

Mathematical Models of Complex Flexible Missile and Software for Control System Design and Simulation

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Abstract: Approaches to the mathematical description of different types of flexible vehicles in view of oscillations of fluid in tanks and moving masses inside the vehicle are observed. Elastic bending of a body surface in interaction with a surrounding medium in a broad band of speed variation are taken into account. Problems of regulator's synthesis, damping of elastic oscillations, and also principles of construction of universal software for research of dynamic properties and simulations of elastic vehicles motion are considered. For implementation of suggested methods and algorithms the specialized program is designed. The software package is supplied with the program modules library. These modules are designed on the basis of mathematical models of the vehicle elements and control system, and also the significant physical phenomena such as flexibility, liquid oscillations, time lag of engines, local aerodynamic effects, etc. Functioning of the program is demonstrated and outcomes of calculations are presented.

Keywords: flight control, control design, simulation, control, flexible, vehicle dynamics, system synthesis

1. INTRODUCTION¹

Possible approaches to the mathematical description of different types of flexible vehicles are observed. Mass and aerodynamic characteristics are changing considerably during the flight of aerospace vehicles. From the point of view of control theory such vehicles are the typical non-linear and non-steady plants. The aim of designer is to create the light construction. For these reason such objects are deformed in flight, and their elastic properties appear. Elastic longitudinal and lateral oscillations of the complex form arise, which frequencies are changing during the flight. Elastic oscillations are usually described by differential partial equations or ordinary differential equations of the great dimension. Deformation of a body results in appearance of the local attack angles and slide angles. As a result of it, the local forces and moments of forces arise. These forces and moments are synchronized with the changes of local angles of attack and slide. The local forces and moments are the reasons of amplification or attenuation of elastic oscillations. This phenomenon is known as aeroflexibility. At excessive development of elastic oscillations the structural failure may take place. Paying attention to these effects has a great importance at control of space stations and space probes, airplanes and other mobile objects liable to the considerable dynamic loads.

Besides the flexibility and aeroflexibility, it is necessary to take into account in the mathematical models of aerospace vehicles the following factors:

1. Dependence of all parameters on time, velocity and altitude of flight, drift of CG, and so on.
2. Distributed and integral aerodynamic forces.
3. Oscillation of liquid in tanks (Sloshing).
4. Inertia of engines.
5. Errors of measuring instruments.
6. Environment stochastic forces and moment of forces.

Mathematical descriptions of all these factors are complex and are based on different physical models. For research of elastic systems the special programs exist, for example, ANSYS, NASTRAN, COVENTOR, FEMLAB, Structural Dynamics Toolbox for use with MATLAB, etc.

In these programs the finite element method is used, which has been well recommended at calculation concerning the simple designs. For dynamic processes investigation and also for simulation of elastic oscillations of the flying vehicles, which consist of hundreds and thousand units of complex form, such approach is unsuitable. For calculation of distributed and integral aerodynamic forces can be used the program Fluent. The program Matlab allows to design of control or stabilization system. It is required to use all models simultaneous for analysis of interconnections and detection of possible resonances.

For with reason, the resulting mathematical model is very complex. There are no known programs to analyze and simulate such complex systems which include subsystems, which base on different physical principles.

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The indicated reasons determine the necessity of development of the specialized program for simulating the motion of flexible objects of the composite form, the analysis of their dynamic properties and design of control systems

In the present paper other approach to modeling and control system design for flexible objects is observed. It is known that the flexible object is described by partial differential equations. The control theory of such objects is complex, bulky and presently is insufficiently designed analytically. There are numerical methods of calculation of the arbitrary quantity of harmonics of flexible vibrations and replacements of partial differential equations by ordinary differential equations of high dimension. For automation of analytical derivation of such mathematical models of flexible aerospace vehicle, for control law synthesis, for analysis and simulation of controlled flight, and also for representation of outcomes of modeling in the two-dimensional and three-dimensional space, the authors have developed the specialized software package.

