SWITCHING SPEED-GRADIENT CONTROL OF PASSAGE THROUGH RESONANCE FOR THE TWO-ROTOR VIBRATION UNIT

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ABSTRACT

The switching control algorithm for passing through resonance zone for the two-rotor vibration unit is proposed. The algorithm is based on speed-gradient method and leads to the significant reduction of the required level of the controlling torque. The dynamics of the overall hybrid system and its robustness against changes of spring stiffness, excentricity of rotors and damping is analyzed by computer simulation.

INTRODUCTION

Vibration units with unbalanced (eccentric) rotors are widely used in the industry. It is well known that the maximum power of driving motor is required during the spin-up mode (Blekhman 2000). The decrease of the spin-up power leads to decrease of nominal power and, therefore to decrease of the weight and the size of the motor. In order to obtain the desired mode of vibration it is necessary to control the rotor speed in a broad range including both pre-resonance and post-resonance regions. However, reduction of the motor power for systems with several degrees of freedom may increase the influence of resonance and lead to appearance of Sommerfeld phenomenon and capture (Andrievsky et al. 2001; Blekhman 2000). Sommerfeld phenomenon is caused by a limited power of motors. It may prevent the system from passing through resonance region and achieving the desired post-resonance value of rotor speed. It means that the problem of passage through resonance arises naturally. It is important for development of new generation of vibration equipment with improved technological characteristics. However, both industrial and laboratory vibration units are described by nonlinear models with many degrees of freedom and because of their complex dynamics the problem is hard to resolve.

The key idea to reduce the power of the unbalanced rotor is to swing the rotor during the spin-up period by feedback control. The control algorithms implementing this idea were proposed in (Kinsey et al. 1992; Kel'zon and Malinin 1992; Malinin and Pervozvanskii 1993; Tomchina and Nechaev 1999, Fradkov and Tomchin 2004). In (Kel'zon and Malinin 1992) and (Malinin and Pervozvanskii 1993) the optimal control method was used leading to complicated and not sufficiently robust controller. Kinsey et. al. (1992) proposed the algorithm based on derivation of the averaged controlled plant equation which is labor-consuming. The algorithms of (Tomchina and Nechaev 1999; Fradkov and Tomchin 2004) are based on the speed-gradient method (Fradkov 1990; Fradkov et al. 1999) and energy-based goal functions. As it was shown in (Fradkov 1996) the speed-gradient algorithms for energy control of conservative systems allow to achieve an arbitrary energy level by means of arbitrarily small level of control power (so called swingability property). Using this approach for systems with losses allows to spend energy only to compensate the losses, and to reduce the power of driving motor significantly. An additional requirement of achieving fast passage through the resonance zone by electrical correction means is also important (Tomchina and Nechaev 1999). In the paper by Tomchina and Nechaev (1999) only the case of one-dimensional motion of the rotor axis was considered. The case of plane motion was studied by Malinin and Pervozvanskii (1993), who designed controller using optimal control method and Fradkov and Tomchin (2004) who proposed a speed-gradient based solution for one-rotor vibration unit.

In this paper the problem of controlling two-rotor vibration unit (Andrievsky et al. 2001) is solved by means of speed-gradient method. The proposed algorithm allows to significantly reduce the required level of the controlling torque. The efficiency of the algorithm is investigated by means of simulation.

PROBLEM STATEMENT

Consider the Two-Rotor Mechatronic Vibration Unit developed in St. Petersburg, see (Blekhman et al. 1999; Andrievsky et al. 2001). The unit consists of three blocks: electromechanical double-rotor bench (Fig. 1,2), electronic transducer amplifier and PC controller.

The electromechanical part contains a pair of unbalanced vibration actuators. The actuators are mounted on the vibration isolated carrier. Each actuator contains the DC electrical motor, Cardan joint and unbalanced rotor. Due to the rotation of the unbalanced rotors the centrifugal forces appear. They can be combined in controlled manner producing a variety of body oscillations. The unit is equipped with eight sensors. The sensors generate signals of the two rotors angular position and speed and the body translations. Details and operating characteristics of the unit are described in (Blekhman et al. 1999; Andrievsky et al, 2001).

