CYBERNETICS AND PHYSICS, VOL. 6, NO. 3, 2017 , 139–144

STUDY OF MULTISTABLE VISUAL PERCEPTION BY STOCHASTIC MODULATION USING A SYNERGETIC MODEL

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Abstract

We consider a synergetic model of multistable visual perception with additive noise and study dynamics of coexisting percepts as a function of a bias parameter. The bifurcation analysis allows us to estimate the region of coexisting percepts. The effect of noise on the attention state manifests itself as intermittent switches between different perception states. Using the recurrence plot we find a threshold value of the noise intensity when the perception selection can be well defined.

Key words

Multistable, perception, bifurcation, synergetic model.

1 Introduction

The term "multistable" used in perception means that an ambiguous stimulus received by the brain can be interpreted in different ways. This phenomenon was intensively studied by psychologists since 1832 [Kuhn and Hawkins, 1963]. Multistable perception can be evoked by visual patterns that are too ambiguous for the human visual system to recognize a single interpretation. The famous examples of such images are the Necker cube, Rubin vase, rabbit-duck, etc. Since the most of ambiguous images have only two possible meanings, this kind of perception is often referred to as *bistable perception*. When a person observes such an image for a long time, the attention intermittently switches between different percepts. This alternation was attributed to neuronal adaptation [Köhler and Wallach, 1944].

Nowadays, there are several hypotheses about possible mechanisms underlying these switches. Some of them suppose the influence of stochastic processes inherent to neural network activity [Moreno-Bote et al., 2007; Pisarchik et al., 2014; Pisarchik et al., 2015; Runnova et al., 2016; Pisarchik et al., 2017]. Other hypotheses [Kelso, 2012] relate the switches to proper dynamics of the brain neural network.

In the theory of complex systems, multistability means the coexistence of several stable states or attractors for the same set of parameters. The stability of an attractor depends on the velocity at which the system comes back to this state after a small disturbance which kicks the system out of the attractor. Theoretically, multistability can be detected by simply varying initial conditions of all system variables. Experimental detection of multistability is a more sophisticated problem. The most common ways to reveal multistability are either to vary a control parameter forth and back in order to find a hysteresis behavior, or to add noise which induces switches between coexisting states [Pisarchik and Feudel, 2014]. In the last case, noise converts the multistable system to a metastable one. This happens with visual perception of ambiguous images where brain noise benefits distinct imterpretations of the same ambiguous image [Moreno-Bote et al., 2007]. Alternation of perception is a stochastic process according to the Markov chain. Any decision would be impossible without noise which produces probabilistic choices.

In the recent paper [Pisarchik et al., 2014], the Necker cube has been used as the essential example of an ambiguous figure, where the contrast of inner lines was taken as a control parameter to measure dynamical hysteresis in perception when the control parameter was increased and decreased. The level of brain noise was estimated from the dependence of the hysteresis size on the velocity of the contrast change. Apart from inherent brain noise, external noise also induced alternation in perceptions. The results of psychological experiments have been interpreted on the base of a stochastic bistable energy model. There are also other models which can be used to reveal essential mechanisms underlying switches between different visual percepts.

In this paper, we focus on the synergetic model, first introduced by Haken [Haken, 1979] for studying bistable visual perception. The rest of the paper is organized as follows. In Sec. 2 we describe the model. Then, in Sec. 3 we construct the perception bifurcation diagram, and in Sec. 4 we analyze stochastic dynamics under the influence of additive noise. In Sec. 5 we make the recurrence plot of the selection of preferential perceptions for different noise intensities. Finally, in Sec. 6 we summarize our results.

2 Model

In 1944, Köhler proposed a psychological hypothesis about perception saturation [Köhler and Wallach, 1944]. He suggested, and many psychologists supported his opinion, that the observed switches between different percepts of ambiguous images result from fatigue, inhibitions or neuronal saturation. This perceptual behavior can be simulated using the synergetic model of human perception of ambiguous patterns [Kohonen, 1989; Haken, 1979; Ditzinger and Haken 1989; Haken, 2004], which results are in a qualitative agreement with our experimental results. This mathematical model is a straightforward extension of a general algorithm for pattern ambiguous recognition on one hand, and saturation of the attention parameter on the other hand [Haken, 1979]. In this paper, we focus on the Ditzinger and Haken synergetic model [Ditzinger and Haken 1989] which consists of four coupled nonlinear differential equation: two variables for saturation attention and two for perception of ambiguous patterns.

