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SPATIAL PINNING CONTROL OF VICSEK'S AGENTS

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Abstract
In this work we study a system of mobile agents that move in an anisotropic space and interact according to the Vicsek model. In particular, the space is divided in two regions: in the first one, agents obey to the traditional Vicsek model, while in the second one, called control region, a motion control law is added. The control law forces the pinned agents, that are the agents moving in the control region, to follow a criterion mediating between the tendency to adopt the average direction of neighboring agents, and that to follow an imposed preferential direction. We show that, for low and medium levels of noise in the system, the control law is effective to drive the system towards a global ordered state, while, for high levels of noise, a strong control action leads to a configuration, for some aspects paradoxical, where all the agents tend to avoid the control region and occupy for most of the time the remaining part of the space.

Key words
Control of complex systems, Vicsek's model.

1 Introduction

The Vicsek model [Vicsek *et al.*, 1995] is probably one of the most studied models for collective behavior [Sumpter, 2010]. It exhibits, in fact, an emergent self-organized global behavior through a simple rule of local interaction among the agents that compound the system. The model, that represents individuals or agents as self-propelled particles moving on a planar space, assumes the only rule that, at each time step, each agent moves with constant absolute velocity and with direction of motion equal to the average of the directions of the neighboring agents, plus a random perturbation. The Vicsek model has been generalized in different ways in order to incorporate several additional features, many of them fundamental for a realistic description of swarm/group dynamics, not accounted for in the original model [Couzin *et al.*, 2002; Buscarino *et al.*, 2009; Couzin *et al.*, 2005; Couzin *et al.*, 2006; Chaté *et al.*, 2008; Buscarino *et al.*, 2006; Buscarino *et al.*, 2007; Wang *et al.*, 2013; Romenskyy *et al.*, 2013; Meschede *et al.*, 2013; Bhattacharya and Vicsek, 2010; Newlands and Porcelli *et al.*, 2008; Ballerini *et al.*, 2008].

In its original formulation the Vicsek model consists of N agents moving on a square plane of linear size L with periodic boundary conditions, according to:

$$\begin{cases} \mathbf{x}_i(t+\Delta \mathrm{t}) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta \mathrm{t} \\ \theta_i(t+\Delta \mathrm{t}) = \langle \theta_i(t) \rangle_r + \Delta\theta_i \end{cases} \quad (1)$$

where $\mathbf{x}_i(t)$ is the position in the plane of the $i-$th agent at time t; $\mathbf{v}_i(t) = v(\cos(\theta_i), \sin(\theta_i(t))$ is its velocity at time t (v is the velocity modulus and $\theta \in [0, 2\pi[$ the agent heading); Δt is the discrete step size. The variable $\Delta\theta_i$ is the noise term, generated at random at each time step by drawing a number with uniform probability from the interval $[-\frac{\eta}{2}, \frac{\eta}{2}]$, so that η represents the noise level. The operation $\langle \theta_i(t) \rangle_r$ is the computation of the average direction of agents within a neighborhood of the $i-$th agent, defined as a disc of radius r. This term, calculated as

$$\langle \theta_i(t) \rangle_r = \arctan\left(\frac{\langle \sin\theta(t) \rangle_r}{\langle \cos\theta(t) \rangle_r} \right), \quad (2)$$

implements the local interaction rule among agents, able to modify the direction of their velocity without affecting its modulus. A modified version of the model that also acts on the modulus of the velocity was introduced by Couzin and co-workers [Couzin *et al.*, 2002]. Considering that individuals in living groups tend to keep a vital space free of neighbors and have different mechanisms of adjustment of their motion direction as function of the distance from their neighbors, the model includes rules for repulsion, attraction and orientation, and is able to reproduce different kinds of collective behaviors observed in animal groups such as swarming, torus formation, or parallel formation [Buscarino *et al.*, 2009].

A further extension of the model was formulated in [Couzin *et al.*, 2005; Couzin *et al.*, 2006] to account for the experimental observations of groups of animals whose direction is driven by a minority of individuals. The presence of experienced (informed) individuals is included in the model by assuming that a subset of individuals has a preferential direction of motion, and defining a motion updating rule that mixes the social criteria (*i.e.,* repulsion, orientation and attraction of neighboring agents) with the individualistic one, that is, the motion towards the preferential direction.