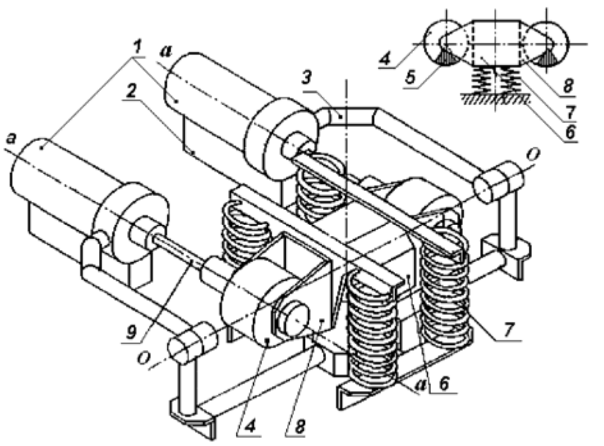

Figure 1: Schematics of the St.Petersburg Two-Rotor Mechatronic Vibration Unit: 1- DC motors; 2- sensors; 3- frame; 4- rotors; 5- eccentrics; 6- vibrating body (platform); 7- springs; 8- forks of vibroactuators; 9- Cardan shafts

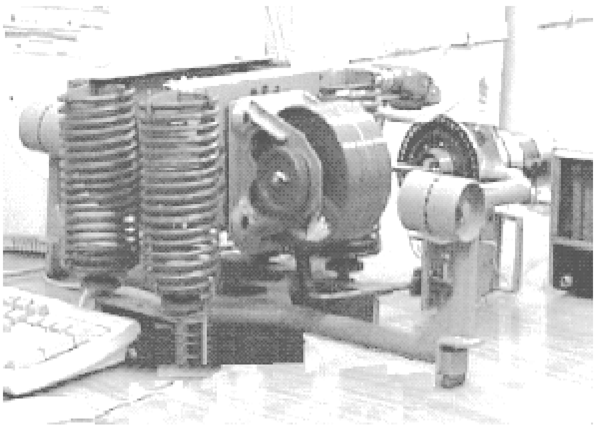

Figure 2: Fragment of the St.Petersburg Two-Rotor Mechatronic Vibration Unit

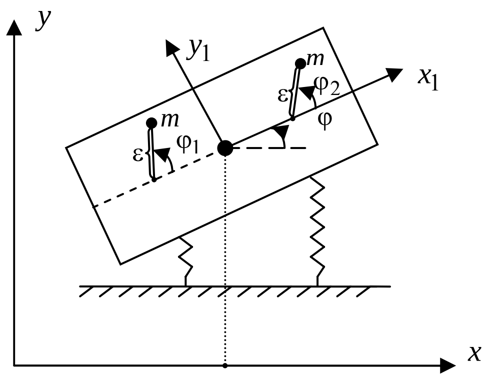

Figure 3: Frames and variables for the modeling of the Two-Rotor Vibration Unit

Using Lagrangian approach, its dynamics are modelled by the following system of differential equations (Andrievsky et al. 2001):

$$\begin{aligned}
&J\ddot{\varphi}_1 + k_\varphi \dot{\varphi}_1 + m\varepsilon g \sin\varphi_1 = \\
&\quad = m\varepsilon(\ddot{x}\cos\varphi_1 + \ddot{y}\sin\varphi_1) + u_1(t) \\
&J\ddot{\varphi}_2 + k_\varphi \dot{\varphi}_2 + m\varepsilon g \sin\varphi_2 = \\
&\quad = m\varepsilon(\ddot{x}\cos\varphi_2 + \ddot{y}\sin\varphi_2) + u_2(t) \\
&(2m+M)\ddot{x} + k_x\dot{x} + c_x x = m\varepsilon(\ddot{\varphi}_1\cos\varphi_1 + \\
&\quad + \ddot{\varphi}_2\cos\varphi_2 - \dot{\varphi}_1^2\sin\varphi_1 - \dot{\varphi}_2^2\sin\varphi_2) \\
&(2m+M)\ddot{y} + k_y\dot{y} + cy + (2m+M)g = m\varepsilon \times \\
&\times(\ddot{\varphi}_1\sin\varphi_1 + \ddot{\varphi}_2\sin\varphi_2 + \dot{\varphi}_1^2\cos\varphi_1 + \dot{\varphi}_2^2\cos\varphi_2),
\end{aligned} \qquad (1)$$

where φ_1, φ_2 – rotor angles, x,y – coordinates of the system center of mass, $u_1(t), u_2(t)$ – control actions (rotating torque of motors), J - moment of inertia of rotors, m - mass of a rotor, M – mass of a platform, ε - eccentricity of the rotor centers of mass, c, c_x - stiffness, k_φ, k_x, k_y - damping factors (see Fig.3).