This synergetic model simulates visual perception of ambiguous images, such as, for example, left and right orientations of the Necker cube shown in Fig. 1. The model is given by the following equations

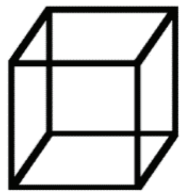

Figure 1. Necker Cube

$$
\begin{aligned}
\dot{\xi}_1(t) &= \xi_1 \Bigg[\lambda_1 - A\xi_1^2 - B\xi_2^2 + \\
&\qquad 4\left(B-A\right)\alpha\xi_2^2 \left(1 - \frac{2\xi_2^4}{\left(\xi_1^2+\xi_2^2\right)^2}\right)\Bigg], \\
\dot{\xi}_2(t) &= \xi_2 \Bigg[\lambda_2 - B\xi_1^2 - A\xi_2^2 - \\
&\qquad 4\left(B-A\right)\alpha\xi_1^2 \left(1 - \frac{2\xi_1^4}{\left(\xi_1^2+\xi_2^2\right)^2}\right)\Bigg], \\
\dot{\lambda}_1(t) &= \gamma\left(1-\lambda_1-\xi_1^2\right) + F(t), \\
\dot{\lambda}_2(t) &= \gamma\left(1-\lambda_2-\xi_2^2\right) + F(t),
\end{aligned} \tag{1}
$$

where ξ_1 and ξ_2 are variables associated with two different cube orientations (Fig. 1), and λ_1 and λ_2 are variables associated with corresponding saturation attentions. In this model, the attention is subjected to a damping mechanism mimicking the effect of saturation and synaptic connections. Here, $A = 1.5$, $B = 2$, and $\gamma = 0.1$ are constant parameters, α is a bias parameter referred to the perception preference, and $F(t)$ is a perturbation factor which represents either harmonic or stochastic modulation.

Figure 2 illustrates the time series of the variables for perception and saturation attention for different values of bias parameter α at $F(t) = 0$. When $\alpha = 0$, there are no preference between two precepts of ambiguous patterns because the pulse widths of the two variables are the same, as seen in Fig. 2(a). We refer these processes to as perceptions without bias. However, even for very small bias $\alpha = 0.064$ (Fig. 2(b)), one of the percepts has preference over the other, because the pulse widths of variables $\xi_{1,2}$ and $\lambda_{1,2}$ are different. This process was previously described by Ditzinger and Haken [Kohonen, 1989; Haken, 1979] as perceptions with different bias. An additional increase in parameter α up to 0.128 gives preference of one percept over the other. It is clearly seen from Fig. 2(c) that the pulse widths of the two perception variables are very different.

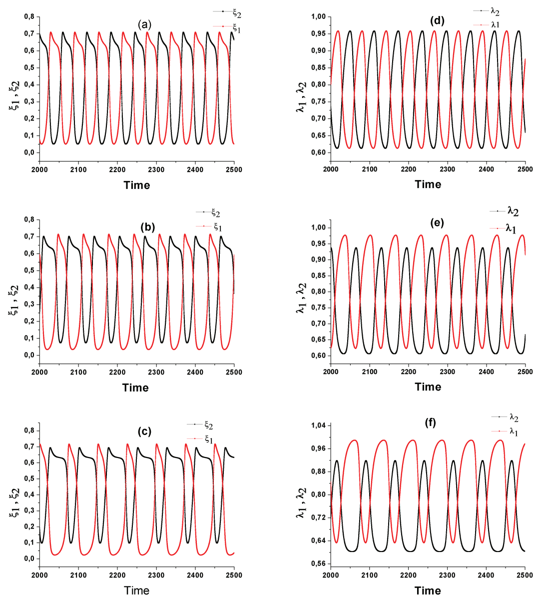

Figure 2. Time series of perception and saturation attention variables for $F(t) = 0$ and bias parameter (a,d) $\alpha = 0$, (b,e) $\alpha = 0.064$, and (c,f) $\alpha = 0.128$.

