# NONLINEAR DYNAMICS OF MINI-SATELLITE RESPINUP BY WEAK INTERNAL CONTROL TORQUES

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**Abstract**
Contemporary space engineering advanced new problem before theoretical mechanics and motion control theory: a spacecraft directed respinup by the weak restricted control internal forces. The paper presents some results on this problem, which is very actual for energy supply of information mini-satellites (for communication, geodesy, radio- and opto-electronic observation of the Earth et al.) with electro-reaction plasma thrusters and gyro moment cluster based on the reaction wheels or on the control moment gyros.

**Key words**
Spacecraft, nonlinear dynamics, attitude control

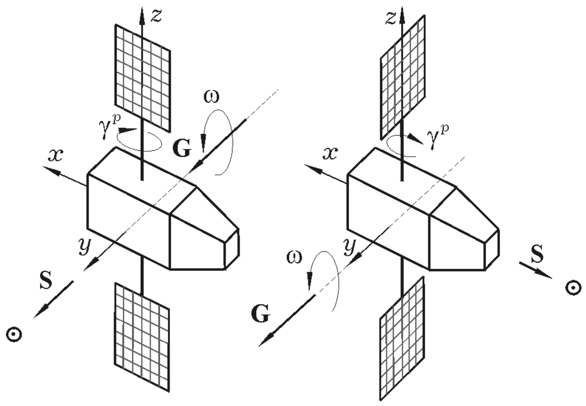


Figure 1. The rotating SC attitude over the Sun

## 1 Introduction

In the current practice the information mini-satellites are equipped with thruster unit based on plasma reaction thrusters (RTs) having high specific pulse and large power consumption. While designing a mini-satellite weighted of 100 to 500 kg it is very attractive to employ plasma RTs only for all modes. The constrains at the problem are as follows (Titov *et al.*, 2003):

• On separating from a launcher, a spacecraft (SC) obtains an initial angular rate up to $20°/\text{s}$. During that SC rotation an electric power required for the on-board equipment is generated by solar arrays panels (SAPs) or by chemical batteries. An energy generated by the SAPs depends on an angle between their normal and direction towards the Sun.

• Plasma RT enjoy small thrust values (about several grams) and large power consumption (magnitude of 1 to 1.5 kW). Small thrusts and therefore small control torques are the cause of a long time period required to damp initial SC rate. The plasma RTs can be activated a specified time period $T_a$ from several *hours* to several *days* after the separation.

• Severe requirements applied to the mass of the attitude & orbit control system (AOCS) installed on a satellite result in the fact that the angular momentum (AM) of a gyro moment cluster (GMC) based on the reaction wheels (RWs) or on the single-gimbal control moment gyroscopes (CMGs) – gyrodines (GDs), is significantly lower then the SC's AM obtained after its separation. The engineering problem is to ensure such motion of a SC separated with *no plasma RTs used*, under which the energetic conditions are met, and then after the specified period $T_a$ to complete a SC orientation towards the Sun by plasma RTs. The approach applied is based on two main assumptions:

1. the plasma RTs are applied to perform two tasks: (i) satellite attitude control and unloading of an accumulated AM, and (ii) satellite orbit control;
2. a small-mass GMC having a small AM is applied at initial mode without joining-up the RTs.

At a separation time moment $t_0$, the SC body AM vector $\mathbf{K}_0 \equiv \mathbf{J}\boldsymbol{\omega}(t_0) = \mathbf{G}_0$ has an *arbitrary* direction, therefore the principle problem is to *coincide* this vector with the SC body's maximum inertia axis $Oy$ using *only* the GMC having small resources for the AM and control torque variation domains. Essentially nonlinear dynamical processes are arising from a moving the *total* AM vector $\mathbf{G}(t)$ of mechanical system

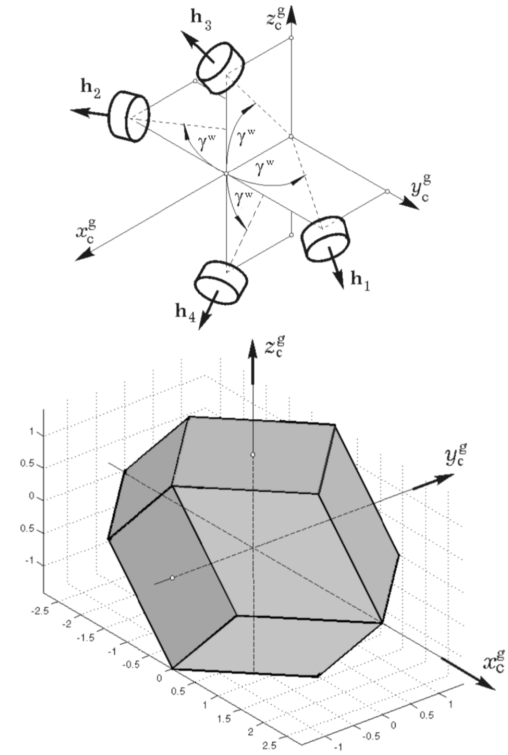


Figure 2. *GE* scheme and envelope of its AM variation domain

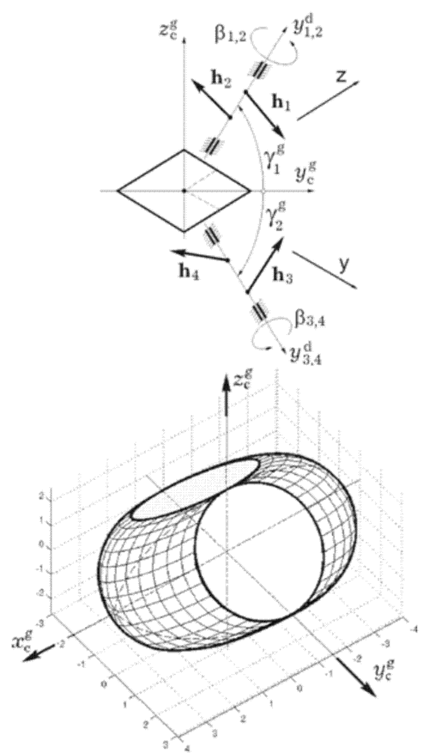


Figure 3. *2-SPE* scheme and envelope of its AM variation domain

with respect to the satellite body reference frame (BRF) $\mathrm{O}xyz$. Moreover, a Sun sensor is switched on, the Sun position is determined within the BRF and, if required, the SAPs are turned by an angle $\gamma^p$, $0 \le \gamma^p \le 270°$. In result, the SC angular rate is set along the axis $\mathrm{O}y$ which is perpendicular to the SAPs rotation axis. Depending on the initial vector $\mathbf{G}$ angular position and direction $\mathbf{S}$ towards the Sun, the SAPs will be illuminated either continuously when the vectors $\mathbf{G}$ and $\mathbf{S}$ have coincided, or periodically if $\mathbf{G}\perp\mathbf{S}$, see Fig. 1. At this phase of the SC mission, the GMC is applied to generate control torques and plasma RTs are *not activated*. At next phase of the AOCS initial modes the RTs are turned on and generate the control torques to damp a SC angular rate.