Finally, among other variations of the Vicsek model we mention the inclusion of particle's symmetry changes, mechanisms of local cohesion, and the characterization of the fluid in which the particles move [Chaté *et al.*, 2008]; the consideration of interactions which are mostly local, yet with a small percentage of long-range interactions [Buscarino *et al.*, 2006; Buscarino *et al.*, 2007]; the exponential weighting of neighbor connections, and the limitedness of the visual fields [Wang *et al.*, 2013]; the introduction of several alignment mechanisms [Romenskyy *et al.*, 2013; Meschede *et al.*, 2013]; the description of processes of collective decision making in short time (such as synchronized landing of bird flocks) [Bhattacharya and Vicsek, 2010]; and the characterization of animal group motion, made possible thanks to technological advances in image capturing and processing [Newlands and Porcelli *et al.*, 2008; Ballerini *et al.*, 2008].

It is well known that different characteristics of the space induce difference in the locomotion patterns of animal groups. For example, the swimming activity of some fish species is influenced by the water temperature, while flight routes of pigeons tend to align to the earth geomagnetic field intensity during homing [Dennis *et al.*, 2007]. Thus, the study of models for collective behavior relying on anisotropic properties of the space may provide useful insights in collective dynamics. Inspired by this evidence, we define the concept of *spatial pinning control.* The idea underlying pinning control is that the control of high dimensional systems can be achieved by controlling a subset of their state variables. The variables selected for control are called the pinned ones, and the pinned set can be either time-invariant [Porfiri and diBernardo, 2008] or time-varying [Porfiri and Fiorilli, 2009]. Several criteria have been defined for the selection of the pinning set. In this work, we assume that the ability of pinning a certain set of variables is related to the spatial characteristics of the environment. That is, a variable is pinned if the agent that owns it lays on a specific region of the available space. A similar framework was introduced in [Frasca *et al.*, 2012] to study synchronization in a system of agents moving in an anisotropic space. Achieving coordination of motion through spatial pinning control is of twofold interest. Firstly, it is a valuable extension of the Vicsek model that incorporates the concept of anisotropic space, extending the modeling capability for the description of natural phenomena. Secondly, the definition of control paradigms which exploit privileged limited zones of the control space may constitute a valuable tool for the realization of distributed control strategies for mobile robots, commanded through wireless systems whose effect is present only in limited portions of the space.

With this in mind, in this work we investigate a system of self-propelled particles where the rule for direction updating depends on their position on the plane. We assume that the plane where the agents move contains a control region where agents can receive a particular control signal that tends to impose a privileged direction in their motion (referring to the model in [Couzin *et al.*, 2005; Couzin *et al.*, 2006], this is equivalent to having a time-varying set of informed individuals). On the other hand, agents external to the control region have no preferential direction and update their direction according to the classical Vicsek model. In particular, we show that, under particular conditions, by acting only on the parameters of the control area, a global control of the system can be attained, that is, agents will move along the preferential direction even outside of the control area. Moreover, we highlight the role of the noise in the global control performance of the model.

The rest of the paper is organized as follows: in Section 2 the model is discussed; in Section 3 numerical results are illustrated and discussed; in Section 4 our conclusions are drawn.

2 Model

We consider N moving agents distributed on a planar space $\Gamma = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq L, 0 \leq y \leq L\}$, with periodic boundary conditions, and define a further region $\Gamma_c \subseteq \Gamma$ as $\Gamma_c = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq L_c, 0 \leq y \leq L_c\}$ with $L_c \leq L$. We refer to Γ_c as the control region. In fact, we assume that the system's agents that, at time t, are located in the control region Γ_c, are forced by an external driving signal to follow a preferential direction $\overline{\theta}$. On the other hand, the command $\overline{\theta}$ is absent outside of the control region. In analogy with the mechanisms of informed individuals adopted in [Couzin *et al.*, 2005; Couzin *et al.*, 2006], a weighted mix between a social criterion (that is, the average direction of the Vicsek model) and the preferential direction is applied.

