathbf{A}(t), \qquad \mathbf{A}(t_0) = \mathbf{A}_0 \quad (11)$$

and to define initial value of matrix $\mathbf{A}$ ($\mathbf{A}(t_0) = \mathbf{A}_0$).

3. IMPACT OF ADDITIVE NOISE

In this section we investigate an additive noise impact on the estimates of the motion vector displacement derived with using the functionalization method. We examine a case of the plane-parallel motion of the CP relative to an underlying surface. In this case the main functional (4) used in the method will have the form

$$\tilde{\Phi}(t) = \iint_D K(\mathbf{r})\, \tilde{E}(\mathbf{r}(t), t)\, ds, \quad (12)$$

where $\tilde{E}(\mathbf{r}(t), t) = E(\mathbf{r}(t), t) + n(\mathbf{r}, t)$ - a video signal available for the instrumental measurement; $n(\mathbf{r}, t)$ – a noise (stochastic function).

We assume that the noise and the image intensity function are not mutually correlated and the indispensable conditions of differentiability of the noise realizations $n(\mathbf{r}, t)$ in respect to time are fulfilled. Deriving the functional (12) in respect to time *t,* then we get

$$\dot{\tilde{\Phi}} = -\upsilon_x \Phi'_x - \upsilon_y \Phi'_y + H'_t, \qquad (13)$$

where $\upsilon_x = V_{XH} f / Z_H$; $\upsilon_y = V_{YH} f / Z_H$;

$$\Phi'_x = \iint_D K'_x(\mathbf{r}) E(\mathbf{r}(t)) ds;$$

$$\Phi'_y = \iint_D K'_y(\mathbf{r}) E(\mathbf{r}(t)) ds;$$

$$H'_t = \iint_D K(\mathbf{r}) \frac{\partial\, n(\mathbf{r}(t), t)}{\partial t} ds.$$

As the stochastic function H cannot be practically measured instrumentally it is impossible to use the equation (13) for the calculation of image velocity vector $\dot{\mathbf{r}} = [\upsilon_x\ \upsilon_y\ 0]^T$. For this aim we will use an approximate form (14) deduced from (13) by eliminating the term H'_t from right-hand side of equation (13) and using $\tilde{E}(\boldsymbol{r}(t))$ instead of $E(\boldsymbol{r}(t))$. Then we get

$$\dot{\tilde{\Phi}}(t) = -\tilde{\upsilon}_x \tilde{\Phi}'_x - \tilde{\upsilon}_y \tilde{\Phi}'_y, \qquad (14)$$

where

$$\tilde{\Phi}'_x = \iint_{\mathrm{D}} K'_x(\mathbf{r}) \tilde{E}(\mathbf{r}(t), t) ds;\ \tilde{\Phi}'_y = \iint_{\mathrm{D}} K'_y(\mathbf{r}) \tilde{E}(\mathbf{r}(t), t) ds;$$

$\tilde{\upsilon}_x = \upsilon_x + \Delta\upsilon_x$ and $\tilde{\upsilon}_y = \upsilon_y + \Delta\upsilon_y$ are estimations of the components of the image motion velocity vector; $\Delta\upsilon_x$ and $\Delta\upsilon_y$ – the estimation errors. All the coefficients in the right-hand side of (14) are measurable in the defined above sense.

To reduce errors $\Delta\upsilon_x$ and $\Delta\upsilon_y$ we will over–constrain the equation (14) using an information from several windows D_i ($i \in \{1 \ldots l\}$), $l >> 2$ and thus compose the system

$$\tilde{\upsilon}_x \sum_{i=1}^n (\tilde{\Phi}'_{xi})^2 + \tilde{\upsilon}_y \sum_{i=1}^n (\Phi'_{xi}\Phi'_{yi} + \Phi'_{xi}H'_{yi} + H'_{xi}\Phi'_{yi} + H'_{xi}H'_{yi})$$
$$= -\sum_{i=1}^n (\dot{\tilde{\Phi}}_i \Phi'_{xi} + \dot{\tilde{\Phi}}_i H'_{xi});$$
$$\tilde{\upsilon}_x \sum_{i=1}^n (\Phi'_{xi}\Phi'_{yi} + \Phi'_{yi}H'_{xi} + H'_{xi}\Phi'_{yi} + H'_{xi}H'_{yi}) + \tilde{\upsilon}_y \sum_{i=1}^n (\tilde{\Phi}'_{yi})^2$$
$$= -\sum_{i=1}^n (\dot{\tilde{\Phi}}_i \Phi'_{yi} + \dot{\tilde{\Phi}}_i H'_{xi}),$$

$$H'_x = \iint_D K(\mathbf{r}) \frac{\partial\, n(\mathbf{r}(t), t)}{\partial x} ds;\ H'_y = \iint_D K(\mathbf{r}) \frac{\partial\, n(\mathbf{r}(t), t)}{\partial y} ds$$