It is well-known (Blekhman 2000; Kononenko 1964), that the "capture" of angular velocity of a rotor (Sommerfeld phenomenon) may take place in the near-resonance zone. The capture phenomenon happens when the level of constant control action $u_i(t) \equiv (-1)^i M_1$, i=1,2 is small. If the level of constant control action $u_1(t) = -u_2(t) \equiv M_1$ is higher than a threshold, the system passes the resonance zone. Simulation results for system (1) are shown in Fig. 4 for the parameter values: J = 0.014 [kg·m^2], m = 1.5 [kg], M=9 [kg], ε = 0.04 [m], k_φ = 0.01 [J·s], $k_x = k_y = 5$ [kg/s], $c = 5300$ [N/m], $c_x = 1300$ [N/m] and the constant control action M_1 = 0.65 [N·m] (inner curves, capture) and M_1 = 0.66 [N·m] (outer curves, passage).

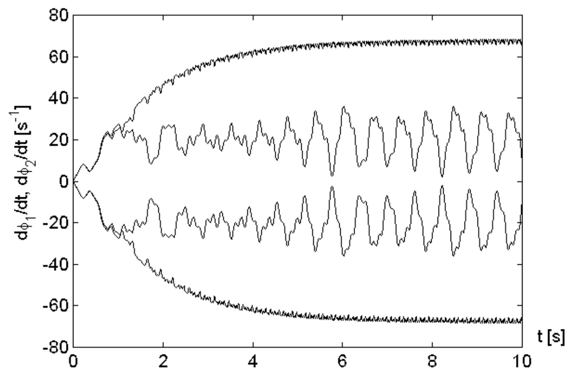

Figure 4: Conventional control, $u_i(t) \equiv (-1)^i M_1$, M_1 = 0.65 [N·m] (inner curves, capture) and M_1 = 0.66 [N·m] (outer curves, passage).

The problem is to design the control algorithm $u = \boldsymbol{U}(z)$, providing the spin-up of unbalanced rotor until the system passes through resonance zone, where $z = [x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2]^{\mathrm{T}}$ - state vector of the control plant. It is assumed that the level of control signal is restricted and does not allow the system to pass through resonance when the control signal is constant.

DESIGN OF CONTROL ALGORITHM

To design the control algorithm we use the speed-gradient method (Fradkov 1990; Fradkov et. al. 1999). A brief description of the method is given below.
Let the control objective be the reduction of the current value of some goal (objective) function $Q(x(t),t)$. However, for continuous-time systems the value $Q(x)$ does not depend directly on control u. Instead of decrease of $Q(x)$, the secondary control goal is posed as decrease of the speed $\dot{Q}(x)=\frac{\partial Q}{\partial x}F(x,u)$, and according to the *speed-gradient* (SG) method the control $u\,(t)$ should be changed along the gradient in u of the speed $\dot{Q}(x)$. The general SG algorithm has the form

$$u=-\Psi\left[\nabla_u \dot{Q}(x,u)\right], \qquad (2)$$

where $\Psi(z)$ is vector-function forming acute angle with its argument z. For affine controlled systems $\dot{x}=f(x)+g(x)u$ algorithm (2) is simplified to:

$$u=-\Psi\left[g(x)^T\nabla Q(x)\right].$$

Special cases are the proportional SG-algorithm

$$u=-\Gamma\nabla_u \dot{Q}(x,u),$$

where Γ is a positive-definite matrix, and the relay SG-algorithm

$$u=-\Gamma\ sign\left[\nabla_u \dot{Q}(x,u)\right].$$

Another version of the SG-algorithm is its differential form

$$\dot{u}=-\Gamma\nabla_u \dot{Q}(x,u).$$

Justification of the SG-method is based on a Lyapunov function V decreasing along trajectories of the closed-loop system. The Lyapunov function is constructed from the goal function: $V(x)=Q(x)$ for finite form algorithms and $V(x,u)=Q(x)+0.5(u-u_*)^T\Gamma^{-1}(u-u_*)$ for differential form algorithm, where u_* is the desired (ideal) value of the control.
At this stage we suppose that the control plant is conservative, i.e. the friction equals to zero. The control goal is formalized as follows: To find controlling function $u(t)$ providing the goal equality $H(x,\dot{x},y,\dot{y},\varphi_1,\dot{\varphi}_1,\varphi_2,\dot{\varphi}_2)=H^*$, where $H(t)$ is a current energy, H^* is the given energy level corresponding to the desired average rotation speed (aligorithm parameter). Then it is possible to choose the goal functional as follows:

$$Q(z)=1/2\left(H(z)-H^*\right)^2,\ z=\left[x,\dot{x},y,\dot{y},\varphi_1,\dot{\varphi}_1,\varphi_2,\dot{\varphi}_2\right]^{\mathrm{T}}.$$

For the controller design purposes it is convenient to use Hamiltonian form

$$\dot{q}=\frac{\partial H}{\partial p},\quad \dot{p}=-\frac{\partial H}{\partial q}+Bu,$$

where $p=p(t)$ is the vector of generalized momenta, $q=[\varphi_1,\varphi_2,x,y]^T$ are generalized coordinates, $H=H(p,q)$ is the Hamiltonian function (total energy of the system), $B=[1,1,0,0]^T$. Then

$$\dot{Q}(z)=\left(H-H^*\right)\left(\dot{\varphi}_1 u_1+\dot{\varphi}_2 u_2\right).$$

and the speed-gradient method applies. One of the standard forms of speed-gradient algorithm is the "relay" one:

$$u_i=-M_0\ sign\left[\left(H-H^*\right)\dot{\varphi}_i\right],\quad i=1,2. \qquad (3)$$

It is worth noticing that the algorithm (3) was designed neglecting the system dynamics. In case of multi-DOF oscillatory system such a design is not sufficient because of interaction between rotors and platform, and because of the Sommerfeld phenomenon. It leads to appearance of fast oscillating motion that make difficult passing through resonance zone. In (Fradkov and Tomchin 2004) new control algorithms were proposed facilitating passage through resonance by means of introducing additional low pass filter. Another pecularity of the algorithm (3) is large variability of the debalance angular velocity because of changes of potential energy due to gravity. To improve the system performance it is suggested to introduce switching into the control algorithm. Then the algorithm takes the form:

$$\begin{cases} u_i=\begin{cases}(-1)^i M_1,\ if\ \gamma=1,, \\ (-1)^i M_1,\ if\ \gamma=0\ \&\ \left(H-H^*\right)\left(\dot{\varphi}_i-\psi_i\right)>0 \\ 0, \quad else,\end{cases} \\ \gamma(t)=\max\limits_{0\le\tau<t}\ sgn\,(H(\tau)-H^*) \\ T_\psi\dot{\psi}_i=-\psi_i+\dot{\varphi}_i, \quad i=1,2, \end{cases} \qquad (4)$$

where $\psi_i(t)$ - filtered variables, $T_\psi>0, T_\psi=\text{const}$.
The value of T_ψ (time constant of the angular velocity filters) should be more then the period of the resonant oscillations. At the same time, if the value of T_ψ is too high, the algorithm works too slowly.

COMPUTER SIMULATION RESULTS

The designed hybrid control system was numerically investigated to analyze the efficiency of the proposed algorithm. Numerical integration was made in MATLAB environment by means of Runge-Kutta method of second order. The choice of integration

method takes into account nonsmoothness of the system model. The value of the fixed step equal to 0.000125 [sec] was chosen so as the relative simulation error does not exceed 5%.
The nominal values of system parameters were chosen as follows: $J = 0.014$ [kg·m^2], $m = 1.5$ [kg], M=9 [kg], $\varepsilon = 0.04$ [m], $k_\varphi = 0.01$ [J·s], $k_x = k_y = 5$ [kg/s], $c = 5300$ [N/m], $c_x = 1300$ [N/m].
The value of the rotating torque M_2 of a motors, which allows system to pass the resonance zone with the proposed algorithm, but not allows system to pass the resonance zone for any $M < M_2$ was calculated.

