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MODIFYING THE PHYSICAL PROCESS OF ULTRASOUND TOMOGRAPHY SCANNING THROUGH COMPRESSIVE SENSING

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Abstract
Latest research papers on ultrasound tomography feature a new way of performing the scanning procedure that is similar to computed tomography. The underlying work of computed tomography consists of scanning thin slices with a rotating narrow X-ray beam. Usually, one needs a large number of projections for many different angles to reconstruct an image. In this work, we consider the application of the compressive sensing framework together with randomization to arrange an economy representation of the ultrasound diagnostics data without essential loss of performance. Using this approach, we will be able to randomly select the angles of scanning instead of obtaining all possible projections. We start with a brief overview of ultrasound tomography techniques, after that we formulate the problem of our interest, and provide the appropriate decision with results of preliminary computer simulations.

Key words
Compressive sensing, randomized algorithms, sparsity, wavelet transform, ultrasound tomography.

1 Introduction

In the 21^{st} century, the amount of information to be processed has increased dramatically, mainly due to mass transitions to processing flows of two-dimensional (2-D) and three-dimensional (3-D) data. The complexity of traditional signal quantization methods grows exponentially with dimensionality. In modern applications for digital photos and video cameras, the traditional requirement for the desired measurement frequency (Nyquist rate, see [Nyquist, 1928]) is so high that too much data must be compressed substantially before being stored or transmitted. In other applications, including display systems (medical scanners and radars) and high-speed analog-digit converters, increasing a measurement frequency has proven to be too costly.

We assume that *knowledge* makes it possible to restore the required information $\mathbf{x}$ from the data $\mathbf{y}$ acquired in the course of experiments or computations. For the sake of simplicity, one can often assume that the essential information about a phenomenon $\mathbf{x} \in \mathbb{X}$ under consideration is related to the available data $\mathbf{y} \in \mathbb{Y}$ by means of understanding the regularities of the phenomenon, i.e., the knowledge (operator)

$$\mathbf{y} = \mathbf{U}\mathbf{x} \ \ (= \mathbf{U}(\mathbf{x})). \tag{1}$$

If the operator $\mathbf{U}$ is invertible, it provides exhaustive knowledge to fully restore $\mathbf{x}$ from $\mathbf{y}$. It is known from matrix algebra that $\mathbf{x} = \mathbf{U}^{-1}\mathbf{y}$ for the case $\mathbf{y}, \mathbf{x} \in \mathbb{R}^T$ and the nonsingular $T \times T$ matrix $\mathbf{U}$.

A case in which the data are subject to the action of an uncontrollable disturbance

$$\mathbf{y} = \mathbf{U}\mathbf{x} + \mathbf{v} \tag{2}$$

is typical for open systems. For an insignificant level of external disturbances $\mathbf{v}$ (or at their damping), the problem of restoring $\mathbf{x}$ from $\mathbf{y}$ comes down to one of inverting the operator $\mathbf{U}$, which usually is accomplished by increasing the number of observations: choosing $m > T$ for $\mathbf{y} \in \mathbb{R}^m$ and $\mathbf{x} \in \mathbb{R}^T$. From the practical point of view, it is extremely interesting to investigate the possibilities for restoring $\mathbf{x} \in \mathbb{R}^T$ from $\mathbf{y} \in \mathbb{R}^m$ for $m \ll T$ which is, of course, unattainable in the general case.

Modern information theory originating from the famous Nyquist-Shannon Theorem (see [Nyquist, 1928; Shannon, 1949]) states that if an analog signal $f : \mathbb{R} \to \mathbb{R}$ from $L_2(\mathbb{R})$ has a limited spectrum, it can be restored uniquely without losses of its discrete readings taken at a frequency greater than the doubled maximal frequency of the spectrum. In many practical applications, the original notion of information $\mathbf{x}$ may be described much more simply than the actual signals f observed by the researcher. For example, to make a decision in some control system, one needs to know that a signal in the form of an acoustic or electromagnetic wave appeared in the registering channel. Of interest is a one-bit answer to the simple *yes/no* question, whereas the arriving and registered signal may take a complex form and be distributed in time and space (multidimensional vector). The rapidly progressing new paradigm of information processing relies on such specificity.