## 2 The problem background

Most satellites contain a GMC to provide gyroscopic stability of a desired attitude of the SC body, problems of gyrostat optimal control (Krementulo, 1977; Chernousko *et al.*, 1980; Somov and Fatkhullin, 1975; Junkins and Turner, 1986) and synthesis of control laws (Zubov, 1975, 1982, 1983) had been studied. V.I. Zubov's results were essentially developed by Ye.Ya. Smirnov (1981) and his successors (Smirnov *et al.*, 1985; Smirnov and Yurkov, 1989). Here a Lyapunov function is applied with small parameter for its crossed term. This idea for mechanical systems rises to G.I. Chetayev (1955). Instead that A.V. Yurkov (1999) applied a large parameter for a position term at the Lyapunov function. The SC spinup problems have been investigated by numerous authors (Hubert, 1981*a*,*b*; Huges, 1986; Guelman, 1989; Hall, 1995*a*,*b*) et al. C.D. Hall (1995*a*) have been obtained a bifurcation diagram for all gyrostat spinup equilibria *manifolds*. Different approaches were applied to *convert* the intermediate axis spin equilibrium to those of major axis spin (to *respinup* the SC body) by variation of the RWs AM (Hubert, 1981*a*,*b*; Huges, 1986; Salvatore, 1991). If *enough* AM is added, the desired spin is globally stable in the presence of energy dissipation (Huges, 1986). However, no literature was found suggesting the SC respinup *feedback* control by the GMC having *small* resources, when the SC body AM vector have a *large* value and an *arbitrary* direction.

In the paper, only principle aspects of strongly nonlinear dynamics related to the robust controlled coincidence of the SC body $\mathrm{O}y$ axis with the SC's AM vector $\mathbf{G}$ are presented. Results early obtained, see Fig. 3 in Somov *et al.* (2005*a*), are direct proofs for large efficiency of the GDs as compared with the RWs. The solution achieved is based on the methods for synthesis of nonlinear robust control (Somov, 2002; Somov *et al.*, 2002) and on rigorous analytical proof for the required SC rotation stability (Somov *et al.*, 2003*b*, 2005*a*). These results were verified by computer simulation of strongly nonlinear oscillatory processes at respinuping a flexible spacecraft.

## 3 Mathematical Models

### 3.1 Spacecraft rigid model

Let us we have a *free* rigid body (RB) with one fixed point O and any GMC. An inertia tensor $\mathbf{J}$ of the RB with a GMC is a arbitrary diagonal one, i.e. $\mathbf{J} = \lceil J_x, J_y, J_z \rfloor \equiv \mathrm{diag}\{J_i, i = x, y, z \equiv 1 \div 3\}$ within the BRF $\mathrm{O}xyz$. Model of the RB motion is presented at well-known vector form

$$\dot{\mathbf{K}} + \boldsymbol{\omega} \times \mathbf{G} = \mathbf{M} \equiv -\dot{\boldsymbol{\mathcal{H}}}, \qquad (1)$$

where $\boldsymbol{\omega} = \{\omega_i\}$ is an absolute angular rate vector of the RB; $\mathbf{K} = \mathbf{J}\,\boldsymbol{\omega}$ is an AM vector of the RB equipped with a GMC; $\mathbf{G} = \mathbf{K} + \boldsymbol{\mathcal{H}}$ is a total AM for mechanical system in the whole; $\boldsymbol{\mathcal{H}}$ is a *column-vector* of a GMC *total* AM determined in the BRF. It is suitable to present any GMC type using a canonical reference frame (CRF) $\mathbf{E}^{\mathrm{g}}_{\mathrm{c}}(x^{\mathrm{g}}_{\mathrm{c}}, y^{\mathrm{g}}_{\mathrm{c}}, z^{\mathrm{g}}_{\mathrm{c}})$. The necessary location of the required $\mathbf{S}$ domain of the GMC AM vector $\boldsymbol{\mathcal{H}}$ variation within the BRF is achieved by the CRF orientation versus the BRF.

For any GMC based on 4 RWs having the own axial inertia moments $J_r$, the model of the system motion can be presented by two vector equations

$$\dot{\mathbf{K}} + \dot{\boldsymbol{\mathcal{H}}} + \boldsymbol{\omega} \times \mathbf{G} = \mathbf{0};\; J_r\, \mathbf{A}^{\mathrm{t}}_{\gamma}\dot{\boldsymbol{\omega}} + \dot{\mathbf{H}} = \mathbf{M}_r, \qquad (2)$$

where $\mathbf{A}_{\gamma}$ is a rectangular matrix; $\boldsymbol{\mathcal{H}} = \mathbf{A}_{\gamma}\mathbf{H}$; columns $\mathbf{H} = \{h_p, p = 1 \div 4\}$ and $\mathbf{M}_r = \{m^r_p\}$ are the own AMs and control torques applied along the rotation axes of the RWs. The AM vector $\mathbf{G} = \mathbf{J}\,\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}$ is represented as $\mathbf{G} = \mathbf{K}_e + \boldsymbol{\mathcal{H}}_e$. Here $\mathbf{K}_e = \mathbf{J}_e\,\boldsymbol{\omega}$ and $\mathbf{J}_e = \mathbf{J} - J_r\mathbf{A}_{\gamma}\mathbf{A}^{\mathrm{t}}_{\gamma}$; $\boldsymbol{\mathcal{H}}_e = \mathbf{A}_{\gamma}\mathbf{H}_e$ and $\mathbf{H}_e \equiv J_r\mathbf{A}^{\mathrm{t}}_{\gamma}\,\boldsymbol{\omega} + \mathbf{H}$. For the denotations $\mathbf{M}^{\mathrm{w}}_e \equiv -\mathbf{A}_{\gamma}\,\dot{\mathbf{H}}_e = -\mathbf{A}_{\gamma}\mathbf{M}_r$ from (2) we obtain the SC motion model in the form

$$\dot{\mathbf{K}}_e + \boldsymbol{\omega} \times \mathbf{G} = -\dot{\boldsymbol{\mathcal{H}}}_e \equiv \mathbf{M}^{\mathrm{w}}_e;\;\; \dot{\mathbf{H}}_e = \mathbf{M}_r, \qquad (3)$$

where first equation has a structure of (1). At other hand, if in (2) we assume vector $\mathbf{M} = \mathbf{M}^{\mathrm{w}} = -\dot{\boldsymbol{\mathcal{H}}} \equiv -\mathbf{A}_{\gamma}\dot{\mathbf{H}}$ to be known for the complete compliance with (1), then the evident definition $\dot{\mathbf{H}}$ enables to calculate the vector $\mathbf{M}_r = \dot{\mathbf{H}} + J_r\mathbf{A}^{\mathrm{t}}_{\gamma}\dot{\boldsymbol{\omega}}$ of required RW control torques in the form

$$\mathbf{M}_r = \dot{\mathbf{H}} + J_r\mathbf{A}^{\mathrm{t}}_{\gamma}\mathbf{J}^{-1}(\mathbf{M}^{\mathrm{w}} - \mathbf{G} \times \boldsymbol{\omega}). \qquad (4)$$