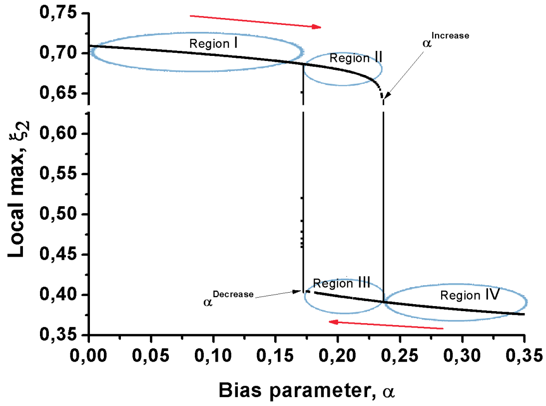

Figure 3. Bifurcation diagram of local maxima of percept ξ_2 as a function of bias parameter α. The red arrows indicate the direction of the parameter change. Regions I-IV shown by blue ellipses represent different perception states.

3 Bifurcation Diagram

In the bistable perception model, two different percepts are associated with two stable steady states or attractors. A change in a control parameter responsible for the ambiguity leads to the deformation of the basins of attraction of these coexisting attractors and finally to a change in their stability that occurs at a critical point, usually a saddle-node bifurcation. Here, we use bias α as a control parameter. In Fig. 3 we plot the bifurcation diagram of the local maxima of percept ξ_2 as a function of bias parameter α at $F(t) = 0$.

As the bias parameter α is increased from 0 to 0.17, the system is in a periodic orbit (Region I in Fig. 3). At $\alpha \approx 0.17$, a steady state attractor appears (Region III) which coexists with the periodic orbit (Region II). When α is further increased, the periodic orbit disappears and only stable steady state remains (Region IV). While one percept is stable for $\alpha < \alpha^{Increase}$, another percept is stable for $\alpha > \alpha^{Decrease}$.

The time series of the system variables corresponding to different regions are shown in Fig. 4. In Region I (for $\alpha < \alpha^{Decrease}$) we observe a periodic orbit only (Figs. 4(a,b), whereas for $\alpha^{Decrease} < \alpha < \alpha^{Increase}$ two regimes (periodic orbit and steady state) coexist (Figs. 4(c-f)). They are found by varying initial conditions presented in Table I. Finally, for $\alpha > \alpha^{Increase}$ only a steady state exists (Figs. 4(g,h)).

Table I. Parameter α and initial conditions used to obtain regimes shown in Fig. 4.

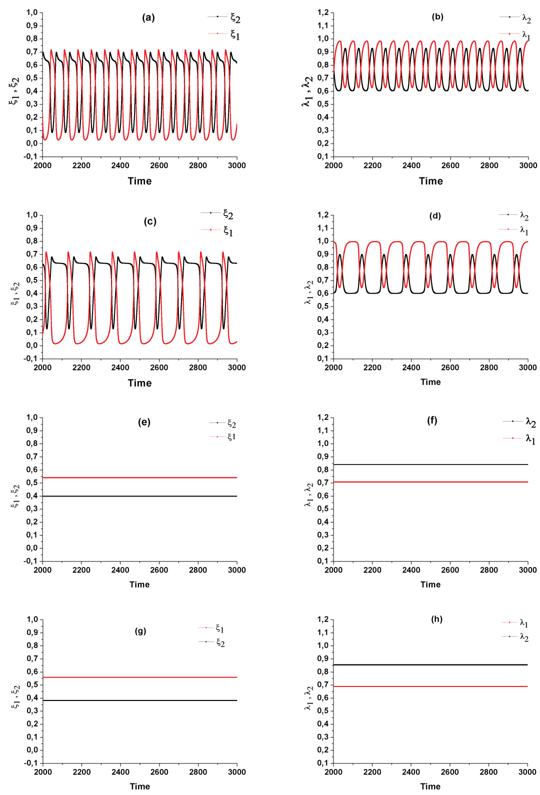

Figure 4. Time series of variables $\xi_{1,2}$ and $\lambda_{1,2}$ corresponding to different regimes shown in Fig. 3: (a,b) monostable steady state at $\alpha = 0.1$ (Region I), (b-f) bistable regime of coexisting (c,d) periodic orbit (Region II) and (e,f) steady state (Region III) at $\alpha = 0.2$, and (g,h) monostable steady state (Region IV) at $\alpha = 0.25$.