2. METHODS OF THE PROBLEM SOLUTION

Authors propose new approach and special program to input construction of aerospace vehicles, to calculate mathematical model, to correct this model on the basis of separate experiments, to simplify separated models for any factors. The program allows using whatever experimental data about properties of the vehicle, presented in the most various formats. Hand-operated input and correction of separate values, and also the automated lead of large arrays of the information is provided. At absence or inaccessibility of a part of experimental data in the program, the models based on various theories or on generalization of experimental data of vehicles are used. The software for simulation of flexible essentially non-steady vehicle motion, synthesis of control systems for such a vehicle, research of dynamic properties by different methods in time and frequency domains, is developed and described in this paper. The basis of the program is the structure which allows analyzing the dynamic responses, simulation and visual information representation of the complex dynamic systems.

All stages of aerospace vehicles design are discussed, including the following problems:

- input of initial constructive data of vehicle,
- determination of controllability and observability for the full and simplified model of a vehicle for real control inputs and arbitrary choice of measured signals,
- choice of flight program and control law,
- automatic linearization relative to arbitrary trajectory,
- automate processes of mathematical models simplification for flexible vehicles and separate physical phenomena (oscillations of a liquid in cavities, time lag of engines, local aerodynamic loadings, etc.) and to control these simplifications,
- execute a system synthesis of control for elastic object in frequency area with the given margin of stability on amplitude and a phase,
- control system synthesis for a vehicle with use of method of Kalman filtration and methods of optimal control,
- simulation of vehicle motion with nonlinear model and control law different complexity,
- investigation of local aerodynamic loads effect on elastic vibrations of a vehicle,
- determination eigenfrequencies of a liquid oscillations in tanks and computation of the local forces which affect on a vehicle body because of these oscillations,
- choice of sensors and actuators characteristics,
- computation charts of relations for any variables of state vector, both from a time, and from other variables of state vector,
- any frequency characteristics plots construction,
- choice of flight program and control law,
- determination of the elastic vibrations of a body and oscillation of liquid in tanks modes and to illustrate these oscillations as animations,
- study of control system sensitivity to vehicle parameters change.

3. MATHEMATICAL MODELS OF PHYSICAL PHENOMENA HAVING PLACE AT FLIGHT

3.1 Solid Dynamics

The rigid part of mathematical model of vehicle is allocated into the separate block, in which the system of differential non-linear equations of vehicle spatial motion is integrated. These equations in vector form in body-axes can be written as

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{F}}{m} - \boldsymbol{\Omega} \times \mathbf{V}, \quad (1)$$

$$\frac{d\boldsymbol{\Omega}}{dt} = \mathbf{I}^{-1}(\mathbf{M} - \boldsymbol{\Omega} \times (\mathbf{I} \cdot \boldsymbol{\Omega})) \quad (2)$$

These equations express the motion of a rigid body relative to an inertial reference frame. Here \mathbf{V} is velocity vector at the center of gravity (CG), $\boldsymbol{\Omega}$ is angular velocity vector about the c.g., \mathbf{F} is total external force vector, \mathbf{M} is total external moment vector, \mathbf{I} is inertia tensor of the rigid body.

Outputs of this subsystem are parameters of vehicle motion as a rigid body.

3.2 Flexibility

Equation of elastic line flexible displacements from the longitudinal neutral axis looks like

$$\Delta \mathbf{M} \ddot{\mathbf{q}} + \Delta \boldsymbol{\Xi} \dot{\mathbf{q}} + \mathbf{q} = \Delta \mathbf{f}, \quad (3)$$

where $\mathbf{q}(t)$ is deflection of elastic line from the longitudinal axis; Δ is symmetrical stiffness matrix; \mathbf{M} is diagonal mass matrix; $\boldsymbol{\Xi}$ is symmetrical structural damping matrix; \mathbf{f} is

distributed load. This equation describes only the flexible displacements of object points in the body-fixed coordinates. The distributed loads resulting to longitudinal moving of object and its rotation are filtered by a matrix of rigidity, do not result in deformation and are taken into account only in the equations of object motion as a solid body. Damping and elastic forces do not affect the moving vehicle as a rigid body, because the condition of dynamic balance is satisfied.

