Shape Control of Smart Structures using Fluidic Actuators

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Abstract - Shape control means control of position or alignment of a certain number of points of the structure so as to track a desired value, which is an important task of smart structures. Shape control is mostly extended by smart materials such as piezoelectric actuators or shape memory alloys. In this paper a new approach for static shape control of beams without the draw back of the smart materials-based scheme is developed. A system of eccentric reversing channels embedded into structures is then developed utilizing this fluidic actuator which acts as a continuous actuator. Shape control of elastic structures. In this system, by adjusting parameters such as fluid's velocity, fluid's density, and the eccentricity path's equation of the channel, shape and deflection curve of the beam can be controlled. Several analytical examples are at last demonstrated considering different eccentricity paths for the channel with and without external load. Distinct characteristics are observed in the deflection curve associated with this kind of fluidic actuators, such that the tip deflection for a cantilever beam embedded with fluidic actuators does not necessarily occurs at the tip. This method can be applied in aircraft wings for getting higher lift or drag at the time of take off or landing.

I. INTRODUCTION

Shape control means control of position or alignment of a certain number of points on the structure so as to track a desired value, which is an important task of smart structures.

Shape control is mostly extended by smart materials such as piezoelectric actuators or shape memory alloys [1-3] but other approaches are either developed like using compliant mechanisms [4] or the principle of stress-free eigenstrain load [5].

The main idea of this paper has arisen from the pipes conveying fluid [6,7]. The author has later developed the dynamic behaviour and stability boundaries of the pipes conveying fluid with flexible support [8] and, under axial load [9].

In this paper a new approach for static shape control of beams without the draw backs the smart materials-based schemes is developed. In the current paper the focus is on the shape control of the elastic structures and the dynamic behaviour is no more interested.

II. PHYSICAL MODEL OF THE FLUIDIC ACTUATOR

Schematic of the fluidic actuator is depicted in Fig. 1. A channel is generated in the beam for passing the fluid.

The eccentricity path of the channel ($e(x)$) can follow any curve e.g. parabola, exponential, sinusoidal, etc. Shape and also deflection at any point of the beam such as tip deflection can then be controlled by changing the fluid's density, velocity or eccentricity.

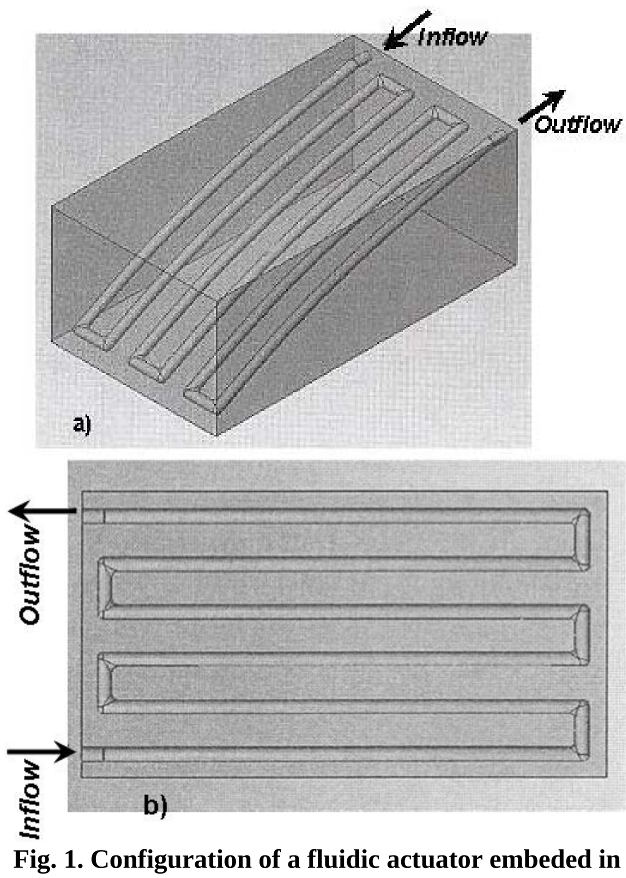

Fig. 1. Configuration of a fluidic actuator embeded in an elastic beam, a) 3d, b) Plane.

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A. *Governing differential equation:*

For deriving the governing differential equation, beam is considered to be an Euler-Bernoulli beam and small lateral motion is assumed.

A unit force per unit length is exerted on the beam caused from the velocity of the moving fluid and the curvature of the channel.