Yet recently a new paradigm of "compressive sensing" (CS) has been introduced in place of traditional signal processing theory. This paradigm makes it possible to restore the sparse information $\mathbf{x}$ with sufficient accuracy (see [Donoho, 2006]). The new methodology is based on a certain — usually randomized — selection of the matrix $\mathbf{U}$ and on the fact that the vector $\mathbf{x}$ resulting from ℓ_1-optimization has at most $s < m$ nonzero components, that is, is strongly sparse. This remarkable fact was established and used by one of the authors for constructing an ℓ_1-optimal stabilizing controller of a non-minimum phase control plant that was initially presented in [Granichin, 1983]. An extended and detailed explanation of why only a small number — equal to the co-dimension of the subspace $\mathbf{Ux} = 0$ — of vector components of the ℓ_1-optimization problem solution is not equal to zero was given in [Barabanov and Granichin, 1984]. More precisely, the CS paradigm uses randomized measurements as nonadaptive linear projections with a measurement operator $\mathbf{H}$ — a random $m \times T$ matrix — retaining the structure of a signal $\mathbf{f} = \mathbf{\Phi x}$. The information $\mathbf{x}$ is recovered using, for example, the methods of ℓ_1-optimization: $\mathbf{x}$ is determined as the solution of a problem such as

$$||\mathbf{x}||_1 \to \min_{\mathbf{x}} \qquad \text{subject to : } \mathbf{y} = \mathbf{Ux}, \qquad (3)$$

where $\mathbf{U} = \mathbf{H\Phi}$ is an operator that converts $\mathbf{x}$ into the set of data $\mathbf{y}$.

The acquisition of information on the basis of compressive sensing may be more efficient than the traditional sampling of rare or compressed signals. Popular estimates using the least square method are inadequate in CS for good signal reconstruction. Therefore, other types of convex optimization are used. The domain of compressive sensing applications has recently gone far beyond the limits of coding/decoding theory and now embraces problems of image classification and processing. In some applications, the actual arriving signals require no restoration at all, and the data are processed only in their compressed form.

Ultrasound tomography is widely applied in medical diagnostics. The development of technology enabled use of a larger number of sensors in the US transducers to obtain better images. Moreover, for better focusing, higher sampling frequencies of the signal received is required, hence leading to a huge pile of data to be processed. However, tomography-produced images are of sparse nature, i.e., there is little information available for the reconstruction. At the same time, the data is overly redundant, leaving space for the development of more efficient data acquisition and analysis technology in the process of tomography diagnostics. One such technology for collecting and reconstructing US tomography images is described below; it is based on the randomization ideology and is deemed to be less time-consuming, yet quality-lossless.

At present, resolution capability of ultrasound tomography is comparable to that of magnetic resonance imaging; in medicine, it is widely used in the diagnostics of soft tissues [Hopp et al., 2014]. Among the advantages of the ultrasound tomography are relatively low cost of equipment and maintenance, portability, safety to a human, non-invasive nature.

To increase the quality and resolution capability of the image obtained in the process of ultrasound diagnostics, both the amount of sensors in the transducer and sampling time are to be increased. This unavoidably leads to the increase of the amount of data transmitted which complicates the overall process. On top of that, distortions of data and presence of noise are usually assumed [Shannon, 1949].

The paper is organized as follows. In Section II basic concepts of travel-time tomography are introduced. In Section III we describe the problem statement. In Section IV the data processing in ultrasound tomography is considered. Simulation results are given in Section V. Section VI contains conclusions.