Full equation of flexible displacement in generalized coordinate \mathbf{z} is set as:

$$\mathbf{G} \mathbf{V} \ddot{\mathbf{z}} + \mathbf{G} \mathbf{D} \mathbf{V} \dot{\mathbf{z}} + \mathbf{V} \mathbf{z} = \sqrt{\mathbf{M}} \Delta \mathbf{f}, \quad (4)$$

where $\mathbf{z}(t)$ is vector of generalized coordinates (modes of oscillations); $\mathbf{G} = \sqrt{\mathbf{M}} \Delta \sqrt{\mathbf{M}}$; $\mathbf{D} = (\sqrt{\mathbf{M}})^{-1} \Xi (\sqrt{\mathbf{M}})^{-1}$; \mathbf{A} is diagonal matrix of eigenvalues of symmetric matrix \mathbf{G} ; \mathbf{V} is orthogonal matrix of eigenvectors: $\mathbf{V}' = \mathbf{V}^{-1}$. Eigenvalues equal to zero correspond to motion of solid body (Mishin, 1990). In more details procedure of transformation and simplification of the equations of flexible oscillations is given in the article (Panferov et al., 2008).

The eigenfrequency of i -mode of free bending oscillation is defined as

$$\omega_i = \lambda_i^{-1/2}. \quad (5)$$

The relation between displacements of elastic line $\mathbf{q}(t)$ and generalized coordinates $\mathbf{z}(t)$ looks like

$$\mathbf{q}(t) = \sum_i \mathbf{h}^{<i>} z_i(t), \quad (6)$$

where \mathbf{H} is matrix of shapes $\mathbf{h}^{<i>}$ of free bending oscillations $\mathbf{H} = (\sqrt{\mathbf{M}})^{-1} \mathbf{V}$.

For the flexible discrete system having the definite number of point mass particles the number of eigenfrequencies accords to the number of particles and can be defined by the equation (6) dimension. Shapes and eigenfrequencies for this system can be found as the exact solutions. For the continuous object, when the tolerance of forms and eigenfrequencies evaluation is given, the sampling frequency sets the number of bending eigenfrequencies.

Simulation of complex model with a big number of modes allows modes with small magnitudes. Elimination of those modes practically not influence on transient processes. For this reason it is advisable to limit the number of modes when simulating the distributed flexible object dynamics by dominant harmonics. Reduced equation of flexible displacement in generalized coordinate \mathbf{z} is such:

$$\mathbf{G} \mathbf{V}_{\{K\}} \ddot{\mathbf{z}} + \mathbf{G} \mathbf{D} \mathbf{V}_{\{K\}} \dot{\mathbf{z}} + \mathbf{V}_{\{K\}} \mathbf{z} = \sqrt{\mathbf{M}} \Delta \mathbf{f}, \quad (7)$$

K is dominant modes numbers: $K = \{i_1, i_2, \dots, i_k\}$, $k < n$; $\mathbf{V}_{\{K\}}$ is matrix that consists of K columns of matrix \mathbf{V} .

The equation (7) in the matrix form describes the singular system of differential equations that cannot be expressed by the highest order derivative. The transformation

$$\ddot{\mathbf{z}} = (\mathbf{G} \mathbf{V}_{\{K\}})^+ \{ - \mathbf{G} \mathbf{D} \mathbf{V}_{\{K\}} \dot{\mathbf{z}} - \mathbf{V}_{\{K\}} \mathbf{z} + \sqrt{\mathbf{M}} \Delta \mathbf{f} \} \quad (8)$$

is used for its numerical integration.