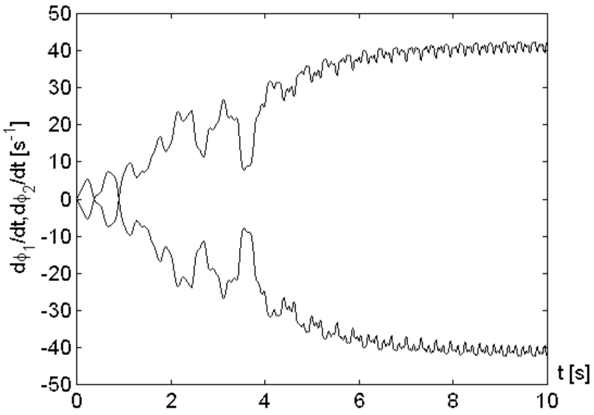

Figure 5: Controlled passage through resonance, $M_2 = 0.42$ [N·m], $T_\psi = 0.35$ [s]

Next step was to investigate behavior of the closed loop system under breaking symmetry of initial conditions. The following simulation results are obtained. In Fig. 6 time histories of rotor angular velocities are shown for initial conditions $\varphi_1(0) = 0.1$, $\varphi_2(0) = 0$.

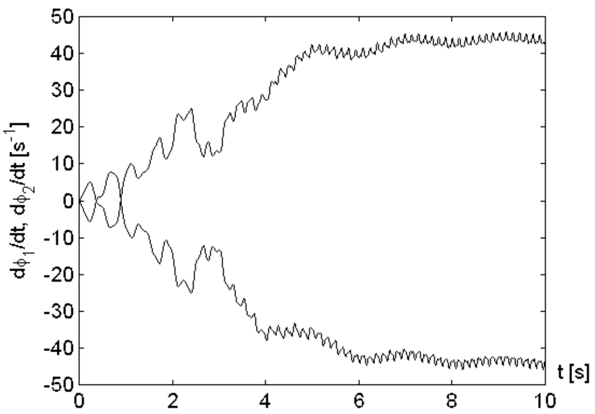

Figure 6: Dynamics of rotor angular velocities for $\varphi_1(0) = 0.1$, $\varphi_2(0) = 0$.

In Fig. 7 time histories of rotor angular velocities are shown for initial conditions $x(0) = 0.01$. It is seen that time histories in the case of passage through resonance are almost the same.

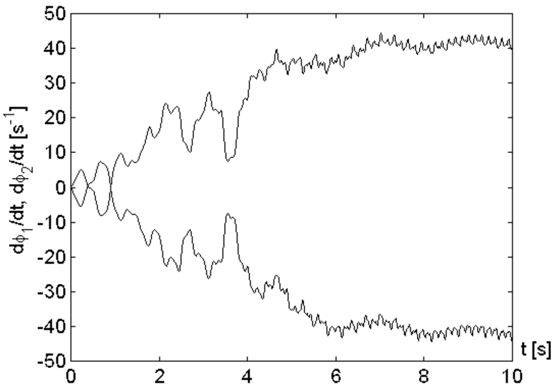

Figure 7: Dynamics of rotor angular velocities for $x(0) = 0.01$.

Further the designed control algorithm was numerically investigated to analyze the efficiency of the proposed algorithm for various values of plant and algorithm parameters.
The torques M_1 and M_2 were calculated for every series of experiments. M_1 is the value of the rotating torque of a motor, which allows system to pass the resonance zone for $u(t) \equiv M_1$, but not allows system to pass the resonance zone for any $M_0 < M_1$. M_2 is the value of the rotating torque of a motor, which allows the system to pass through the resonance zone for relay control algorithm (4), but not allows it for any $M_0 < M_2$.
Firstly, the influence of the stiffness c on system dynamics was investigated for nominal values of other plant parameters. The dependence of the minimal value of control action, allowing the passage through resonance, on stiffness c is shown in Fig. 8. It is clear that the efficiency is small when c is small. However value of rotating torque M_1 can be reduced in 2 times if the shaft torsional stiffness c increases.