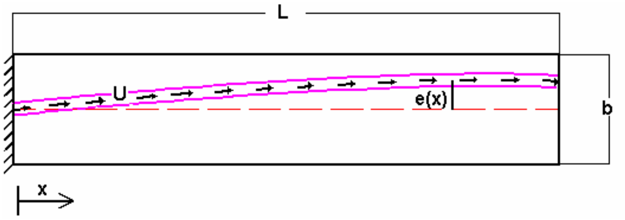

Fig. 2- A smart clamped beam with embedded channel (the actuating tube doubles back four times).

The fluid flows through the curved channel and thus applies a unit force per unit length on the beam which is a function of fluid's velocity and curvature of the beam.

For calculating curvature of the beam, both, deflection curve of the beam and also the curvature of the eccentricity path of the fluid's channel should be considered. The governing equation of the elastic structure is then obtained [10]

$$EI\frac{\partial^4 W}{\partial X^4} + m_f V^2 \frac{\partial^2 (W+E)}{\partial X^2} = m\frac{\partial^2 W}{\partial t^2} \tag{1}$$

Vibration analysis of this system is similar to the pipe's conveying fluid, which is extended before by the author [9]. In this paper, we are focused on the shape control of the structure and the dynamic behaviour of the system is not interested any more. Thus a steady-state flow is considered and the right term of (1) is neglected.

$$\frac{d^4 W}{dX^4} + \frac{m_f V^2}{EI}\frac{d^2 W}{dX^2} = -\frac{m_f V^2}{EI}\frac{d^2 (E)}{dX^2} \tag{2}$$

where $m_f V^2$ is momentum of the fluid and E(x) is a known function that denotes equation of the curved channel.

Then the nondimensional equation of the system is introduced by defining these nondimensional terms.

$$x = \frac{X}{L}, \qquad w = \frac{W}{L}, \qquad e = \frac{E}{L}, \quad u = VL\sqrt{\frac{m_f}{EI}} \tag{3}$$

where, L is the length of the beam. Then the nondimensional governing equation of the smart beam with embedded channel is rewritten

$$\frac{d^4 w}{dx^4} + u^2\frac{d^2 w}{dx^2} = -u^2\frac{d^2 (e)}{dx^2} \tag{4}$$

A clamped beam is considered in this paper for demonstrating shape control of elastic structures using fluidic actuators. The boundary conditions of a cantilever beam is

$$x = 0 \rightarrow \begin{cases} w = 0 \\ \dfrac{dw}{dx} = 0 \end{cases}, \quad x = 1 \rightarrow \begin{cases} M = 0 \\ V = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{d^2 w}{dx^2} = 0 \\ \dfrac{d^3 w}{dx^3} = 0 \end{cases} \tag{5}$$

B. *Solution of the governing equation*

In this section, an analytical solution is given for any arbitrary eccentricity path equation and the use of fluid on shape control of beams is clarified by several examples. (4) is rewritten in the form of a second degree differential equation

$$v'' + u^2 v = -u^2 e \quad , \quad v(x) = w''(x) \tag{6}$$

where *v* is the moment along the beam. Taking Laplace transform from (6)

$$(s^2 + u^2)V(s) - s.V(0) - \dot{V}(0) = u^2\, E(s) \tag{7}$$

The boundary condition for *V* is known and is equal to zero at the free end of a cantilever beam, thus the x coordinate is here considered to start from the free end to the clamped end. Then using convolution integral

$$w''(x) = v(x) = -u.\int_0^x \sin u(x-\xi).e(\xi)d\xi \tag{8}$$

(8) should be integrated twice for obtaining deflection of the beam. The boundary condition of w is now known at the clamped end, thus the x coordinate is changed to its first state by using this transformation

$$w''(1-x) = -u.\int_0^x \sin u(x-\xi).e(\xi)d\xi \tag{9}$$

After integrating (9) and imposing the boundary conditions for the beam and the eccentricity path, shape the beam, which is a function of velocity and x coordinate is obtained.

In the next section several eccentricity paths are utilized for demonstrating characteristics of a fluidic actuator and controlling shape of the beam using this actuator.

III.NUMERICAL EXAMPLES

Different equations for the eccentricity path of the fluid are considered to clarify the usage of fluid in the shape control.

A. *Parabola*

As the first example, the eccentricity path equation of the channel is considered to be a polynomial of the second degree.

$$E(x) = C\,X^2 \tag{10}$$

The channels parameter is designed to gain (See Fig. 2)

$$\begin{aligned} &E(0) = 0 \\ &E(L) = \frac{b}{4} \end{aligned} \qquad \Rightarrow C = \frac{b}{4L} = \frac{1}{4}a \tag{11}$$

where b and a are the width and aspect ratio of the beam respectively.

$$\frac{d^4w}{dx^4} + u^2\frac{d^2w}{dx^2} = -u^2\frac{d^2(e)}{dx^2} = -2u^2.C \tag{12}$$

This differential equation is solved considering the boundary conditions and the deflection curve of the beam is obtained

$$w(x,u) = -\frac{2C}{u^2}\cos u(1-x) - C\,x^2 + \frac{2C}{u}\sin u\;\, x + \frac{2C}{u^2}\cos u \tag{13}$$

Shape of the beam is then plotted in Fig .3 for several nondimensional velocities of the fluid. Deflection of the beam is a function of the nondimensional velocity of the beam as well as x.