The elastic line displacements can be divided into two components $\mathbf{q} = \tilde{\mathbf{q}} + \hat{\mathbf{q}}$, as:

$$\hat{\mathbf{q}} \in L((\sqrt{\mathbf{M}})^{-1} \mathbf{V}_{\{K\}}), \quad \tilde{\mathbf{q}} \perp L((\sqrt{\mathbf{M}})^{-1} \mathbf{V}_{\{K\}}). \quad (9)$$

In the equation (9) the elastic line displacements $\hat{\mathbf{q}}$ and its derivatives $\dot{\hat{\mathbf{q}}}$ and $\ddot{\hat{\mathbf{q}}}$ are taken into account.

3.3 Aerodynamics and local loads

The distributed and concentrated forces appear because of formation and a break-down of a vortex on the vehicle surface. Local aerodynamic effects substantially depend on the velocity and altitude of flight, the form of a mobile object, angular orientation and flexible deformations of a body. Even at a constant velocity of flow on the vehicle surface the vortices are generated. It results in the composite and time-varying distribution pattern of local loads on a surface of object. At high speeds of flight there are local spikes of pressure in the separate parts of vehicle. For their modeling it is important to define zones of the applying of large local loads and their time history. Usually these zones are arranged close to transitions from conical to cylindrical surface forms or to places of joints of surfaces with more composite form. In designing of vehicle the aim to avoid such connections is usually set, but it is not possible to remove them completely. Here the models for description of the most typical local loads from vortices are resulted. Large local loads arise near to junctions of separate structural members. Usually these are places of transition from a conic surface to cylindrical, places of connection of cylinders of miscellaneous diameter. Vortex flows will be produced a little bit below streamwise places of details bonding, intensity of vortexes and frequency of their separation largely depends on conditions of flight. More particularly these models are described in (Brodsky et al., 2004; Nebylov et al., 2005a, b; Caldwell et al., 2000).

Distributed and integral aerodynamics forces are calculated in this program block. Parameters of vehicle motion as rigid body and bending oscillations for each flight moment are taken into account. Distributed coefficients are evaluated for each point along the longitudinal axis of vehicle. Distributed aerodynamic coefficients $C_n(x)$ and allocated values C_{n_i} are linked with integral coefficients C_n , $C_m(x_{cg})$, $C_{mq}(x_{cg})$ at arbitrary disposition of center gravitation x_{cg} , by the equations:

$$C_n = \sum_i C_{n_i} = \int_0^l C_n(x) dx, \quad (11)$$

$$C_m(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg}) = \int_0^l C_n(x)(x - x_{cg}) dx, \quad (12)$$

$$C_{mq}(x_{cg}) = \sum_i C_{n_i}(x_i - x_{cg})^2 = \int_0^l C_n(x)(x - x_{cg})^2 dx. \quad (13)$$

Local angle of attack a_i^* at a point with coordinate x_i on the line of vehicle longitudinal axis with account of flexible oscillations is

$$a_i^* = a + \frac{x_{cg} - x_i}{V_i} \dot{\theta} - \frac{\dot{q}_i}{V_i} + \frac{\partial q_i}{\partial x_i}, \quad (14)$$

where $\partial q_i / \partial x_i$ is slope of elastic line in current time; \dot{q}_i is velocity of shape of elastic line. Here V_i is local air velocity; a is angle of attack of solid body; ρ is air density.

The force f_i , distributed along longitudinal axis of vehicle and integral drag force F_x are evaluated by the block as:

$$f_i = \rho \frac{V_i^2}{2} C_{n_i} a_i^* \quad F_x = \frac{\rho \cdot V^2}{2} C_d a. \quad (15)$$

The described approach allows calculating distributed pressure on the surface of the vehicle. Visualization of the results of calculations of the pressure for nonsymmetrical flow in the processes of flight is shown in Fig. 1.