2 Travel-time Tomography

Propagation speed of sound in tumors is usually higher than that in normal tissue. This enables efficient reconstruction of tissue densities in the desired domain of the body using equations that involve the propagation paths of the signal and the measured times of travel between the sensors which are located at the perimeter of a ring-type transducer [Quan and Huang, 2007]. It is this *travel-time tomography* principle that underlies the functioning of modern US diagnostics equipment; specifically, this approach was applied to the breast cancer diagnostics [Hormati et al., 2010].

The amount of raw data obtained with travel-time tomography typically depends quadratically on the number of sensors in the transducer, since the impulse emitted consecutively by each of the sensors is received by all the rest of them. This imposes heavy requirements on the computational unit of the tomograph and increases the processing time. Modern US-image reconstruction methods process data iteratively with time complexity of an iteration being of the order of $O(N \log N)$ [Chen, Donoho, and Saunders, 2001].

Travel-time ultrasound tomography is a well-studied technique, which is broadly discussed in research and application papers [Kunyansky, 2011; Quan and Huang, 2007; Hopp et al., 2014]; also, data reconstruction methods which perform well in the presence of noise were studied [Hormati et al., 2010]. Needless to say, accurate determination of arrival times is crucial for better reconstruction [Li et al., 2009]. The overall process of travel-time ultrasound tomography can be schematically depicted as follows: Signal emission from a US-sensor$\longrightarrow$ Acquisiton of data from other sensors $\longrightarrow$ Extraction of travel times $\longrightarrow$ Speed map reconstruction $\longrightarrow$ Shaping the image.

The primary goal of acoustic tomography is to recover the parameters of the analyzed media from the characteristics of sound propagation. First, an exact model is needed that describes adequately the underlying physical system; second, high-precision measurements are required. Then the unknown parameters of the speed propagation model are estimated by solving the inverse problem. The quality of reconstruction directly depends on the adequacy of the physical model, accuracy of the measurements, and on the specific method for solving the inverse problem.

Propagation of energy of acoustic signals in the anisotropic media can be described by the partial differential equation

$$\nabla^2 p(r,t) - \frac{1}{F^2(r)}\frac{\partial^2 p(r,t)}{\partial t^2} = s(r,t) \qquad (4)$$

where $p(r,t)$ is the pressure at the point r at the time instant t, $F(r)$ is an unknown speed propagation model, and $s(r,t)$ is the initial signal. By knowing the initial signal and solving the inverse problem, the model $F(r)$ is detected that best fits the measurements $p(r,t)|_\Omega$ obtained from the known positions Ω.

Simulation of the primal problem using wave propagation is extremely computationally laborious, that is why principles of geometric acoustics are exploited in travel-time tomography. Under the assumption that the frequency of the signal is high enough, the path that it traveled can be found by using Fermat's principle [Schuster, 1904]. Then the inverse problem is to reconstruct the distribution $F(r)$ of the speed of sound from the travel times read from the sensors.

Importantly, in contrast to the X-ray tomography where signals propagate along straight lines, ultrasound signals in anisotropic media propagate differently and depend on speed distribution.

Though acoustic travel-time tomography originates from seismology, at present there is quite a bit of research that testifies to the efficiency of US tomography in medical diagnostics, in particular, in breast cancer diagnostics [Duric et al., 2007].

3 Inverse Problem and Image Reconstruction

The signal travel time Y between the emitter and receiver can be computed as

$$Y = \int_\Gamma \frac{1}{F(r)} ds, \qquad (5)$$

where Γ is the propagation path and $F(r)$ is the speed of sound at point r; notably, the path Γ that the signal travels depends on the speed distribution $F(r)$ in the media, and the travel time nonlinearly depends on the speed of sound in the media. Equation 5 can be represented in the discrete-time form by considering it over a fine enough N-cell rectangular grid and assuming that the speed of sound is constant in every cell:

$$Y = A(F) \cdot F, \qquad (6)$$

where F is the $N \times 1$-vector of speed distribution, $A(F)$ is the $M \times N$-matrix of paths that the signal travels over, and Y is the $M \times 1$ vector of the signal arrival times obtained from the sensors. With k sensors in the system, we have $M = k(k-1)$ various paths.