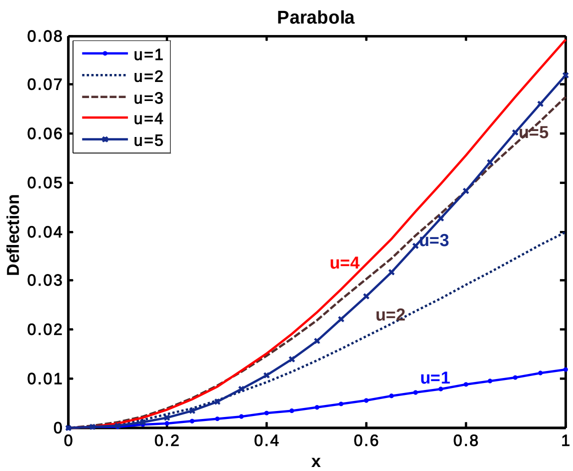

Fig. 3. Deflection of an elastic clamped beam with embedded channel following a parabola eccentricity path

This kind of beams can be considered as actuators or even grippers since deflection of its tip can be controlled by tuning the fluid velocity. Thus, tip deflection of the beam is separately studied.

$$w(1) = \frac{2C}{u^2}(-1 + u.\sin u + \cos u) - C = f(u) \tag{24}$$

Tip deflection of the beam with parabola eccentricity path equation is plotted in Fig.4, which indicates that increasing velocity will not always result in larger deflection. For other kinds of equations shape and tip deflection of the beam is computed as follows and plotted.

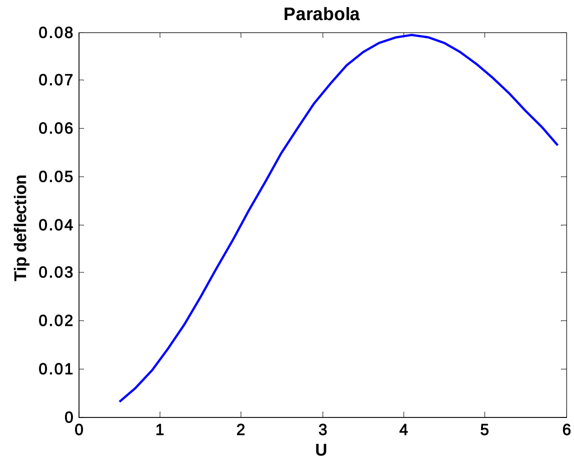

Figure 4 Tip deflection of the Clamped beam- Parabola

B. Exponential

The eccentricity path equation of the channel is considered to be an exponential

$$e(x) = e^{(\ln(a/4).x)} \tag{15}$$

The channel parameters are designed to gain (See Fig. 2)

$$E(0) = 0\ , E(L) = \frac{b}{4} \tag{16}$$

where b and a are the width and aspect ratio of the beam respectively. This differential equation is solved considering the boundary conditions, and the deflection curve of the beam is obtained

$$\begin{aligned} w(x) = \frac{\alpha^2 u}{u^2+\alpha^2}.\{&-\frac{1}{u}\cos(u(1-x)) + \frac{\alpha}{u^2}\sin(u(1-x)) - e^{\alpha(1-x)}.\frac{u}{\alpha^2} + \\ &(\sin(u) + \frac{\alpha}{u}\cos(u) - e^{\alpha}.\frac{u}{\alpha})\,x + \frac{1}{u}\cos(u) - \frac{\alpha}{u^2}\sin(u) + e^{\alpha}.\frac{u}{\alpha^2}\} \end{aligned} \tag{17}$$

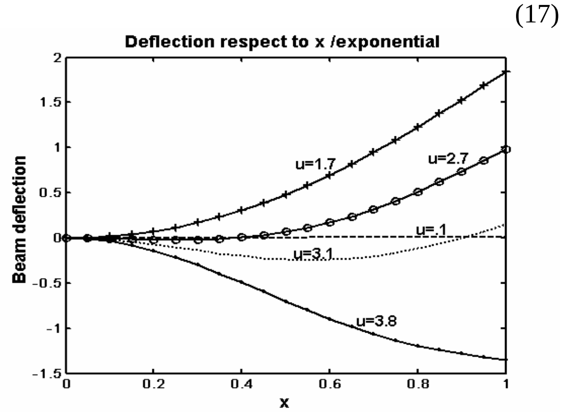

Fig.5. Shape of the elastic clamped beam- Exponential

Shape of the beam is then plotted in Fig .5. Noticing shape of the beam at u=3.1, it's observed that the maximum deflection has not occurred at the tip.