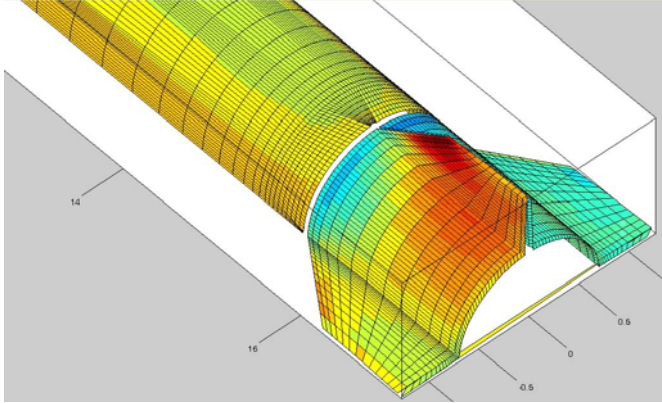


Fig. 1. Visualization of the nonsymmetrical pressure

4. STRUCTURE OF CONTROL SYSTEM

In the design stage the state-space model reduction for stabilization and guidance systems is used. The complexity of the models used in describing the aeroelastic effects via the equations of motion discussed previously makes design of stabilization and guidance systems an extremely difficult problem.

To start this process it is necessary to linearize the system dynamics near the nominal trajectory. Reduction of the linear model is then performed and the desired manner in which the reduced-order linear model approximates the full-order model. At the stage of control system synthesis it is important to represent accurately the system frequency response in the passband of the closed loop system. There are frequencies both above and below the critical frequency range which may not need to be well modeled. The frequency range of interest is very important for applying model simplification.

There are many methods by which the linear elastic vehicle models can be simplified. Several of these methods are used in the software package. The purpose of these simplifications is to design the robust controller. Truncation deletes some of

the modes or states from the full-order model. Balanced reduction minimizes frequency response error and has the certain advantages associated with obtaining the desired accuracy. Symbolic simplification addresses the impact of various physical parameters on the system responses and ignores those ones that have a little influence. Some advantages and disadvantages for each of these methods exist. After designing the robust or adaptive controller for simplified model, the analysis of real accuracy with wholeness is executed.

5. DESIGNING OF THE GUIDANCE SYSTEMS

The nonlinear flight dynamics equations of high dimension are not convenient for designing the control law for vehicle maneuvers. It is more acceptable to separate this problem into two stages. In the first stage the control law for damping of the certain flexible oscillations is synthesized. Usually these oscillations are in the passband of actuator. The plant with such control law is considered as a rigid plant and its mathematical model is simplified. This simplified model of the plant is used in the second stage of the control system synthesis. It is possible to write in the common case for arbitrary small interval of time for SISO system after linearization:

$$W(s) = \frac{k(s + \omega_{11}) \dots (s + \omega_{1i})(s^2 + 2\xi_{21}\omega_{21}s + \omega_{21}^2) \dots (s^2 + 2\xi_{2k}\omega_{2k}s + \omega_{2k}^2)}{(s + \omega_{31}) \dots (s + \omega_{3j})(s^2 + 2\xi_{41}\omega_{41}s + \omega_{41}^2) \dots (s^2 + 2\xi_{4l}\omega_{4l}s + \omega_{4l}^2)}$$

where k, ξ_{mn}, ω_{mn} are constant numbers. For numerical calculations the following transfer functions (TF) of the rigid W_{30}^R and flexible W_{30}^F models of vehicle were used:

$$W_{30}^R = \frac{-0.525(s^2 + 0.05401s + 0.0007627)}{(s - 0.03135)(s + 0.01475)(s^2 + 0.01588s + 1.766)}$$

$$W_{30}^F = W_{30}^R \frac{(s^2 + 0.4057s + 887)(s^2 + 134.8s + 2.225e004)(s^2 - 129s + 2.295e004)}{(s^2 + 1.138s + 2587)(s^2 + 2.787s + 1.304e004)(s^2 + 8.92s + 3.594e004)}$$