The goal of the inverse problem is to find an estimate of the speed distribution vector which explains the travel time in 6 in the best way. One of the traditional algorithms for solving this problem is the conjugate gradient method with ℓ_1-regularization [Hormati et al., 2010]. We thus arrive at the minimization problem

$$\min_F \|A(F) \cdot F - Y\|_2^2 + \lambda \|\Psi^\top F\|_1, \qquad (7)$$

where Ψ is a basis that brings F to a sparse representation, and λ is a weight coefficient.

Next, to compare the results of a reconstruction obtained by using compressive sensing with the original model (data), we have to choose the measure of difference between two sparse images (i.e., a metric in the space of sparse images).

Experimentally it was found that the relative error metric showed itself inadequate, while the Frobenius norm defined as $\|F\|_2 = \left(\sum_{i,j} |f_{ij}|^2\right)^{1/2}$ demonstrated a reasonable behavior.

The following reduced-scale model was used when testing the compressive sensing paradigm in the area of ultrasound diagnostics data:

1. the diameter of the ring-type transducer is $d = 6\text{cm}$
2. the overall amount of sensors evenly located on the perimeter is $k = 100$
3. the amount of receivers among them is $k_{Rx} = \frac{1}{3}k$
4. the reconstruction grid is $N = 64 \times 64 = 4096$
5. the number of cross-sections is $z_{total} = 15$
6. the system under diagnostics is represented by the media (water) with the speed of sound $f_w = 1500\text{mps}$ (meters per second) and an object (tumor) to be detected placed in it, having speed of sound $f_t = 2600\text{mps}$

4 Data Processing in Ultrasound Tomography

To get a better image, one needs a large number of sensors and high signal sampling rate, which obviously necessitates the processing of huge amount of data obtained from the sensors; this can be evaluated as

$$V = k^2 z Y_{max} \nu, \tag{8}$$

where V is the overall number of measurements, k is the number of sensors, z is the number of cross-sections to be analyzed, Y_{max} is the signal travel time with account for echo return loss, ν is the sampling rate.

A modern commercial ultrasound tomograph The SoftVue has the following characteristics:

1. Master server: Two processors quad-core Intel Xeon E5620, with 192Gb RAM
2. Backup server: Two processors quad-core Intel Xeon E5620, with 96Gb RAM, and two GPU Nvidia Tesla M2070

Table 1 presents the amount of data (in Gb) obtained from sensors per one cross-section.

Table 1. Amount of data obtained from sensors

sampling rate vs # sensors	256	512	1024
10	0.21	0.86	3.44
12	0.26	1.03	4.13
14	0.3	1.2	4.81

When using a single server, it takes several minutes for the reconstruction (manufacturer's information as per [Roy et al., 2013]).

4.1 Analysis of the Speed Map in the Wavelet Framework

Ultrasound tomography images are known to have sparse character. Among the efficient transformation tools leading to a sparse representation of the reconstructed speed map F are wavelet transforms. In the context of the problem under consideration, the Daubechies wavelet D4 was proved to be particularly efficient.

Figures 1–2 exemplify use of the wavelet D4 as applied to the reconstruction of the cross-section $z = 3$ of the reduced-scale model described above. Speed map reconstruction from the signal is depicted in Fig. 1. The result of the transform $X = \Psi^{-1}F$ is shown in Fig. 2.