If the noise and the image are such that the functionals Φ'_x, Φ'_y, H'_x, H'_y are not mutually correlated, then the asymptotic forms (when $l \to \infty$) for $\Delta\upsilon_x$ and $\Delta\upsilon_y$ will be

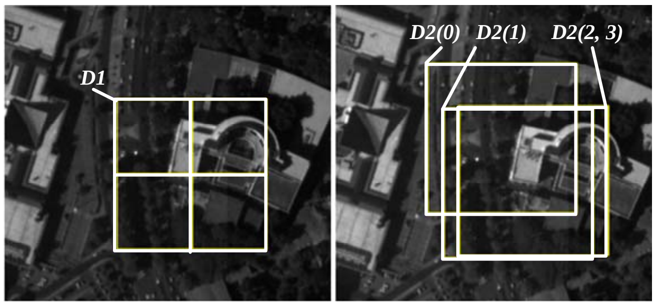
$$\Delta\overline{\upsilon}_x = -\upsilon_x \frac{1}{1 + \sum_{i=1}^n (\Phi'_{xi})^2 / \sum_{i=1}^n (H'_{xi})^2},$$
$$\Delta\overline{\upsilon}_y = -\upsilon_y \frac{1}{1 + \sum_{i=1}^n (\Phi'_{yi})^2 / \sum_{i=1}^n (H'_{yi})^2}. \qquad (15)$$

It can be seen from (15) that the velocity estimations obtained with the functionalization method have negative sign shifts directly proportional to the image velocity and reciprocally proportional to a ratio of the signal power to the noise power.

Moreover, it follows from (15) that the velocity estimations shifts can be eliminated with the use of a compensating measurement method mechanically or electronically implemented.

4. ITERATIVE TECHNIQUE

One of the ways to implement the compensating measurement method is to registry the images in two sequential in time image frames. We suppose that analyzing window $D1$ is placed on the first image frame that was registered at time t_0. The position of $D1$ is demonstrated in Fig. 3a. The second image frame is registered in time interval Δt at time $t_1 = t_0 + \Delta$. Let it another analyzing window be positioned at any arbitrary place $D2(0)$ on the second frame as it is shown in Fig. 3b. Our aim is to overlap the windows in these two frames by means of moving the window in the second frame, so that the images in the windows should match each other in the best way.



a) b)

Fig. 3 Iterative technique of image motion parameters estimation

The iterative scheme we put forward is the following. At every *k*-th step of the procedure we use the information obtained from windows $D1$ and $D2(k)$ to calculate the estimations of vector $\tilde{s}(k) = [\tilde{s}_x(k)\ \tilde{s}_y(k)]$ of a mutual displacement of the two windows with the use of the method. In this case to form right-hand side of FR (9) we use the first difference instead of a temporal derivative of the main functional (12). Volumes of the main functional are taken from windows $D1$ and $D2(k)$.

In order to form over-constrained system FR we will use additional analyzing windows covering up the main windows $D1$ and $D2(k)$ with sets $D1_i$ и $D2_i(k)$ (i=1,2,…,M) of the identical windows of fewer dimensions.

The example of such covering is shown in Fig.3a. In the example each of the coverings consists of five windows and consequently we get the set of five FR equations. The solution of the over-constrained FR system is attained by Least-Squares method. At the next step of the iterative scheme the analyzing window $D2_i(k)$ is warped towards $D1_i$ with the use of current estimate $\tilde{s}(k) = [\tilde{s}_x(k)\, \tilde{s}_y(k)]^T$ of the windows displacement and then we get a new position for window $D2_i(k)$, let it be denoted as $D2_i(k+1)$. This procedure can be written as

$$\widetilde{\mathbf{s}}\ (k+1)= \widetilde{\mathbf{s}}\ (k) + \Delta\mathbf{s}(k), \qquad (16)$$

where $\Delta\mathbf{s}(k)=[\Delta s_x(k)\ \ \Delta s_y(k)]^T$; $\widetilde{s}\ (0)=s_0$; k=1, 2,…- a number of iteration.

At every iteration we get increment of the displacement estimation $\Delta\mathbf{s}(k) = [\Delta s_x(k)\ \Delta s_y(k)]^T$. The procedure iterates till displacement $\widetilde{s}\ (k)$ becomes smaller than the preliminarily specified radius ρ ($\rho^2=\Delta s_x^2(k)+\Delta s_y^2(k)$, $k=k_{max}$) of an accessibility tube ($\widetilde{s}\ (k)\ \le\rho$) and will not leave the tube in the sequel.