These TFs correspond to 30th second of hypothetical vehicle flight. Time response of the transversal acceleration of the inertial measuring unit of flexible vehicle as result of impulse deflection of rudder, in is shown Fig. 2. This time response contains all three modes of flexible oscillations. These oscillations are the reason of appearance big stresses in the body of the vehicle. For this the reason it is necessary suspend these oscillations in the processes flight control. Motion of rigid part of the vehicle, which corresponds to TF W_{30}^R , is not arose. It allows to separate the problems of synthesis of controllers for vehicle's rigid and flexible parts.

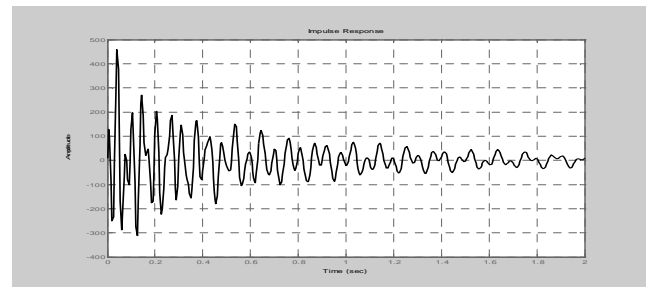


Fig. 2. Time response of acceleration on impulse

Frequency response of the flexible vehicle is shown in Fig. 3.

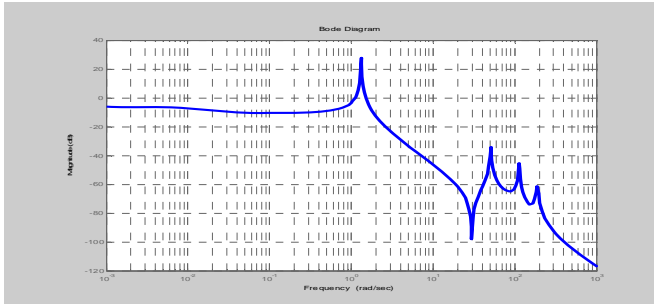


Fig. 3. Frequency response of the flexible vehicle

Separated multipliers in numerator and denominator of the TF describe separated components of the plant motion and separated modes of flexible oscillations. This TF can be transformed in the sum of simple fractions

$$W(s) = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2} + \dots + \frac{r_n}{s - p_n} + r(s). \quad (16)$$

For real plants the order of TF numerator is less than order of a denominator and for this reason $r(s) = 0$. The real poles p_i correspond to aperiodic components of plant movement. The complex pairs of poles correspond to oscillatory movement of object or modes of elastic oscillations. These pairs of poles are combined for deriving the TF of oscillatory parts with the real factors. Further, each TF will be transformed to system of the differential equations of the first order, and the obtained equations are united in uniform system of the equations of a following kind:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \quad (17)$$

$$y(t) = Cx(t) + v(t) \quad (18)$$

Physical interpretation of all components of state vector is very clear and it allows selecting the weighting coefficients for damp the separated flexible oscillations in the following functional

$$J = M \left\{ \int_0^{\infty} [x^T(t)Q_x x(t) + u^T(t)R_u u(t)] dt \right\}, \quad (19)$$

where matrixes Q_x and R_u are matrixes of weighting coefficients. The optimal control law is well known and can be written in the following form

$$u(t) = -R_u^{-1} B^T S \hat{x}(t), \quad (20)$$

where matrix S is the positive-definite solution of Riccati matrix algebraic equation

$$SA + A^T S - SBR_u^{-1} B^T S + Q_x = 0 \quad (21)$$

and the estimation of state vector is calculated in real time by integration of the following equations at the known initial conditions:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(t)(y(t) - C\hat{x}(t));$$

$$L(t) = R^{-1}(t)CP(t);$$

$$\dot{P}(t) = AP(t) + P(t)A^T - P(t)C^T R^{-1}(t)CP(t) + GQ(t)G^T.$$

The BodeMag diagram for flexible vehicle (blue) and for aggressive optimal control (green) for one of instant of flight are compared in Fig. 4. It is necessary to emphasize, that the synthesis of controller is fulfilled with proposition, that all own frequencies of the flexible vehicle are known exactly.