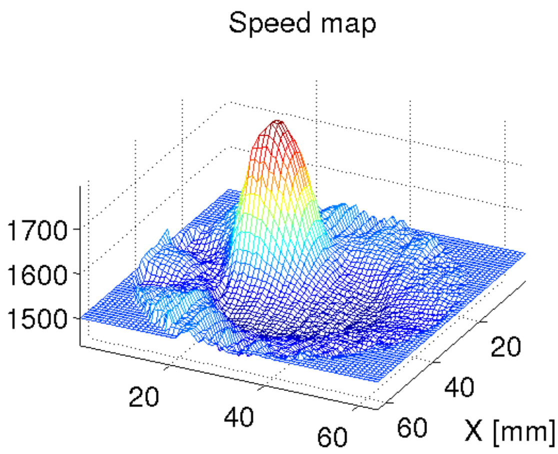

Figure 1. Speed map reconstruction from the signal

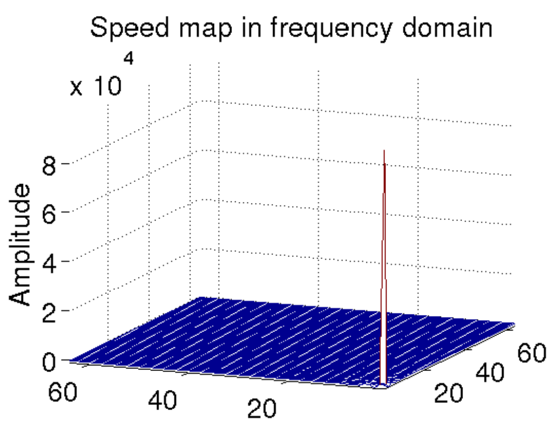

Figure 2. Speed map after wavelet transform

The overall number of the coefficients in the reduced-scale model is $N = 4096$. Sparsity of the signal is easily seen. Indeed, Fig. 3 demonstrates that use of just 100 to 200 coefficients allows for nearly complete reconstruction.

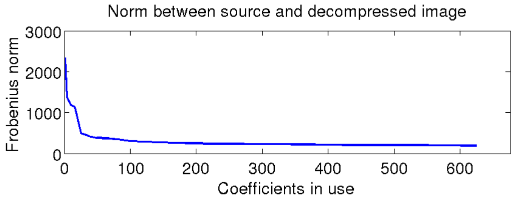

Figure 3. Density map reconstruction error using incomplete coefficient set of the wavelet representation

In the compressive sensing context this means that the signal can be considered as being s-sparse with the feasible values of s lying between $s = 100$ and $s = 200$. We chose to pick $s = 12^2 = 144$.

4.2 Scaling the Grid

Given k sensors, the overall amount of equations obtained from using the maximum possible number of projections of the image is equal to

$$M = k(k-1). \tag{9}$$

Compressed sensing ideology is useful here in the sense that, having once detected the sparsity level s, a scaling of the reconstruction grid for F can be performed, leading to the required number of the equations

$$m \approx 4s \log \frac{N}{s} \tag{10}$$

without need for performing extra experiments. Thus, Fig. 4 sheds light on the superiority of the compressed sensing paradigm over many other approaches; namely, the number of equations required for the reconstruction of a pre-specified quality, grows only logarithmically as compared to the traditional use of the complete set of M equations.

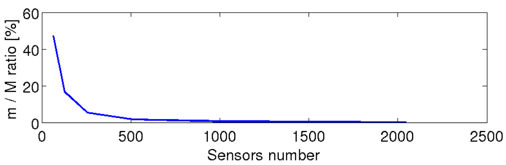

Figure 4. Portion of the equations sufficient for reconstruction

5 Simulation Results

We applied the following two randomization schemes:

1. The entries of the vector Y are picked at random, which corresponds to the $m \times N$ matrix A with the only 1 in each row at a random position, and the rest of the entries are zeros.
2. The matrix $A \in \mathbb{R}^{m \times N}$ is generated randomly with entries having either the Bernoulli distribution $\text{Prob}\{x = -1\} = \text{Prob}\{x = 1\} = 1/2$, or the distribution $\text{Prob}\{x = -1\} = 1/6, \text{Prob}\{x = 0\} = 2/3, \text{Prob}\{x = 1\} = 1/6)$.