Fig. 4	α	ξ_1	ξ_2	λ_1	λ_2
(a),(b)	0.3	0.558	0.382	0.689	0.854
(c),(d)	0.2	0.540	0.399	0.708	0.841
(e),(f)	0.2	0.125	0.677	0.817	0.735
(g),(h)	0.1	0.083	0.697	0.800	0.737

4 Stochastic Modulation

Most natural systems have a stochastic component which affects its dynamics. Many researchers from different areas of science demonstrate a great interest to the interaction between stochastic and deterministic processes. In particular, in multistable systems noise can change the stability of some states, thus making the system metastable when the phase-space trajectory visits different attractive regions of the phase space. In this case, we deal with so-called *multistate intermittency* [Pisarchik et al., 2012; Hramov et al., 2016]. Intermittency is a common behavior in nonlinear dynamics, characterized by irregular bursts (turbulent phase) interrupted by a steady state (laminar phase). In multistable systems, intermittency can be induced by noise. In the case of bounded noise, which usually takes place in natural systems, the effect depends on the noise intensity. While weak noise does not eliminate attractors and only changes the probability of their appearance (preference), strong noise mixes attracting basins of the coexisting states so that the trajectory visits different states, thus resulting in a new intermittent attractor, sometimes called attractor hopping [Pisarchik and Feudel, 2014].

To study the influence of noise, we define the perturbation term in Eq. (1) as follows

$$F(t) = \eta\varsigma, \tag{2}$$

where $\varsigma \in [-1, 1]$ is a random number and η is the noise intensity. In the presence of random modulation Eq. (2), the solutions of stochastic Eq. (1) have a probabilistic character. For $\alpha = 0.2$ (Region II), weak noise with $\eta < 0.012$ does not induce intermittency of the variables ξ_1 and ξ_2. It only changes statistical properties of the system providing preference to one of the attractors over the other, as shown in Fig. 5(a). In this case, ξ_1 and ξ_2 exhibit almost coherent behaviors rarely interrupted by some disturbances during short time intervals. For stronger noise with intensity $0.012 < \eta < 0.094$, intermittent switches between the coexisting states arise, as clearly seen in Fig. 5(b)). During relatively large time windows, ξ_1 stays at a zero state (partial amplitude death) while ξ_2 exhibits noisy oscillations near 0.6. In terms of perception, these intervals can be interpreted as the perception of one cube orientation only. For stronger noise ($\eta < 0.12$), the duration of the dead states increases as the noise intensity increases (Fig. 5(c)).

Thus, the increasing noise intensity gives preference to one of the attractors. In terms of perception this means that one percept dominates over the other.

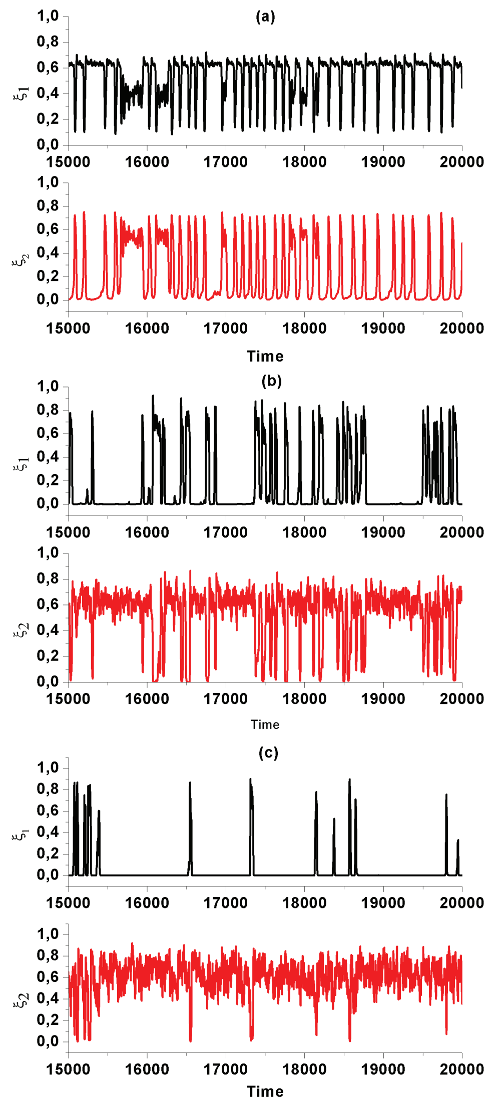

Figure 5. Time series of ξ_1, ξ_2 at $\alpha = 0.2$ in Region II in Fig. 3 for (a) $\eta = 0.01$, (b) $\eta = 0.09$, and (c) $\eta = 0.1$.