For each RW the control torque and own AM are limited as per a module, i.e. $\forall t \in \mathrm{T}_{t_0} \equiv [t_0, \infty)$

$$|m^r_p(t)| \le \mathrm{m}^{\mathrm{m}}; \quad |h_p(t)| \le \mathrm{h}^{\mathrm{m}},\; p = 1 \div 4, \qquad (5)$$

where parameters $\mathrm{m}^{\mathrm{m}}$ and $\mathrm{h}^{\mathrm{m}}$ are *specified* positive constants. Obviously, with an arbitrary matrix $\mathbf{A}_{\gamma}$ these constrains are converted into *fixed* convex domains of allowable variation for the AM vector $\boldsymbol{\mathcal{H}} = \mathbf{A}_{\gamma}\mathbf{H}$ and the control torque vector $\mathbf{M}^{\mathrm{w}} = -\mathbf{A}_{\gamma}\mathbf{M}_r$ attributed to the GMC type. As an example, in Fig. 2 the *General Electric* (*GE*) scheme is presented at the *normalized* to $\mathrm{h}^{\mathrm{m}}$ form. At the denotations $\mathsf{x} = \mathsf{x}_1 + \mathsf{x}_2$; $\mathsf{y} = \mathsf{y}_1 + \mathsf{y}_2$; $\mathsf{z} = \mathsf{z}_1 + \mathsf{z}_2$; $\mathbf{h} = \boldsymbol{\mathcal{H}}/\mathrm{h}^{\mathrm{m}} \equiv \{\mathsf{x}, \mathsf{y}, \mathsf{z}\}$, $S_{\gamma} \equiv \sin\gamma^{\mathrm{w}}$ and $C_{\gamma} \equiv \cos\gamma^{\mathrm{w}}$ we have the relations

$$\mathbf{h} = \begin{bmatrix} (x_1 + x_2)/\mathrm{h}^{\mathrm{m}} \\ S_{\gamma}(h_1 - h_2)/\mathrm{h}^{\mathrm{m}} \\ S_{\gamma}(h_3 - h_4)/\mathrm{h}^{\mathrm{m}} \end{bmatrix}; \mathbf{A}_{\gamma} = \begin{bmatrix} C_{\gamma} & C_{\gamma} & C_{\gamma} & C_{\gamma} \\ S_{\gamma} & -S_{\gamma} & 0 & 0 \\ 0 & 0 & S_{\gamma} & -S_{\gamma} \end{bmatrix},$$

where $x_1 = C_{\gamma}(h_1 + h_2)$ and $x_2 = C_{\gamma}(h_3 + h_4)$.

Applied *2-SPE* (*2 Scissored Pair Ensemble*) scheme on 4 GDs with *own* AM $h_g$ is presented in Fig. 2. Here within the CMG precession theory the AM vector $\boldsymbol{\mathcal{H}}$ have the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = h_g\mathbf{A}_{\gamma}\mathbf{h}$ with constant non-singular matrix $\mathbf{A}_{\gamma}$, where a normed vector $\mathbf{h} = \sum \mathbf{h}_p(\beta_p)$ made up from units $\mathbf{h}_p(\beta_p)$, vector column $\boldsymbol{\beta} = \{\beta_p\}$ presents the GD's precession angles, at last vector column $\mathbf{h} \equiv \{\mathsf{x}, \mathsf{y}, \mathsf{z}\}$, where $\mathsf{x} = \mathsf{x}_{12} + \mathsf{x}_{34}$; $\mathsf{x}_{12} = \mathsf{x}_1 + \mathsf{x}_2$; $\mathsf{x}_{34} = \mathsf{x}_3 + \mathsf{x}_4$; $\mathsf{y} = \mathsf{y}_1 + \mathsf{y}_2$; $\mathsf{z} = -(\mathsf{z}_3 + \mathsf{z}_4)$; $\mathsf{x}_p = C_{\beta_p}$; $\mathsf{y}_p = S_{\beta_p}$; $\mathsf{z}_p = S_{\beta_p}$. At the command column $\mathbf{u} = \{\mathrm{u}_p\}$ the vector of the GMC output torque have the form

$$\mathbf{M}^{\mathrm{g}} = -\dot{\boldsymbol{\mathcal{H}}}(\boldsymbol{\beta}, \dot{\boldsymbol{\beta}}) = -h_g\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta})\mathbf{u};\;\; \dot{\boldsymbol{\beta}} = \mathbf{u}, \qquad (6)$$

where $\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \mathbf{A}_{\gamma}\mathbf{A}_h(\boldsymbol{\beta})$, and matrixes $\mathbf{A}_{\gamma}$ and $\mathbf{A}_h(\beta) = \partial\mathbf{h}(\boldsymbol{\beta})/\partial\boldsymbol{\beta}$ are presented as

$$\mathbf{A}_{\gamma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & S_{\gamma} & S_{\gamma} \\ 0 & -C_{\gamma} & C_{\gamma} \end{bmatrix};\; \mathbf{A}_h = \begin{bmatrix} -\mathsf{y}_1 & -\mathsf{y}_2 & -\mathsf{z}_3 & -\mathsf{z}_4 \\ \mathsf{x}_1 & \mathsf{x}_2 & 0 & 0 \\ 0 & 0 & -\mathsf{x}_3 & -\mathsf{x}_4 \end{bmatrix}$$

for the denotations $S_{\gamma} \equiv \sin\gamma^{\mathrm{g}}$ and $C_{\gamma} \equiv \cos\gamma^{\mathrm{g}}$. The GDs' angles vary within the full range, but the domain $\boldsymbol{\mathcal{S}}$ of the GMC's AM vector $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})$ variations is limited. The "control" $u_p(t)$ of each GD is module-limited by *given* positive parameter $\mathrm{u}^{\mathrm{m}}$ :

$$|u_p(t)| \le \mathrm{u}^{\mathrm{m}},\; p = 1 \div 4,\;\; \forall t \in \mathrm{T}_{t_0}. \qquad (7)$$

These constrains are converted into $\beta$-*dependent* convex variation domain for a control torque $\mathbf{M} = \mathbf{M}^{\mathrm{g}} = -\dot{\boldsymbol{\mathcal{H}}}(\boldsymbol{\beta}, \dot{\boldsymbol{\beta}})$ in the model (6).

### 3.2 Spacecraft flexible model

Model of a *free* flexible SC motion is presented also at the vector-matrix form with standard notations

$$\begin{bmatrix} \mathbf{J} & \mathbf{D}_q \\ \mathbf{D}^{\mathrm{t}}_q & \mathbf{A}^q \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \ddot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} - \boldsymbol{\omega} \times \mathbf{G} \\ -\{a^q_j(\frac{\delta^q}{\pi}\Omega^q_j\dot{q}_j + (\Omega^q_j)^2 q_j)\} \end{bmatrix}, \quad (8)$$

$\mathbf{q} = \{q_j\}; \mathbf{A}^q = \lceil a^q_j \rfloor; \mathbf{G} = \mathbf{G}^{\mathrm{o}} + \mathbf{D}_q\dot{\mathbf{q}}; \mathbf{G}^{\mathrm{o}} = \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}\boldsymbol{\beta}).$

## 4 The problem statement

Considering the model (1), let denote an AM vector of a RB at initial time moment $t_0$ as $\mathbf{K}_0$. Let the vector of a GMC's total AM at the initial time be equal to zero, i.e. $\boldsymbol{\mathcal{H}}_0 \equiv \boldsymbol{\mathcal{H}}(t_0) = \mathbf{0}$. A norm of the vector $\mathbf{K}_0$ is assumed to be limited with the *given* constant, i.e. $\|\,\mathbf{K}_0\,\| \le k^*_{\mathrm{o}},\; k^*_{\mathrm{o}} > 0$, but the direction of this vector within the BRF is *arbitrary*. Therefore, at the time $t = t_0$ the total AM vector related to the whole mechanical system $\mathbf{G}_0 = \mathbf{K}_0$ with $\|\,\mathbf{G}_0\,\| \equiv g_{\mathrm{o}} \le g^*_{\mathrm{o}} = k^*_{\mathrm{o}}$. The inertial parameters of the RB are assumed to be known, the same for the possibility to measure the vector $\boldsymbol{\omega}(t)$ and the vector $\boldsymbol{\mathcal{H}}(t)$. Let establish of a fixed unit vector $\mathbf{f} = \mathbf{e}_y = \{0, 1, 0\}$ or $\mathbf{f} = -\mathbf{e}_y = \{0, -1, 0\}$ is given within the BRF – the unit of a RB having the *largest* moment of inertia or the one opposite.