As for the Vicsek model in Eqs. (1)-(2), we in-

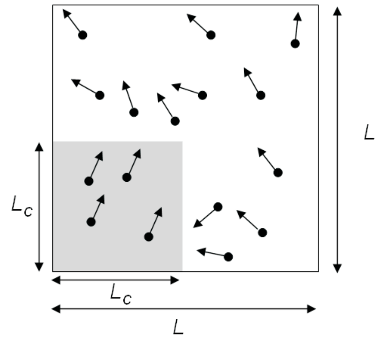

Figure 1. Scheme illustrating the spatial pinning control of Vicsek's agents.

dicate the position of the $i-$th agent at time t as $\mathbf{x}_i(t) = \left[x_i\ y_i\right]^T$, and its velocity as $\mathbf{v}_i(t) = v(\cos(\theta_i), \sin(\theta_i(t))$, where v is the velocity modulus, which is constant in time and identical for every agent. To incorporate the definition of the control law discussed above, the motion update rule is modified as follows:

$$\begin{cases} \mathbf{x}_i(t+\Delta\mathrm{t}) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta\mathrm{t} \\ \theta_i(t+\Delta\mathrm{t}) = (1-w_i(t))\left(\langle\theta_i(t)\rangle_r + \Delta\theta_i\right) + w_i(t)\overline{\theta} \end{cases} \quad (3)$$

where $\overline{\theta} \in [0, 2\pi[$ represents the preferential direction $w_i(t) \in [0,1]$ is the weight between the social criterion and the preferential direction. Since only agents in Γ_c are controlled towards the preferential direction, $w_i(t)$ is given by

$$w_i(t) = \begin{cases} \overline{w} & \text{if } \mathbf{x}_i(t) \in \Gamma_c \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

It is evident that the effect of the control region on the global system performance depends on the magnitude of the control action $\overline{w}$, and on the linear size of the control region, L_c. Thus, these two variables will be considered as system parameters. The considered framework is schematically illustrated in Fig. 1.

To characterize the system behavior, two order parameters are considered. The first one quantifies the capability for the agents of moving in the same direction. This is the average normalized velocity calculated as:

$$v_a = \langle \frac{1}{Nv} \left| \sum_{i=1}^{N} \mathbf{v}_i \right| \rangle_t \quad (5)$$

where $\langle\rangle_t$ denotes average over time. Clearly, $v_a \in [0,1]$. The value $v_a = 0$ indicates that the agents are not coordinated, while $v_a = 1$ indicate that all the agents are moving in the same direction.

The second one represents the capability for the agents of following the preferential direction, and is computed as [Couzin *et al.*, 2005; Couzin *et al.*, 2006]:

$$v_d = \langle 1 - \frac{\angle(\sum_{i=1}^{N} \frac{1}{N} \frac{\mathbf{v}_i(t)}{|\mathbf{v}_j(t)|} - \mathbf{g})}{\pi} \rangle_t \quad (6)$$

where $\angle$ represents the direction of the resulting vector and $\mathbf{g}$ is the unit vector pointing towards the preferential direction $\overline{\theta}$. Clearly, $v_d \in [0,1]$ represents the agreement between the average direction of the ensemble and the preferential one. In fact, v_d is closer to 1 as long as all the agents tend to align to the preferential direction, and, on the contrary, it approaches zero when all the agents tend to move opposite to the preferential direction.

3 Results

Our analysis mainly focuses on the system behavior with respect to different values of the noise level η, and to the two parameters defining the control law, L_c and $\overline{w}$. Even though other parameters are important for the system characterization, such as the number of agents N, their density $\rho = \frac{N}{L^2}$, the velocity modulus v, the interaction radius r, their role is more related to the general characteristics of the Vicsek model, rather than to the peculiarities of the proposed approach, and has been extensively characterized in the literature [Vicsek *et al.*, 1995; Bhattacharya and Vicsek, 2010]. Thus, their effect is not explicitly discussed in our work, and in the following we keep them constant to the following values: $N = 100$, $v = 0.03$, $r = 1$, $\rho = 4$ (and so $L = 5$). Without any lack of generality the preferential direction has been fixed as $\overline{\theta} = \frac{\pi}{4}$, and the time step to $\Delta t = 1$.