Since these smart beams can be applied as actuators and the tip deflection is one of the most important parameters of an actuator, we have studied the tip deflection of the beam separately. This deflection can be controlled by tuning fluid's velocity.

$$w(1) = \frac{\alpha^2 u}{u^2+\alpha^2}.\{-\frac{1}{u}-\frac{u}{\alpha^2}+\sin(u)+\frac{\alpha}{u}\cos(u) - e^{\alpha}.\frac{u}{\alpha}+\frac{1}{u}\cos(u)-\frac{\alpha}{u^2}\sin(u)+e^{\alpha}.\frac{u}{\alpha^2}\} \qquad (18)$$

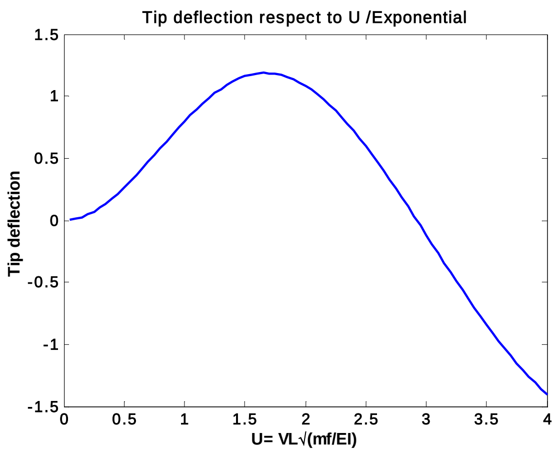

Fig. 6. Tip deflection respect to nondimensional velocity- Exponential

Fig. 5 and 6, indicate that increasing velocity will not always result in larger deflection.

C. Sinusoidal

$$e(x) = a/2.\sin(\frac{\pi}{6}x) \qquad (19)$$

$$w(x,u) = -\frac{a.u}{(\frac{\pi}{6})^2-u^2}\{(\frac{\pi}{6})^3.\sin u(1-x) - u^3.\sin\frac{\pi}{6}(1-x) + (-(\frac{\pi}{6})^3.u.\cos u + (\frac{\pi}{6}).u^3\cos(\frac{\pi}{6})).x - (\frac{\pi}{6})^3.\sin u + u^3.\sin\frac{\pi}{6}\} \qquad (20)$$

$$w(1) = -\frac{a.u}{(\frac{\pi}{6})^2-u^2}\{-(\frac{\pi}{6})^3.u.\cos u + (\frac{\pi}{6}).u^3\cos(\frac{\pi}{6}) - (\frac{\pi}{6})^3.\sin u + u^3.\sin\frac{\pi}{6}\} \qquad (21)$$

In the equation of the shape of the clamped beam with an embedded channel following sinusoidal eccentricity path, sinusoidal and polynomial functions are observed.

D. Calculation of Maximum deflection

The maximum deflection of a clamped beam with exponential eccentricity path is calculated by differentiating the deflection curve equation (Eq.(15)). It observed that by increasing the fluid' velocity, the shape of a beam has an exterimum which is mathematically a minimum. Comparing the absolute value of the deflection at this point with the tip deflection, it has pointed out that the maximum deflection is the tip or is the minimum of the deflection curve of the beam.