*The problem* consists in designing the GMC control law for achieving such condition of a gyrostat (1) with *specified accuracy* at any time moment $t = T_{\rm f}$:

$$\mathbf{K}_{\rm f} = \mathbf{J}\,\boldsymbol{\omega}_{\rm f};\ \boldsymbol{\omega}_{\rm f} = \omega_{\rm f}\mathbf{f};\ \boldsymbol{\mathcal{H}}_{\rm f} = \mathcal{H}_{\rm f}\mathbf{f}, \qquad (9)$$

where $\mathbf{K}_{\rm f} \equiv \mathbf{K}(T_{\rm f})$; $\boldsymbol{\mathcal{H}}_{\rm f} \equiv \boldsymbol{\mathcal{H}}(T_{\rm f})$; $\boldsymbol{\omega}_{\rm f} \equiv \boldsymbol{\omega}(T_{\rm f})$ and module $\mathcal{H}_{\rm f}$ of the total GMC AM's is established, in particular, as $\mathcal{H}_{\rm f} = 0$. Taking into account the identity $J_y\ \omega_{\rm f} + \mathcal{H}_{\rm f} = g_{\rm o}$, one can find the obvious relation $\omega_{\rm f} = (g_{\rm o} - \mathcal{H}_{\rm f})/J_y$.

After solving this *vital* problem, it is necessary to ensure the *distribution* of the AM $\boldsymbol{\mathcal{H}}$ and control torque $\mathbf{M} = \mathbf{M}^{\rm w}$ or $\mathbf{M} = \mathbf{M}^{\rm g}$ vectors between four RWs or GDs, accordingly. It is desirable to have the *explicit* distribution law (DL) allowing to obtain all movement characteristics for each electromechanical actuator based on the *analytical* relations. The GMC with *collinear* GD gimbal axes obtains a significant advantage: all its *singular* states are *passable* (Somov *et al.*, 2003*a*). At 4 GDs the same approach is possible only for *2-SPE* scheme, see Fig. 2. It is also necessary to consider a respinup of the flexible spacecraft structure through using four GDs.

## 5 Synthesis of main control law

An AM vector $\mathbf{G}(t) = \mathbf{J}\,\boldsymbol{\omega}(t) + \boldsymbol{\mathcal{H}}(t)$ of the *whole* mechanical system with no external torques has its value unchanged within any *inertial* reference frame (IRF), in accordance with the theoretical mechanics principles. The unit vector $\mathbf{g}(t) \equiv \{{\rm g}_i(t)\} = \mathbf{G}(t)/g_{\rm o}$ is also a *fixed* one within the IRF, but within the BRF this unit is *moving* in accordance with equation

$$\dot{\mathbf{g}}(t) = -\,\boldsymbol{\omega}(t) \times \mathbf{g}(t). \qquad (10)$$

Let us the following be calculated within the BRF when the system moves as per the *measured values* of the $\boldsymbol{\omega}(t)$ and $\boldsymbol{\mathcal{H}}(t)$ vectors:

- position of an AM unit vector $\mathbf{g}(t)$;
- position of a vector $\boldsymbol{\xi}(t) = \mathbf{g}(t) \times \mathbf{f}$;
- for $\parallel \boldsymbol{\xi}(t) \parallel = S_\varphi(t) \equiv \sin\varphi(t) \geq \varepsilon_1 = \text{const}$ the *unit* vector value $\mathbf{e}_\xi(t) = \boldsymbol{\xi}(t)/\parallel \boldsymbol{\xi}(t) \parallel$;
- a cosine of angle between the units $\mathbf{g}$ and $\mathbf{f}$, namely $C_\varphi(t) \equiv \cos\varphi(t) = \langle \mathbf{f}, \mathbf{g}(t) \rangle$.

A mismatch between the actual and required values of the SC rate vector is presented as

$$\boldsymbol{\eta}(t) = \delta\boldsymbol{\omega}(t) \equiv \boldsymbol{\omega}(t) - \omega_{\rm f}\,\mathbf{f}. \qquad (11)$$

Let us assume that at time $t_0$ there is also calculated an indicator $a_{\rm f} = \text{Sgn}\ C_\varphi(t_0)$ of the unit vector direction $\mathbf{f}$ by the *definition*

$$\text{Sgn}\ x = 1 \text{ for } x \geq 0 \quad \text{and} \quad \text{Sgn}\ x = -1 \text{ for } x < 0,$$

and then we determine unit vector $\mathbf{f} = a_f \mathbf{e}_y$. At the denotation $\boldsymbol{\zeta}(t) = \mathbf{g}(t) - \mathbf{f}$ as a nearby measure for the unit vectors $\mathbf{g}$ and $\mathbf{f}$, it is suitable to use a scalar function

$$v_p(t) \equiv v_p(\boldsymbol{\zeta}(t)) = \boldsymbol{\zeta}^2(t)/2 = 1 - \langle \mathbf{f}, \mathbf{g}(t) \rangle >> 0. \qquad (12)$$

This function have positive values under $\mathbf{g}(t) \neq \mathbf{f}$ and obtains zero value at the above vectors coincided only. With above selection of unit vector $\mathbf{f} = a_f\{0, 1, 0\}$, we *always* have $v_p(t_0) \leq 1$. Taking into account standard vector *identities* $\langle \mathbf{a}, (\mathbf{b} \times \mathbf{c}) \rangle \equiv \langle \mathbf{b}, (\mathbf{c} \times \mathbf{a}) \rangle \equiv \langle \mathbf{c}, (\mathbf{a} \times \mathbf{b}) \rangle$ and $\dot{\boldsymbol{\zeta}}(t) \equiv -\,\boldsymbol{\omega}(t) \times \mathbf{g}(t)$ by (10), we have derivative of the function $v_p$ (12) as follows

$$\dot{v}_p = \langle \boldsymbol{\zeta}(t), \dot{\boldsymbol{\zeta}}(t) \rangle = \langle \boldsymbol{\xi}(t), \boldsymbol{\eta}(t) \rangle. \qquad (13)$$