Figs. 2 and 3 show the order parameters v_a and v_d with respect to L_c and $\overline{w}$ for different values of the noise level η. We observe that for low values of the noise level, global control can be attained also at low values of L_c and $\overline{w}$, while for increasing values of the noise level global control requires large values of both L_c and $\overline{w}$. So, in the case of low noise level, further illustrated in Fig. 4, where several snapshots of a simulation are shown, a small control region suffices to control the global behavior of the system. In the limit case of zero noise ($\eta = 0$), illustrated in Fig. 5, in the presence of a small control region it is interesting to note that, first, the agents reach a coordinated state (an ordered motion of all the particles towards the same spontaneously chosen direction) as the result of the spontaneous organization of the Vicsek model and, then, they align towards the preferential direction $\overline{\theta}$. Thus, a low noise level does not prevent the achievement of a global controlled state, originating from the interplay between the self-organization properties of the Vicsek model and the application of a weak control law.

We next consider intermediate values of the noise level ($\eta = \{2, 3\}$). In this case, it is still possible to obtain high values of v_a and v_d, although not equal to one

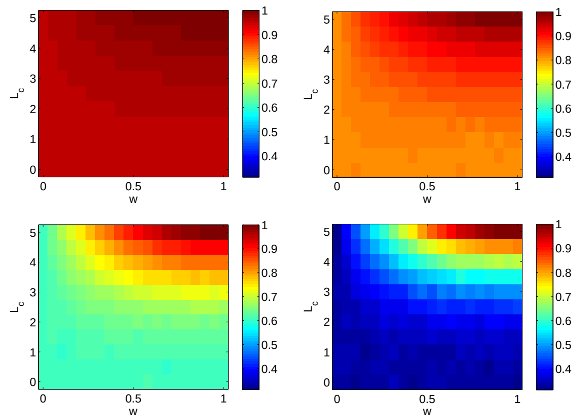

Figure 2. Order parameter v_a with respect to L_c and $\overline{w}$ for different values of the noise level η: (a) $\eta = 1$; (b) $\eta = 2$; (c) $\eta = 3$; (d) $\eta = 4$. Results are averaged over 20 different realizations.

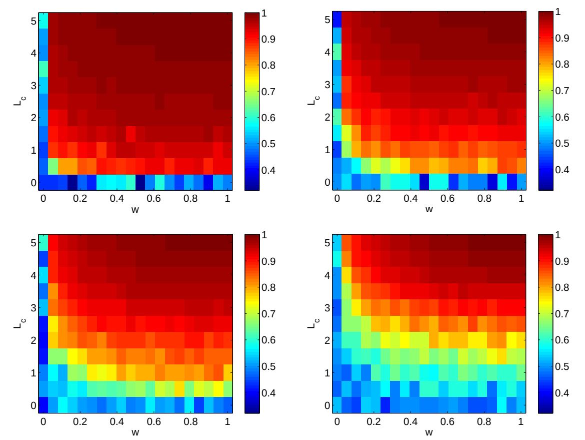

Figure 3. Order parameter v_d with respect to L_c and $\overline{w}$ for different values of the noise level η: (a) $\eta = 1$; (b) $\eta = 2$; (c) $\eta = 3$; (d) $\eta = 4$. Results are averaged over 20 different realizations.

(that is, a quite ordered motion towards the preferential direction), but this requires large values of both L_c and $\overline{w}$. This is evident in Fig. 6, where we compare the behavior of the system, under the same level of noise ($\eta = 2$), for three case studies obtained by choosing different sets of parameters of the control law. When a strong control law is applied (relatively high values of L_c and $\overline{w}$) the system shows a coordinated behavior (Fig. 6(a)), while, when one of the two conditions (large area, high value of $\overline{w}$) does not hold, then global control is not attained (in Fig. 6(b) the control area is too small, despite the high value of $\overline{w}$, while in Fig. 6(c) $\overline{w}$ is too small, despite the large control area).