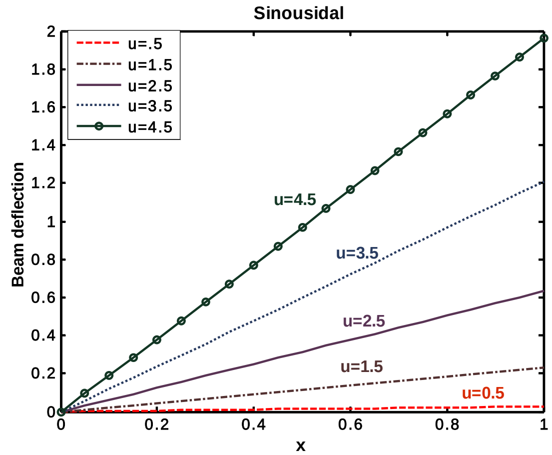

Fig.7. Shape of the elastic clamped beam- Sinusoidal

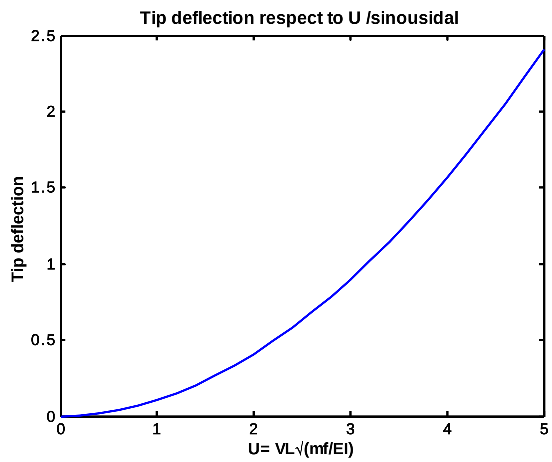

Fig. 8. Tip deflection respect to nondimensional velocity- Sinusoidal

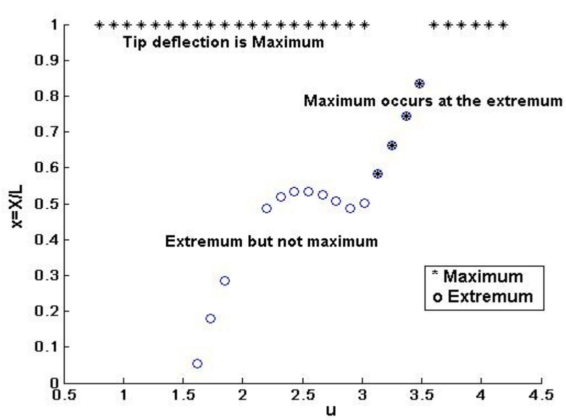

Figure 6- Calculation of the maximum deflection

E. External Loading

Smart structures under extra loadings are considered here to demonstrate another feature of the fluidic actuators. By tuning the fluid's parameters such as velocity an elastic structure can maintain its straight shape under extra loading. Smart structures may be utilized as actuators or grippers and consequently they should operate under extra loads. In this section the velocity which results in zero deflection is carried out.

As common external loading which is a concentrated load exerted at the tip end of the beam is considered here.

Deflection curve of the cantilever beam under a concentrated load is

$$W = \frac{PX^2}{6EI}(3L - X) \tag{3}$$

The equation is nondimentialized using the following parameters.

$$w = \frac{p}{2}x^2(3 - x) \tag{4}$$

$$\text{where } w = \frac{W}{L} \quad x = \frac{X}{L} \quad and,\ p = \frac{PL^2}{3EI} \tag{5}$$

Fig.9. depicts shape of a clamped beam with exponential eccentricity path for the channel. The beam is under an external load at the tip. By tuning the fluid's velocity it is observed that at u=.441, the beam will maintain its straight shape. Other desired shapes for the beam or a preferred tip deflection can also be obtained by adjusting the fluid's parameters.

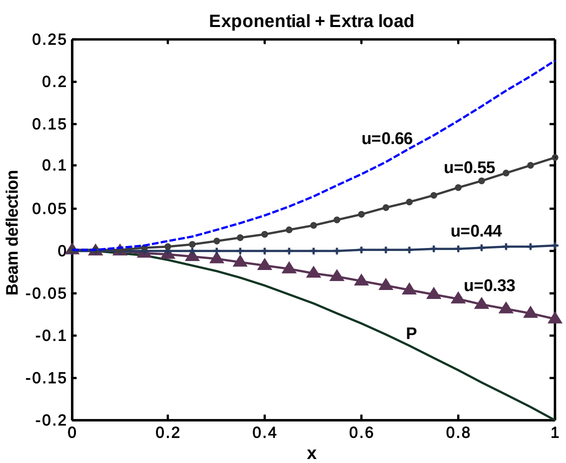

Fig. 9. Shape of the beam under extra loading is controlled

IV.CONCLUSIONS

In this paper, a new idea is developed to control the shape of elastic structures. A channel for passing fluid which can follow different curves is embedded into the system that doubles back. Adjusting the parameters such as fluid's density, velocity and also shape of the curved channel leads to desired shape of the structure.

Several analytical examples with different eccentricity curves are developed in order to demonstrate use of fluidic actuators in shape control of elastic structures. Distinct characteristic of this fluidic actuators are also discussed like the maximum deflection and also the velocity that the maximum tip deflection occurs. As we have shown, the tip deflection is not surely the maximum deflection in these structures.

This method can be applied in aircraft wings for getting higher lift or drag at the time of take off or landing.

ACHNOLEGMENTS

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