Vectors $\boldsymbol{\xi}(t)$ and $\boldsymbol{\zeta}(t)$ are connected by identities

$$\boldsymbol{\xi}^2 \equiv \boldsymbol{\zeta}^2(1 - \boldsymbol{\zeta}^2/4);\ \boldsymbol{\zeta}^2 \equiv 2\boldsymbol{\xi}^2/(1 + (1 - \boldsymbol{\xi}^2)^{1/2}), \qquad (14)$$

moreover the vector $\boldsymbol{\xi}(t)$ is moving by equation

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\eta} - \boldsymbol{\phi};\ \boldsymbol{\phi} \equiv \omega_{\rm f}\boldsymbol{\zeta} + \mathbf{g}\langle \mathbf{f}, \boldsymbol{\eta} \rangle + (\boldsymbol{\eta} + \omega_{\rm f}\mathbf{f})\boldsymbol{\zeta}^2/2. \qquad (15)$$

Taking into account that due to (11) $\dot{\boldsymbol{\omega}}(t) = \dot{\boldsymbol{\eta}}(t)$ and the relations

$$\mathbf{G}(t) = g_{\rm o}\mathbf{g} = g_{\rm o}\mathbf{f} + g_{\rm o}(\mathbf{g}(t) - \mathbf{f}) = \mathbf{K}_{\rm f} + \boldsymbol{\mathcal{H}}_{\rm f} + g_{\rm o}\boldsymbol{\zeta}(t);$$
$$\boldsymbol{\nu} \equiv \mathbf{J}\boldsymbol{\eta} - g_{\rm o}\boldsymbol{\zeta} = -(\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_{\rm f}); \quad \dot{\boldsymbol{\nu}} = \mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{G},$$

the equation (1) is presented in simplest form

$$\dot{\boldsymbol{\nu}} \equiv \mathbf{J}\dot{\boldsymbol{\eta}} - g_{\rm o}\dot{\boldsymbol{\zeta}} = \mathbf{M} = -\dot{\boldsymbol{\mathcal{H}}}. \qquad (16)$$

The function $v_e(\boldsymbol{\nu}) \equiv \boldsymbol{\nu}^2/(2j_h) = (\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_{\rm f})^2/(2j_h)$ defines a GMC kinetic energy at its motion with respect to required equilibrium in the BRF, where any constant $j_h > 0$ presents its the inertia properties.

The RB movement required $\mathbf{O}_\eta \equiv \{\boldsymbol{\xi} = \mathbf{0}; \boldsymbol{\eta} = \mathbf{0}\}$ is the same $\mathbf{O}_\nu \equiv \{\boldsymbol{\xi} = \mathbf{0}; \boldsymbol{\nu} = \mathbf{0}\}$ due to the identities (14). For denotation $\rho^2(t) \equiv \parallel \boldsymbol{\xi}(t) \parallel^2 + \parallel \boldsymbol{\eta}(t) \parallel^2$ in the first let consider any small domain

$$\boldsymbol{\mathcal{O}} \equiv \{\parallel \boldsymbol{\xi} \parallel < \varepsilon_1\} \cap \{\parallel \rho \parallel < \varepsilon_\rho = \text{const}\},$$

within which *no constrains* for the control torque $\mathbf{M}$ vector have occurred. To justify the structure of the control torque $\mathbf{M}$ law into the equation (16), we introduce the *Lyapunov* function

$$\text{V} = a\,b\,v_p(\boldsymbol{\zeta}) + (a/j_h)\langle \boldsymbol{\nu}, \mathbf{P}\boldsymbol{\xi} \rangle + v_e(\boldsymbol{\nu}), \qquad (17)$$

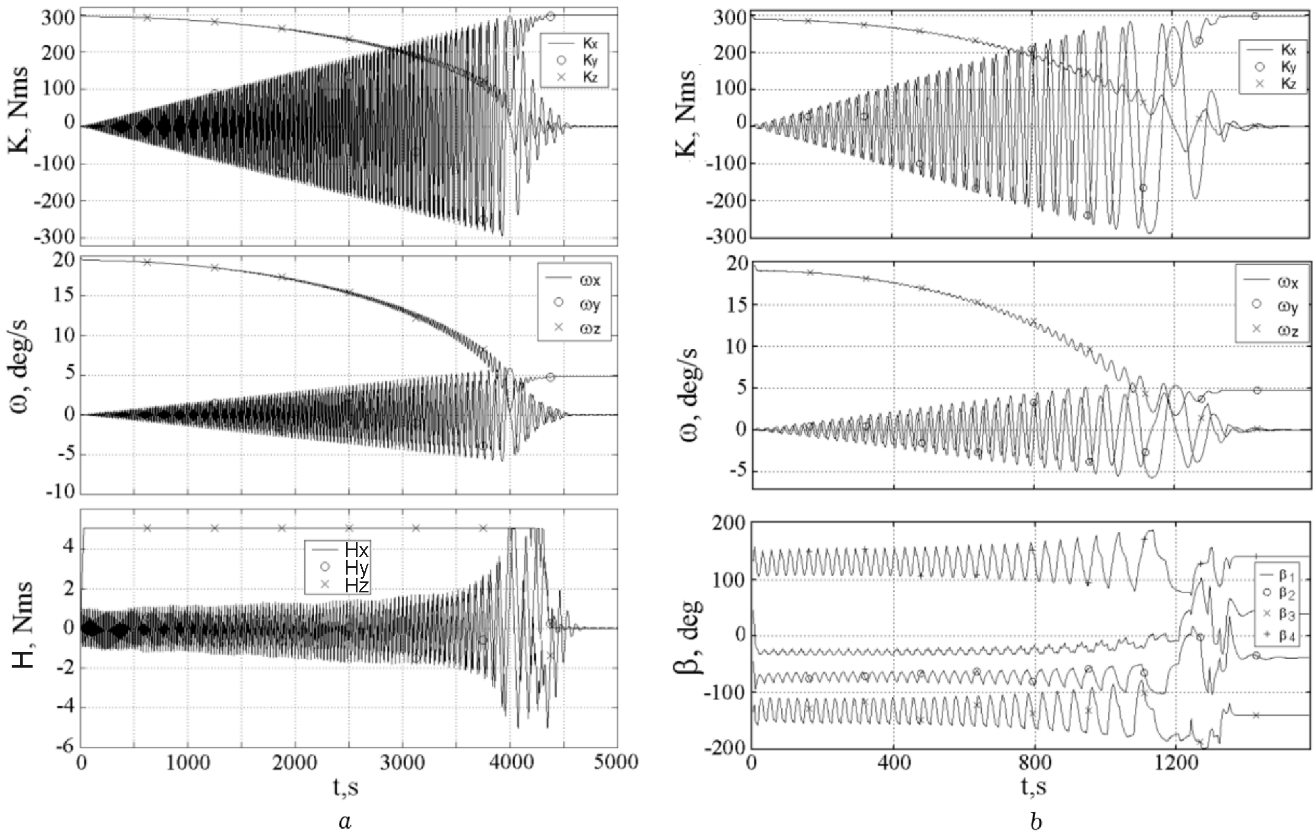


Figure 4. Dynamics of the RB respinup: $a$ — by 3 reaction wheels; $b$ — by 4 GDs on scheme *2-SPE*.