For high values of the noise level, we observe that control is not attained even in the presence of a quite large control region and high values of $\overline{w}$. In Fig. 2(d) v_a becomes close to the unit only when $L_c \to L$ and $\overline{w} \to 1$, which is trivial, since all the area is controlled. Apart from this limit condition, we observe that, when the control region is large and $\overline{w}$ is high, a low value of the order parameter v_a indicates that the agents tend to

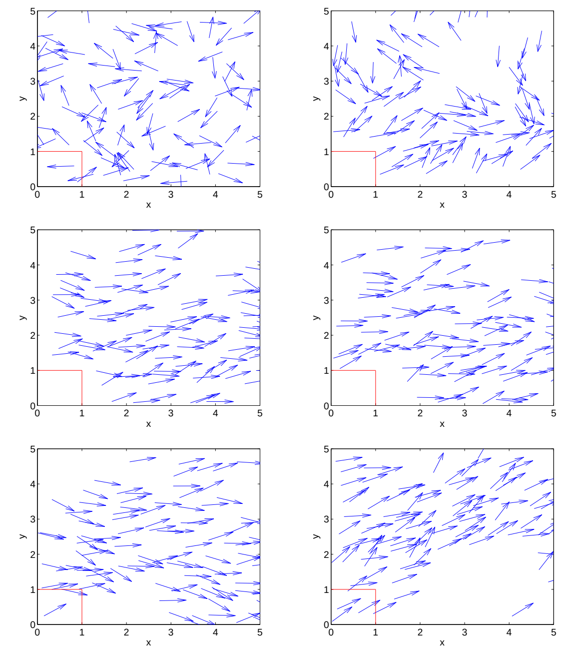

Figure 4. Snapshots of a simulation of $N = 100$ Vicsek's agents with spatial pinning control for $\rho = 4$, $L_c = 1$, $\overline{w} = 0.5$ and $\eta = 1$: (a) $t = 0$; (b) $t = 10$; (c) $t = 25$; (d) $t = 35$; (e) $t = 72$; (f) $t = 150$.

stay away from the control region. In fact, agents in the control region are forced to follow an ordered motion towards the preferential direction, while their motion is not ordered outside of the control region. In this case, it can be noted that when agents approach the control region, they escape very soon from it, since they tend to follow a straight trajectory in the preferential direction, and tend to spend more time in the external region, since their escape time is ruled by a Brownian motion. This is evident in Fig. 7, where the control region is for most of the time free of agents. This emergent behavior is due to the fact that, under these conditions, agents are subjected to two strong and opposite forces: the control action and the tendency towards a disordered motion, as imposed by setting a large value of η. To mediate between these two forces, agents self-organize to occupy a region of the space which allows them to avoid the control action. Under these conditions, therefore, the only effect of the control signal is to deplete a region of the space, without any success in inducing an ordered motion of agents.

We now provide a mathematical argument to explain the behavior of the system observed in the previous simulations. To this aim, we study the control of a set of agents under some ideal conditions. The first assumption is that each agent knows the heading of all other ones, that is, the radius r is large enough to cover the whole space, so that $\langle\theta_i(t)\rangle_r$ is identical for each agent, that is, $\langle\theta_i(t)\rangle_r = \Theta$, $\forall i$. Secondly, we assume that agents move fast enough for considering that, in average and at each time instant, the control action is applied to a fraction of agents proportional to the ra-