5. COMPUTER MODELING

There was used a computer validation of the introduced iterative technique of image registration. The modeling system enclosed three modules: a module of video signals generation, a module elaborating estimations of two image frames displacement and a module for overlapping the two image frames.

The computational model of the video signal corresponded a mixture of a pure video signal and an additive random noise. The random noise used in experiments had a uniform frequency distribution function. Amplitude of the noise was specified in lower order bits (l.o.b). The main functional (4) had the pyramid-like weight function. Results of the computer modeling are represented in Fig.4. Experiments were undertaken to determine a relationship of a mathematical expectation of image overlapping error ρ (accessibility tube) and a parameter A_n (amplitude) of the noise random function. A dynamic range of the video signals belonged to an interval of [0-127] l.o.b. The initial displacement of the two images been under registration varied from 2 to 50 pixels. The main analyzing window used in the experiments measured 128 by 128 pixels. Number of iterations sufficient to attain the accessibility tube shown in Fig. 4 did not exceed $k_{max} \le 5$ (3 at an average). A non-nil error at noise amplitude A_n=0 is a result of a spatial discretization and digitization of the video signals.

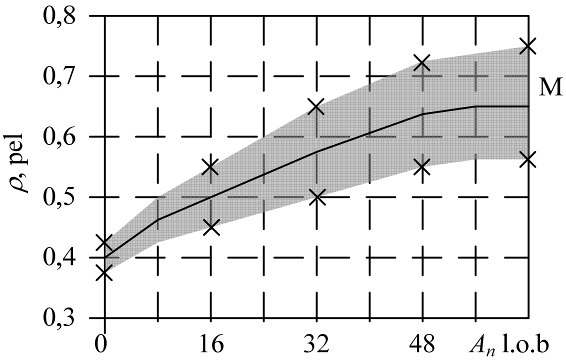


Fig. 4. Experimental data. Relationship of mathematical expectation M of image overlapping error ρ (radius of accessibility tube) and parameter A_n (amplitude) of the noise, $D_{0,96}$ - (gray background) confidence interval.

6. CONCLUSION

The fuctionalization method introduced in the report may be classified as a generalization of the GM methods based on the BCC equation. The BCC equation itself can be simply derived from the FR equation if the measure of definition domain D of the main functional used in the method tends to zero.

Main advantage of the functionalization method is that it does not involve calculations of spatial derivatives of the image intensity function. Moreover, the iterative technique of the method eludes temporal derivative calculations of the image intensity, as well. This feature brings essentially new power to the image registration and motion estimation methods.

REFERENCES

Abakumov, A.M., Kuznetsov, P.K., Mishin, V.Y., Semavin, V.I. (1980). Technique of object motion parameters estimation. In: *Proc. of All Union Symp. "Optical scanners and instruments on their basis",* 1, 50-51. Barnaul. In Russian.

Ageev, V.E., Voytenko, V.N., Anaikin, O.A., Kuznetsov, P.K., Semavin, V.I. (1992). Device for motion parameters estimation. USSR Patent 1742729. In Russian.

Barron, J., Fleet, D., Beauchemin, S. (1994). Performance of optical flow techniques. *International Journal of Computer Vision,* 12(11), 43-77.

Black, M. J. and Anandan, P. (1993). A framework for the robust estimation of optical flow. *ICCV'93*, 231-236.

Horn, B.K.P. and Schunck, B. (1981). Determining Optical Flow. *Artificial Intelligence,* (17), 185–203.

Irani, M. and Anandan, P., Vision algorithms: Theory and Practice. In Proc. of International Workshop on Vision Algorithms, 267-278. Springer-Verlag.

Kuznetsov, P.K., and Semavin, V.I. (1990). Technique of motion parameters calculation of brightness fields. *Izvestiya VUZ. Ser. Instrument engineering*, (6), 26-30. In Russian.

Kuznetsov,P.K., Semavin, V.I., MishinV.Y., Vladimirov, M.V. (1994). Method of functional transformation in problem of motion parameters estimation of brightness fields. In: *Proc. of Samara Technical University.* 1. 66-76. In Russian.

Lucas, B.D.. and Kanade, T. (1981). An Iterative Image Registration Technique with an Application to Stereo Vision. *IJCAI '81,* 674-679.

Schalkov, R.J. (1983). Image Motion Analysis Using Concept of Weak Solution to Distributed Parameter Systems. In *Proc. of IEEE Conference on Computer Vision and Pattern Recognition,* 232– 239. Washington.