where scalar parameters $a > 0$, $b > 0$ and $\mathbf{P}$ is a constant definitely-positive matrix. Taking into account that $\boldsymbol{\zeta}^2 \equiv 2\boldsymbol{\xi}^2/(1+(1-\boldsymbol{\xi}^2)^{1/2})$ due to identity (14) and well-known Schur lemma for a symmetric composite matrix, the function $\mathrm{V}(\boldsymbol{\xi},\boldsymbol{\eta})$(17) is definitely positive with respect to the vector variables $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ into domain $\boldsymbol{\mathcal{O}}$ for *large* value of parameter $b$ and *small* value of parameter $a$. The derivative of this function with (13) and (16) taken into account have the form

$$\dot{\mathrm{V}} = ab\langle\boldsymbol{\xi},\boldsymbol{\eta}\rangle + [\langle\mathbf{M},\boldsymbol{\mu}\rangle + \langle\boldsymbol{\nu},\mathbf{P}\dot{\boldsymbol{\xi}}\rangle]/j_h, \tag{18}$$

where vector $\boldsymbol{\mu} \equiv \boldsymbol{\nu} + a\mathbf{P}\boldsymbol{\xi}$. For domain $\boldsymbol{\mathcal{O}}$ the GMC control law is selected in the form

$$\mathbf{M} = \mathbf{M}_\xi \equiv -qj_h\mathbf{D}\boldsymbol{\mu} = -m\,[\,\boldsymbol{\xi} + k\mathbf{D}\boldsymbol{\nu}] \tag{19}$$

with parameters $q > 0$, $m = qj_h a > 0$, $k = 1/a > 0$ and definitely-positive matrix $\mathbf{D} = \mathbf{P}^{-1}$.

**Theorem** *For the RB movement required $\mathbf{O}_\eta$ of the system's model (15), (16) with the control law (19) the property of exponential stability*

$$\rho(t) \le \beta\,\rho(t_0)\exp(-\alpha(t-t_0)), \tag{20}$$

where $\alpha, \beta = \mathrm{const} > 0$, *is guaranteed for arbitrary vector of initial conditions* $\{\boldsymbol{\xi}(t_0), \boldsymbol{\eta}(t_0)\} \in \boldsymbol{\mathcal{O}}_0 \subseteq \boldsymbol{\mathcal{O}}$ *at chosen large value* $q(g_{\mathrm{o}})$.

**Proof** The derivative (18) of function (17) by the relation (15) taken into account is presented as

$$\begin{aligned}\dot{\mathrm{V}} = &-qa^2\langle\boldsymbol{\xi},\mathbf{P}\boldsymbol{\xi}\rangle + a(b\,\langle\boldsymbol{\xi},\boldsymbol{\eta}\rangle - 2q\langle\boldsymbol{\xi},\mathbf{J}\boldsymbol{\eta}\rangle)\\ &-q\langle\boldsymbol{\nu},\mathbf{D}\boldsymbol{\nu}\rangle + (a/j_h)\langle\boldsymbol{\nu},\mathbf{P}(\boldsymbol{\eta}-\boldsymbol{\phi}(\boldsymbol{\eta},\boldsymbol{\zeta}))\rangle,\end{aligned} \tag{21}$$

where vector $\boldsymbol{\nu} = \mathbf{J}\boldsymbol{\eta} - g_{\mathrm{o}}\boldsymbol{\zeta}$ and the function $\boldsymbol{\phi}(\cdot)$ was defined in (15). Taking into account

$$\langle\boldsymbol{\nu},\mathbf{D}\boldsymbol{\nu}\rangle = \langle\mathbf{DJ}\boldsymbol{\eta},\mathbf{J}\boldsymbol{\eta}\rangle - 2g_{\mathrm{o}}\langle\mathbf{DJ}\boldsymbol{\eta},\boldsymbol{\zeta}\rangle + g_{\mathrm{o}}^2\langle\mathbf{D}\boldsymbol{\zeta},\boldsymbol{\zeta}\rangle$$

and analogous representations of the terms $\langle\boldsymbol{\nu},\mathbf{P}\boldsymbol{\eta}\rangle$, $\langle\boldsymbol{\nu},\mathbf{P}\boldsymbol{\zeta}\rangle$, $\langle\boldsymbol{\nu},\mathbf{P}\boldsymbol{\phi}\rangle$ in (21), and also identities (14), one makes sure of the majoring $\dot{\mathrm{V}} \le -\mathrm{W}(\boldsymbol{\xi},\boldsymbol{\eta})$, where scalar function $\mathrm{W}(\boldsymbol{\xi},\boldsymbol{\eta})$ is definitely positive with respect to variables $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ for *large* values of parameters $b$ and $q$, *depending* on total AM value $g_{\mathrm{o}}$, thanks to the Schur lemma. By analogy with Smirnov (1981) there is proved that $\mathrm{W}(t) \to 0$ at $t \to \infty$ and function $\mathrm{V}(t)$ is decreased *monotonically*. Standard estimates (Smirnov and Yurkov, 1989; Yurkov, 1999) are derived from majoring functions V and W by quadratic forms

$$a_1\rho^2 \le \mathrm{V} \le a_2\rho^2, a_1 > 0;\;\; b_1\rho^2 \le \mathrm{W} \le b_2\rho^2, b_1 > 0,$$

from where the condition (20) is appeared with the parameters $\alpha = b_1/(2a_2)$ and $\beta = (a_2/a_1)^{1/2}$. □

Due to the identity $\boldsymbol{\nu} \equiv \mathbf{J}\boldsymbol{\eta} - g_{\mathrm{o}}\boldsymbol{\zeta} = -(\boldsymbol{\mathcal{H}} - \boldsymbol{\mathcal{H}}_{\mathrm{f}})$ the control law (19) is appeared in very *simple* form

$$\mathbf{M}_\xi(t) = -m[\,\boldsymbol{\xi}(t) - k\mathbf{D}(\boldsymbol{\mathcal{H}}(t) - \boldsymbol{\mathcal{H}}_{\mathrm{f}})\,]$$

interior to nearest neighborhood of required gyrostat state $\mathbf{O}_\eta$. Outside this neighborhood the control law is not effective because of various *equilibrium manifolds* (Hall, 1995*a*) which exist at conditions

$$\mathbf{M}_\xi = \mathbf{J}\dot{\boldsymbol{\eta}} - g_o\dot{\boldsymbol{\zeta}} \equiv \mathbf{0};\, \mathbf{J}\boldsymbol{\eta} - g_o\boldsymbol{\zeta} = \mathbf{c};\, a\mathbf{P}\boldsymbol{\xi} = -\mathbf{c}$$

with a *constant* vector $\mathbf{c} \neq \mathbf{0}$. Therefore other *simple* control laws are needed for fastest the SC respinuping without *sticking* its motion on any equilibrium manifold differing from the state $\mathbf{O}_\eta$. For the denotations

$$\mathbf{M}^{\mathrm{r}}_\xi(t) \equiv -m\,[\,\mathbf{e}_\xi(t)\mathrm{Sgn}C_\varphi(t) - k\mathbf{D}(\boldsymbol{\mathcal{H}}(t) - \boldsymbol{\mathcal{H}}_{\mathrm{f}})\,];$$
$$\mathbf{M}^{\mathrm{r}}(t) \equiv -\mathrm{m}^*\,\{\mathrm{Sgn}\,\mathrm{g}_i(t),\;\; i = x, y, z\},$$

where $\mathrm{m}^*$ is a *large* constant parameter, developed control law has the form

$$\mathbf{M} = \begin{cases} \mathbf{M}_\xi(t) & \parallel \boldsymbol{\xi}(t) \parallel < \varepsilon_1; \\ \mathbf{M}^{\mathrm{r}}_\xi(t) & \varepsilon_1 \leq \parallel \boldsymbol{\xi}(t) \parallel \leq \varepsilon_2; \\ \mathbf{M}^{\mathrm{r}}(t) & \parallel \boldsymbol{\xi}(t) \parallel > \varepsilon_2, \end{cases} \quad (22)$$

where for example, the parameters $\varepsilon_1 = 0.1$ (angle $\varphi = 6^\circ$) and $\varepsilon_2 = 0.5$ (angle $\varphi = 30^\circ$).