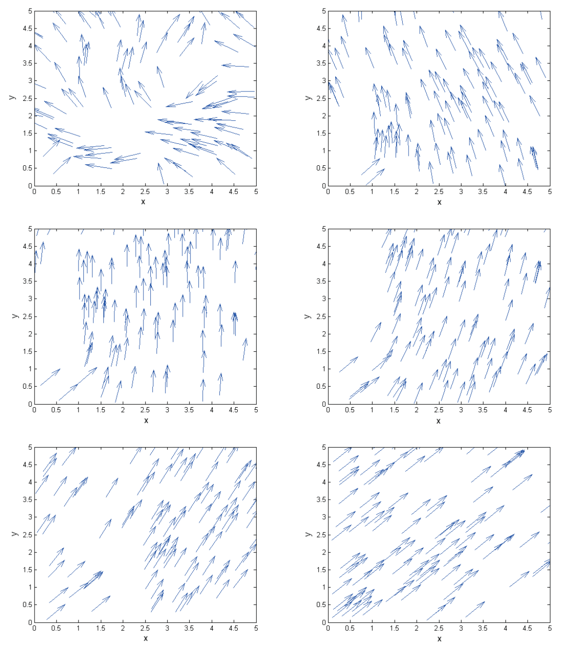

Figure 5. Snapshots of a simulation of $N = 100$ Vicsek's agents with spatial pinning control for $\rho = 4$, $L_c = 1$, $\overline{w} = 0.9$ and $\eta = 0$: (a) $t = 5$; (b) $t = 50$; (c) $t = 100$; (d) $t = 150$; (e) $t = 250$; (f) $t = 400$.

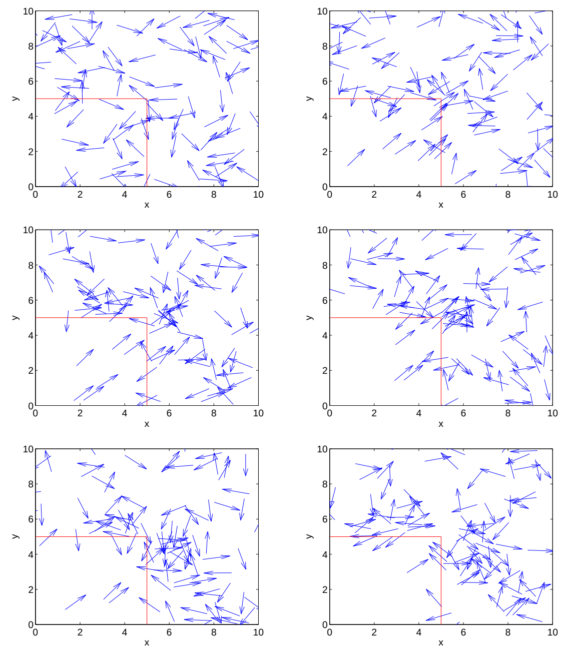

Figure 7. Snapshots of a simulation of $N = 100$ Vicsek's agents with spatial pinning control for $\rho = 1$, $L_c = 5$, $\overline{w} = 0.9$ and $\eta = 4$: (a) $t = 0$; (b) $t = 50$; (c) $t = 100$; (d) $t = 155$; (e) $t = 250$; (f) $t = 350$.

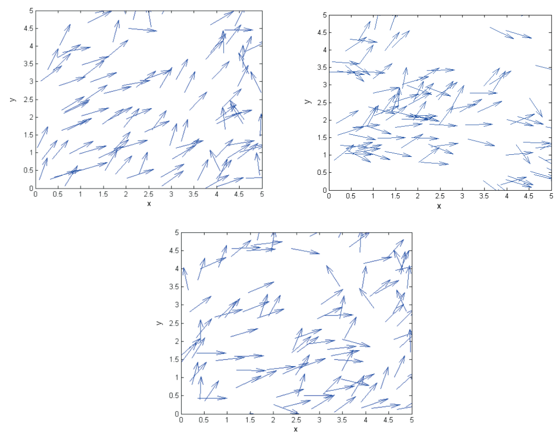

Figure 6. Snapshots at $t = 400$ for three different sets of parameters of the control law for $\rho = 4$ and $\eta = 2$: (a) $L_c = 4$, $\overline{w} = 0.5$; (b) $L_c = 1, \overline{w} = 0.9$; (c) $L_c = 3, \overline{w} = 0.1$.

tio between the control region and the whole area, that is, L_c^2/L^2. This consideration allows us to formulate the assumption of *fast switching* [Stilwell *et al.*, 2006; Frasca *et al.*, 2008]. According to the fast switching approach, it is possible to analyze the average behavior (in time) of the system, by assuming that *every* agent is controlled by an equivalent control parameter $\overline{w}L_c^2/L^2$ at each time instant, instead of considering that L_c^2/L^2 agents are controlled through the actual command $\overline{w}$. Obviously, both hypotheses represent favorable conditions for control, that are met only in limit cases. Nevertheless, we show in the following that even in the most favorable conditions, the noise level constitutes a strong limiting factor for achieving self-coordination of agents.

