Time-Delayed Feedback control of Noise-Induced Dynamics in a Model of a Neural Network

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We study the effect of time-delayed feedback control [1] on the noise-induced dynamics [2] of an ensemble of N globally coupled excitable neural oscillators, each being modeled by the FitzHugh-Nagumo [3] system with independent Gaussian white noise source. Both coupling and feedback are introduced through the mean field, and the equations read

$$\begin{aligned} \epsilon_i \frac{dx_{2i-1}}{dt} &= x_{2i-1} - \frac{x_{2i-1}^3}{3} - x_{2i} + \gamma(M_x - x_{2i-1}) \\ \frac{dx_{2i}}{dt} &= x_{2i-1} - a_i + \sqrt{2T}\xi_i(t) + K(M_{y\tau} - M_y) \\ M_x &= \frac{1}{N}\sum_{i=1}^{N} x_{2i-1}, \quad M_y = \frac{1}{N}\sum_{i=1}^{N} x_{2i}, \quad M_{y\tau} = M_y(t-\tau), \end{aligned} \tag{1}$$

where $i = 1, 2, 3, \ldots, N$. Here, x_{2i-1} is associated with membrane potential, x_{2i} is a recovery variable, γ is coupling strength, T is noise intensity, τ is time delay, $M_{y\tau}$ is a delayed mean field, K is feedback strength, and $\xi_i(t)$ are independent sources of Gaussian white noise with $\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t-t')$ where δ_{ij} is Kronecker delta and $\delta(t-t')$ is Dirac-delta function.

The effect of delayed feedback is studied both analytically and numerically [4]. For the theoretical study we exploit a method of [5] introduced in [6], which allows one to write down a closed system of cumulant equations for the mean values M_x and M_y, variances D_x and D_y and covariance D_{xy} of the system variables defined as

$$\begin{gathered} M_x = \langle x_{2i-1} \rangle, \quad M_y = \langle x_{2i} \rangle, \quad D_x = \langle (x_{2i-1} - M_x)^2 \rangle, \\ D_y = \langle (x_{2i} - M_y)^2 \rangle, \quad D_{xy} = \langle (x_{2i-1} - M_x)(x_{2i} - M_y) \rangle, \end{gathered}$$

where $\langle\rangle$ denote averaging over the ensemble of oscillators. It is assumed that the third and higher order central moments are relatively small, which allows one to obtain the following system of cumulant equations with account of delayed feedback

$$\begin{aligned} \epsilon \frac{dM_x}{dt} &= M_x - {M_x}^3 - M_y - M_x D_x, \\ \frac{dM_y}{dt} &= M_x + a + K\big[M_{y\tau} - M_y\big], \\ \epsilon \frac{D_x}{dt} &= 2\big[D_x(1 - \gamma - {M_x}^2 - D_x) - D_{xy}\big], \\ \frac{dD_y}{dt} &= 2\big[D_{xy} + T\big], \\ \epsilon \frac{dD_{xy}}{dt} &= D_{xy}(1 - \gamma - {M_x}^2 - D_x) - D_y + \epsilon D_x. \end{aligned} \tag{2}$$

Among other effects, it was found out that time-delayed feedback in the mean field can effectively suppress the mean field dynamics at small values of K, provided that the coupling strength γ is close to its critical value at which the mean field starts to oscillate. This effect is illustrated in Fig. 1 where the suppression factor S [7] is shown against τ at three different value of K. At large K suppression is insignificant. This can be explained by analogy with the effect of time-delayed feedback control on a deterministically chaotic Rössler system [8].

Next, a more realistic situation is considered when different oscillators are subjected to noise with different intensities, and the qualitative picture remains same. In addition, we applied control force to a portion of the population of neurons only and studied its effect on the full population. The effect of delayed feedback in both these cases is qualitatively similar to that in the idealistic case considered above. So we infer that there is some universality in its action as applied to the collective behavior of coupled excitable units with noise-induced dynamics.

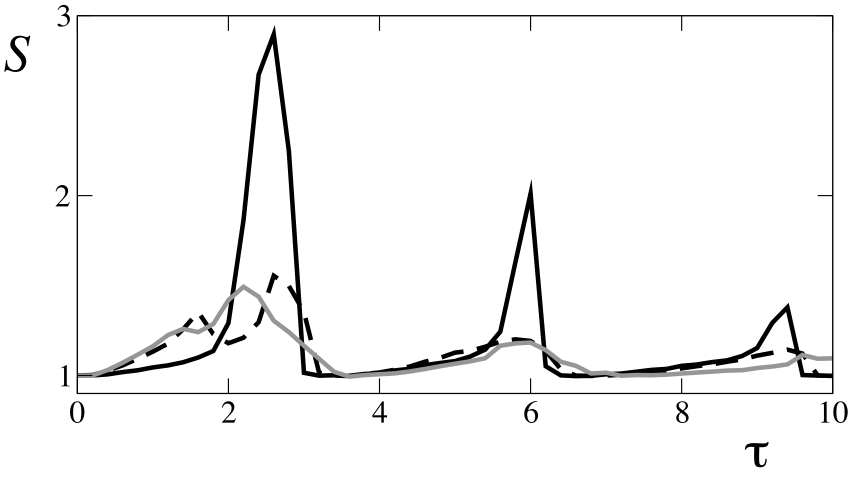

FIG. 1: Suppression factor S versus τ for three different values of feedback strength K in Eqs. (1): $K = 0.1$ (solid black line), $K = 0.5$ (dashed line) and $K = 1$ (grey line).

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