## 6 Distribution laws for a GMC

The most vital control aspect for the GMC having the excessive structure is the selection of a distributing law (DL) of the required total GMC's AM between electromechanical actuators. It is desirable to have an *explicit* DL based on *analytical* relations.

Unlike well-known RW DLs based on pseudo-inversion of matrix $\mathbf{A}_r = J_r\mathbf{A}_\gamma$, fundamental idea of the employed DL is in achieving the strict *uniformity* in terms of the saturation resources for the RWs pairs. In normalized form such DL is described by the relation

$$df_\rho(\mathbf{h})/dt = \Phi_\rho(f_\rho(\mathbf{h})) \equiv -\mathrm{Sat}(\phi_\rho, \mu_\rho f_\rho(\mathbf{h}), \quad (23)$$

where $f_\rho(\mathbf{h}) = \tilde{x}_1 - \tilde{x}_2 + \rho(\tilde{x}_1\tilde{x}_2 - 1)$; $\tilde{x}_1 \equiv \mathsf{x}_1/\mathsf{q}_\mathsf{y}$; $\tilde{x}_2 \equiv \mathsf{x}_2/\mathsf{q}_\mathsf{z}$; $\mathsf{q}_\mathsf{s} = (4C_\gamma^2 - \mathsf{s}^2)^{1/2}$, $\mathsf{s} = \mathsf{y}, \mathsf{z}$; $0 < \rho < 1$, and $\phi_\rho$, $\mu_\rho$ are positive parameters. The GMC angular momentum is distributed as per condition $f_\rho(\mathbf{h}) = 0$ firstly among the RW pairs

$$\mathsf{q} \equiv \mathsf{q}_\mathsf{y} + \mathsf{q}_\mathsf{z};\, b \equiv \mathsf{x}/2;\, c = (\mathsf{q}_\mathsf{y} - \mathsf{q}_\mathsf{z})b + \rho(\mathsf{q}_\mathsf{y}\mathsf{q}_\mathsf{z} - b^2);$$
$$\Delta \equiv (\mathsf{q}/\rho)(1 - (1 - 4\rho\, c/\mathsf{q}^2)^{1/2});$$
$$\mathsf{x}_1 = (\mathsf{x} + \Delta)/2;\;\; \mathsf{x}_2 = (\mathsf{x} - \Delta)/2$$

and then among two RWs in each pair. To define the column $\mathbf{M}_r$ the relation $\mathbf{A}_\gamma\dot{\mathbf{h}} = \dot{\boldsymbol{\mathcal{H}}}$ is supplemented with the equation $\langle \mathbf{a}_f(\mathbf{h}), \dot{\mathbf{h}} \rangle = \Phi_\rho(f_\rho(\mathbf{h}))$, where $\mathbf{a}_f(\mathbf{h}) = \partial f_\rho(\mathbf{h})/\partial\,\mathbf{h}$. As a result, we obtain four *linear* equations having positive determinant for all *internal* points within $\mathbf{S}$ domain. Thus, the vector $\mathbf{M}_r$ can be *analytically* calculated if vector $\boldsymbol{\mathcal{H}}$ is known.

Normed AM vector $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{A}_\gamma^{-1}\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})/h_g$ is distributed between GD's pairs by the DL

$$f_\rho(\boldsymbol{\beta}) = (\tilde{x}_1 - \tilde{x}_2) + \rho\,(\tilde{x}_1\,\tilde{x}_2 - 1) = 0$$

with $\tilde{x}_1 = \mathsf{x}_{12}/\mathsf{q}_\mathsf{y}$; $\tilde{x}_2 = \mathsf{x}_{34}/\mathsf{q}_\mathsf{z}$; $\mathsf{q}_\mathsf{s} = (4 - \mathsf{s}^2)^{1/2}$, $\mathsf{s} = \mathsf{y}, \mathsf{z}$. For $\rho = 2\sqrt{6}/5$ this DL ensures *global maximum* of *Grame* determinant $\mathsf{G} = \det(\mathbf{A}_h\mathbf{A}_h^{\mathrm{t}}) = 64/27$ and *maximum* module of the warranted control torque vector $\mathbf{M} = \mathbf{M}^{\mathrm{g}}$ (6) in an *arbitrary* direction for the "park" state $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{0}$, as well as large singularitiless central part inside of the GMC AM's variation domain

$$\mathbf{S} = \{\mathsf{x}^2 + \mathsf{y}^2 + \mathsf{z}^2 - 2\mathsf{q}_\mathsf{y}\mathsf{q}_\mathsf{z} < 8;\, |\mathsf{y}| < 2;\, |\mathsf{z}| < 2\}$$

and only curves in the set of smoothly passed GMC internal singularities $\mathbf{Q}_{\mathsf{yz}}(\boldsymbol{\beta}) = \mathbf{Q}^p_\mathsf{y} \cup \mathbf{Q}^p_\mathsf{z}$, where

$$\mathbf{Q}^p_\mathsf{s} = \mathbf{Q}^*_\mathsf{s} \cap \mathbf{S}^*_\mathsf{s};\, \mathbf{S}^*_\mathsf{s} = \{\mathsf{s} = 0;\, |\mathsf{s}_1| = |\mathsf{s}_2| = 1\}, \mathsf{s} = \mathsf{y}, \mathsf{z};$$
$$\mathbf{Q}^*_\mathsf{y} = \{(\mathsf{x}_{34}/(2\rho))^2 + (\mathsf{z}/2)^2 = 1;\, \mathsf{x}_{34} < 0\};$$
$$\mathbf{Q}^*_\mathsf{z} = \{(\mathsf{x}_{12}/(2\rho))^2 + (\mathsf{y}/2)^2 = 1;\, \mathsf{x}_{12} > 0\}.$$

At "right-sided differential relay-hysteresis" tuning of the DL due to $D^+ f_\rho(\boldsymbol{\beta}) = \Phi_\rho(f_\rho(\boldsymbol{\beta}), \mathbf{h}(\boldsymbol{\beta}))$ with positive constants $\phi_\rho, \mu_\rho$ and $l_\rho$, where

$$\Phi_\rho(\cdot) \triangleq \begin{cases} -\mathrm{Sat}(\phi_\rho, \mu_\rho\, f_\rho(\boldsymbol{\beta})) & \mathbf{h} \in \mathbf{S} \setminus \mathbf{Q}_{\mathsf{yz}}; \\ \phi_\rho \mathrm{Relh}(a_\mathsf{s}, l_\rho, r_\mathsf{s}) & \mathbf{h} \in \mathbf{Q}^p_s, \mathsf{s} = \mathsf{y}, \mathsf{z}, \end{cases}$$

and the relay hysteresis function

$$\mathrm{Relh}(a, l_\rho, x) = (1,\, if\, x > -l_\rho) \vee (-1,\, if\, x < l_\rho)$$
$$\text{with } \mathrm{Relh}(a_\mathsf{s}, l_\rho, r_\mathsf{s}(\boldsymbol{\beta}(t_0))) = a_\mathsf{s} \in \{-1; 1\}, \mathsf{s} = \mathsf{y}, \mathsf{z};$$
$$r_\mathsf{y} = M_\pi(\beta_1 - \beta_2 - \pi);\quad r_\mathsf{z} = M_\pi(\beta_3 - \beta_4 - \pi);$$
$$M_\pi(\alpha) \equiv (\alpha, if|\alpha| \leq \pi) \vee (\alpha - 2\pi\,\mathrm{Sign}(\alpha), if|\alpha| > \pi).$$