Under the previously mentioned conditions, the parameter v_a is computed as:

$$v_a = \langle \frac{1}{N} \left| \sum_{i=1}^N e^{j\theta_i} \right| \rangle_t = \\ = \langle \frac{1}{N} \left| \sum_{i=1}^N e^{j(1-\overline{w}\frac{L_c^2}{L^2})\Theta} e^{j(1-\overline{w}\frac{L_c^2}{L^2})\Delta\theta_i} e^{j\overline{w}\frac{L_c^2}{L^2}\overline{\theta}} \right| \rangle_t \quad (7)$$

and thus

$$v_a = \langle \frac{1}{N} \left| \sum_{i=1}^{N} e^{j(1-\overline{w}\frac{L_c^2}{L^2})\Delta\theta_i} \right| \rangle_t \quad (8)$$

In Eq. (8) v_a is function of $\overline{w}$, $\frac{L_c^2}{L^2}$ and of the distribution of $\Delta\theta_i$, so ultimately of η. According to Eq. (8), even in the most favorable case, v_a is limited by the noise. In fact, v_a can take values close to one only if $(1 - \overline{w}\frac{L_c^2}{L^2})\Delta\theta_i \simeq 0$, that is, if noise is small or, otherwise, if both $\overline{w}$ and L_c^2/L^2 take values close to one. Thus, a small value for even only one of the two parameters (either $\overline{w}$ or $\frac{L_c^2}{L^2}$), suffices to vanish the effect of the control action when a significant level of noise is present in the system.

In analogy with Fig. 2, we show in Fig. 8 the behavior of v_a, calculated from (8), for four different values of

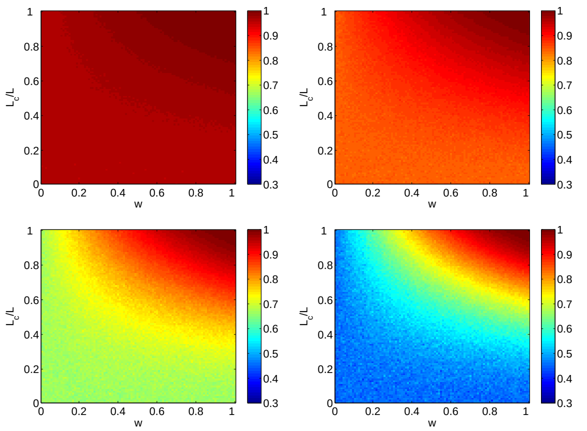

Figure 8. Order parameter v_a, calculated from (8), with respect to L_c/L and $\overline{w}$ for different values of the noise level η: (a) $\eta = 1$; (b) $\eta = 2$; (c) $\eta = 3$; (d) $\eta = 4$.

the noise level. The good agreement between the results from numerical simulations of the system of Vicsek's agents and Eq. (8) indicates that noise is fundamental in shaping the behavior of the order parameter.

4 Conclusions

In this work, we have investigated a system of Vicsek's agents, interacting on a planar anisotropic space. In particular, we assumed the existence of a limited control region where the information on a preferential direction to follow is transmitted. Our analysis revealed that the best results in terms of motion coordination are obtained when the control action operates in synergy with the self-organizing capabilities of the Vicsek's agents. In fact, for low levels of noise a global control can be attained even with a weak control action: under these conditions, at first agents self-organize towards a spontaneously emerging direction and, subsequently, as soon as the information about the preferential direction is spread through the system, they align along the desired direction of motion. Coordination also occurs in the presence of intermediate levels of noise, but, when noise is large, even a strong control action leads to poor performance and, opposite to what may be expected, it has a counter intuitive effect, with the net result of depleting the control region. We supported our numerical analysis with a mathematical argument dealing with the most favorable case, yet able to provide a useful insight on the mechanisms underlying the observed phenomena.

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