The experiment was performed with the reduced-scale model using various numbers of projections. Use of equation 10 leads to $m \approx 60$, which is sufficient for a good reconstruction. Keeping in mind that the number of receiving sensors $k_{Rx} < k$, we obtain the required number of projections $m_\pi = \frac{m}{k_{Rx}} \approx 3$. The results are presented in Fig. 5.

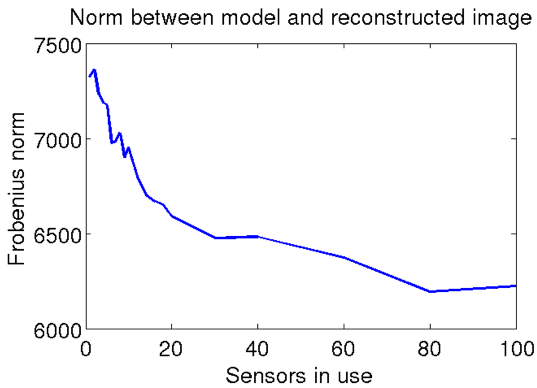

Figure 5. Deviation (in the Frobenius norm) of the reconstructed signal from the original data (speed map) as function of projections

Finally, the figures below depict the reconstruction of the image when using $m_\pi = 60$ projections (Fig. 6) and $m_\pi = 100$ projections (Fig. 7).

The preliminary results presented in this section look promising; in the nearest future, experiments over a full-scale model with $k = 1,024$ sensors will be performed.

6 Conclusions

In this paper we have applied the Compressive Sensing approach to the problem of ultrasound diagnostics data processing. For this purpose, we have investigated the sparsity properties of the obtained images, as well as how increasing the resolution of the image affects the numerical characteristic of the sparsity s. After that, we have studied a modification of the algorithm for the tomographic image reconstruction and tested it on the artificial models. Our experiments have shown

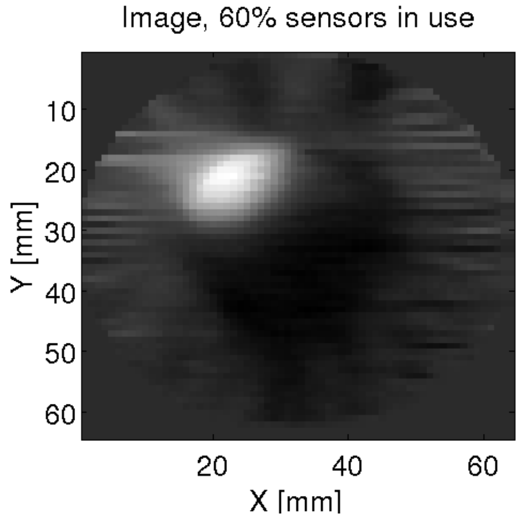

Figure 6. Sparse representation of the speed map using wavelet transform: 60 projections

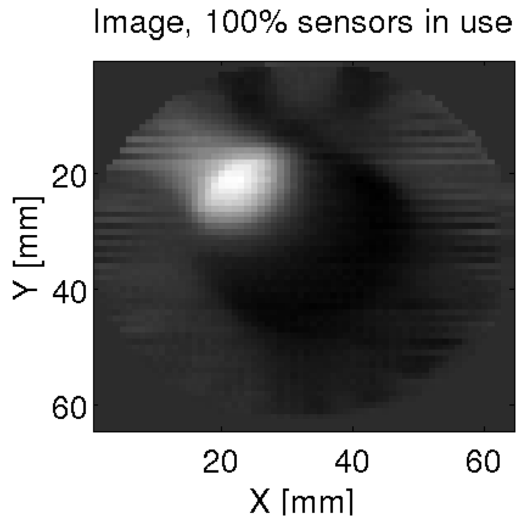

Figure 7. Sparse representation of the speed map using wavelet transform: 100 projections

that it is possible to reconstruct an image without a significant loss of quality despite using incomplete data. The use of Compressive Sensing approach may significantly reduce the amount of processed data, which will in turn significantly weaken the requirements for the computing power of the computed tomography scan and reduce the time spent on each examination.

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