This distribution law ensures its belonging to the *imaginary* singular set $\mathbf{Q}_{\mathsf{yz}}(\boldsymbol{\beta})$ only at *separate* time moments, and *bijectively* connects the vector $\mathbf{M}^{\mathrm{g}}$ with vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$.

## 7 Computer Simulation

Based on the above control laws, the RB motion have been simulated with the following parameter values: $J_x = 2900$, $J_y = 3600$ and $J_z = 870$ kgm$^2$ (Somov *et al.*, 2003*c*). Fig. 4 summarizes the simulation results for initial position of the SC AM vector $\mathbf{G}(t_0)$ with module $g_o = 300$ Nms along the unit $\mathbf{g}(t_0) = \{0, 0, 1\}$ within the BRF and its final position coincided with the unit $\mathbf{f} = \{0, 1, 0\}$. For clearness here the simplest canonical GMC schemes were applied:

- canonical scheme on 3 RWs with the constrains $\mathrm{m^m} = 0.15$ Nm and $\mathrm{h^m} = 5$ Nms;
- the *2-SPE* scheme on 4 GDs with angle $\gamma^{\mathrm{g}} = \pi/4$, $\mathrm{h}_g = 7.5$ Nms and constrain $\mathrm{u^m} = 10$ deg/s.

Some results on the *flexible* spacecraft dynamics during its respinup by four GDs with the same parameters, are presented in Fig. 5.

## 8 Recent Research

Optimization (Somov, 2000) and robust gyromoment control problems (Somov, 2001; Matrosov and Somov, 2004) were also considered for respinup of the flexible spacecraft. In addition to Somov *et al.* (2005*b*,*c*) problems of the SAPs guidance on the Sun were studied. Moreover the SC inertia tensor is changed into the BRF and the GMC's control torque vector $\mathbf{M} = \mathbf{M}^{\mathrm{g}}$ is re-calculated for the principle central axes for variable SC inertia tensor.

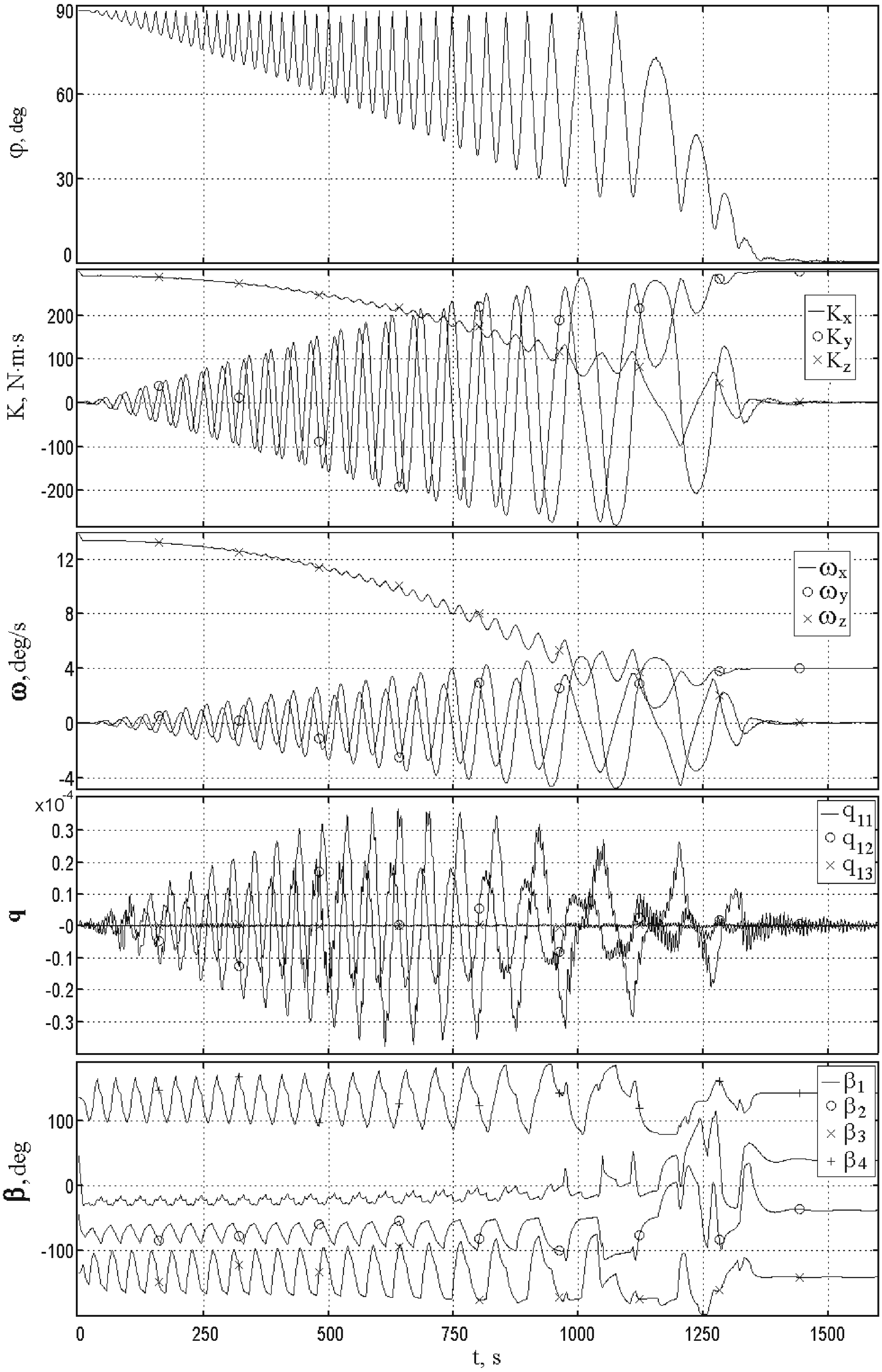


Figure 5. Dynamics of the flexible SC respinup by 4 GDs with own AM $h_g = 7.5$ Nms and constrain $u^m = 10$ deg/s

## Conclusion

Principle aspects of nonlinear dynamics related to the controlled coincidence of any SC body axis with the SC AM vector by the RWs or the GDs were presented. Method for synthesis of nonlinear control law and analytical proof of stability for the required spacecraft rotation mode were developed. Some optimization and robust gyromoment control problems were also discussed for respinuping a flexible spacecraft.

Obtained results were verified by the careful computer simulation of strongly nonlinear processes.


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