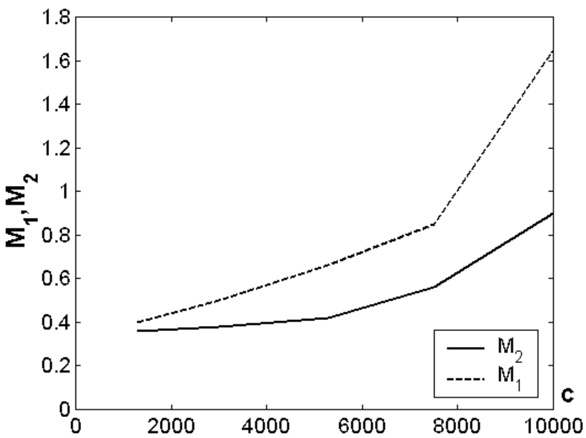

Figure 8: The influence of the stiffness c on system dynamics

Further the influence of the damping factor k_φ on system dynamics was investigated for nominal values of other plant parameters. The dependences of the torques M_1 and M_2 on the damping factor k_φ are shown in

Fig.9. It is seen that dependence on k_φ is almost linear both for the constant control action (dotted line) and relay control algorithm (solid line). It is clear that the efficiency of the algorithm is high for different values of k_φ.

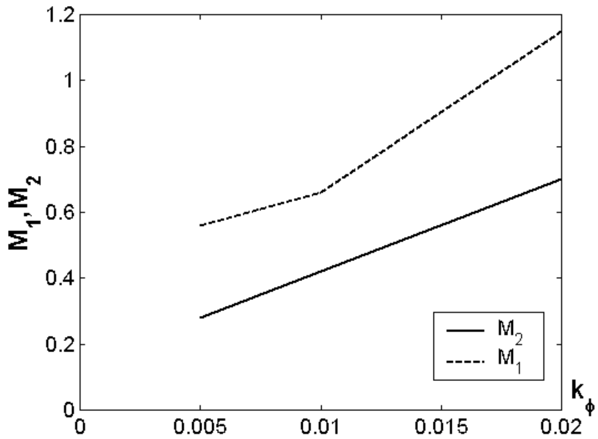

Figure 9: The influence of the damping factor k_φ on system dynamics

Finally, the influence of the eccentricity of a rotor ε on system dynamics was investigated for nominal values of other plant parameters. The dependence of the torques M_1 and M_2 on the eccentricity ε are shown in Fig. 10. It is seen that dependence on ε is almost linear both for the constant control action (dotted line) and relay control algorithm (solid line). It is clear that the efficiency is higher when ε is higher and the value of rotating torque can be reduced in 2 times.

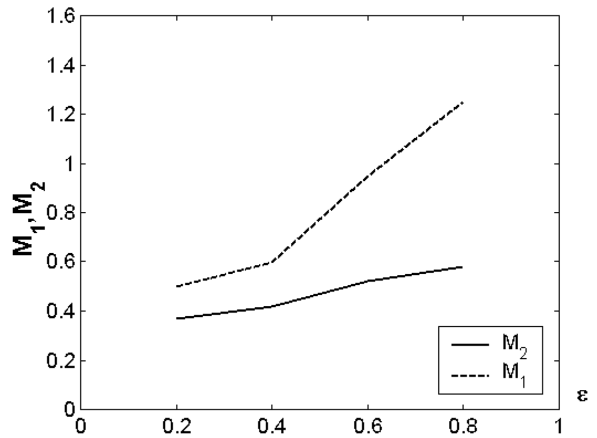

Figure 10: The influence of the eccentricity ε on system dynamics

CONCLUSION

New switching control algorithm for passing through resonance of two-rotor vibration units is proposed based on speed-gradient method. Computer simulations show that the use of the proposed algorithm significantly decreases the level of the controlling torque required to pass through the resonance zone. The algorithm is simple and has only two design parameters, though the system possesses complex behavior. The system performance has low sensitivity with respect to breaking the symmetry of initial conditions. The algorithm efficiency is significant for broad range of plant parameters.

Future research will be devoted to further examination of robustness properties of the proposed system and its comparison with adaptive approaches.

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