5 Recurrence Plot

It is seen from Fig. 5 that there exists a threshold value of noise amplitude $\eta = \eta_t$ at which percept ξ_1 prevails over percept ξ_2. To study the effect of noise on the system dynamics, we build the recurrence plot of percept ξ_1. The recurrence plot shows moments of time at which the phase space trajectory visits roughly the same area of phase space. In this paper, we consider the projection of the phase-space trajectory to the perception variable ξ_1. In other words, we construct a map of times when $\xi_1(i) \approx \xi_1(j)$. Such recurrence plot $R(i, j)$ is a time the perception takes the same value as before. Here, we record the recurrence/non-recurrence

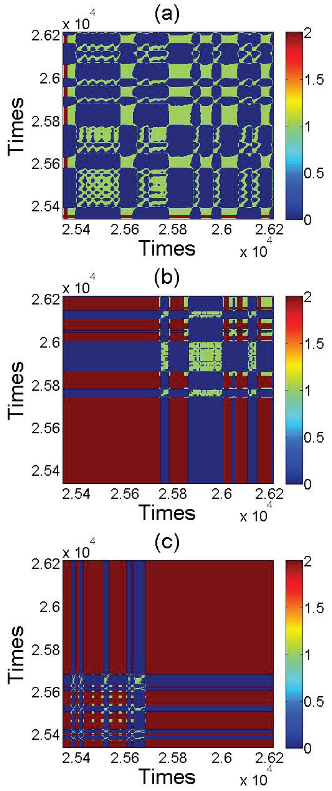

Figure 6. Recurrence plots of percept ξ_1 in Region II for noise intensities (a) $\eta = 0.012$, (b) $\eta = 0.096$, and (c) $\eta = 0.12$. The red dots ($R(i,j) = 2$) indicate dead states ($\xi_1 = 0$), whereas blue and green dots show the oscillatory states ($\xi_1 > 0$) with $R(i,j) = 0$ and $R(i,j) = 1$, respectively.

by ternary function

$$R(i,j) = \begin{cases} 2, & \text{if } \|\xi_1(i) - \xi_1(j))\| \le \varepsilon, \\ 1, & \text{if } \|\xi_1(i)\| > \varepsilon \text{ and } \|\xi_1(j)\| > \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\varepsilon = 0.01$. $R(i,j)$ takes 2 when $\xi_1 = 0$, i.e. we deal with intermittent amplitude death. Instead, the values $R(i,j) = 0$ and 1 mean that the system switches from one state to another ($\xi_1 > 0$).

Figure 6(a-c) shows the recurrent plots of ξ_1 for three different noise amplitudes $\eta = 0.012$, 0.084 and 0.12 at $\alpha = 0.2$ (Region II in Fig. 3). At small noise (Fig. 6(a)), there is a very small number of red dots ($R(i,j) = 2$) and a large number of green and blue dots. This means that there is no amplitude death. For stronger noise (Fig. 6(b,c)), one can clearly see an increase in the number of red dots, i.e., the regions corresponding to the dead states become larger.

6 Conclusion

Perception multistability has been studied using the synergetic model. Our results have confirmed the hypothesis that the alternation between competing percepts is associated with activation of different states of neural activity driven by the bias parameter. We have shown that additive noise induced the preference of one of the percepts. We have found that when the noise intensity exceeded a certain threshold value, one of the percepts disappeared during some time intervals, resulting to partial oscillation death. The length of the time windows during which one percepts had the preference over the other increased as the noise intensity was increased. This intermittent regime has been analyzed using the recurrence plot approach. We believe that the testing methodology proposed in this work can help in understanding pathological brain states because the states with weak and strong stability may contribute to brain pathologies.

Acknowledgements

This work was supported by the project "Equipamientos de los laboratorios de investigacion de los cuerpos academicos de CULAGOS con orientacin optoelectronica", R-0138/2016, Acuerdo RG /019/2016-UdeG, Mexico. D. A. M.-G. acknowledges support from Consejo Nacional de Ciencia y Tecnologia, Mexico, CONACYT (project No. 590537).

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