In practice these frequencies are not known exactly and often vary in time. In these cases, it is required to use an additional algorithms of estimation of own frequencies.

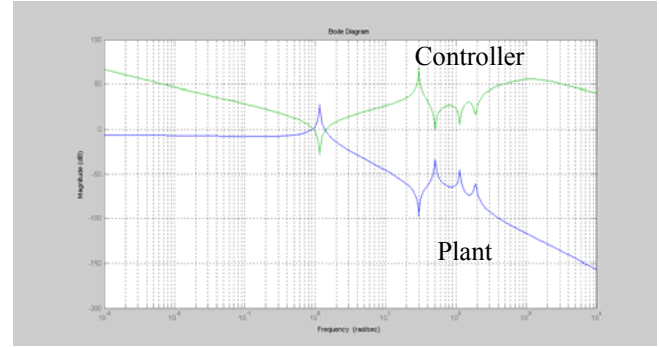


Fig. 4. BodeMag diagram for flexible vehicle (plant) and for aggressive optimal control.

Results of calculation of open loop Bode diagram for flexible vehicle with the optimal controller are shown in Fig. 5. Optimal controller corrects completely all flexible oscillations.

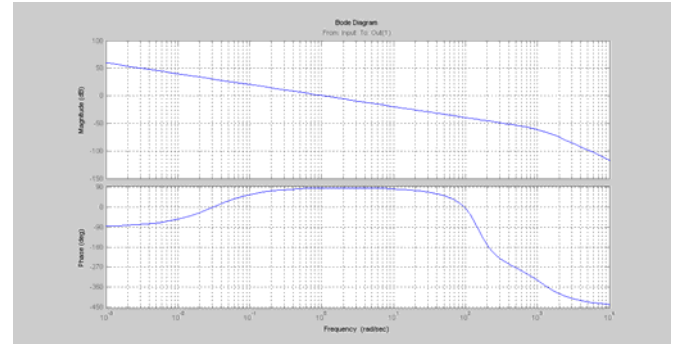


Fig. 5. Open loop Bode diagram for flexible vehicle for aggressive optimal control.

The time response on unit input step of close loop for pitch flexible vehicle with aggressive optimal control one instant of time is represented in Fig. 6.

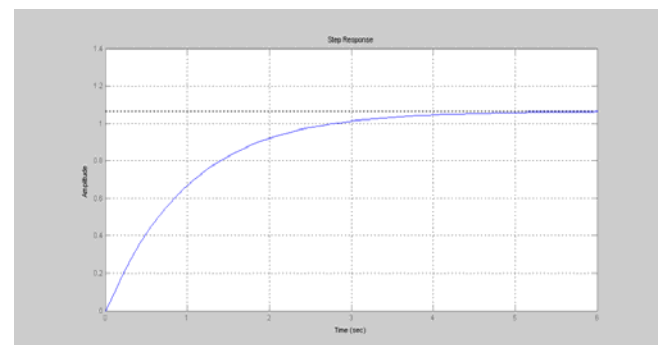


Fig. 6. Time response of closed system

6. CONCLUSIONS

The uniform mathematical model is suggested for simulation of flexible aerospace vehicle flight. It consists of some particular models describing such phenomena as solid dynamics, flexibility, aerodynamics and local loads, sloshing effects and other factors.

The approach to regulator synthesis for elastic object control is offered. The procedure of synthesis is separated into two stages. At the first stage the control law for damping of the certain flexible oscillations is synthesized. At the second stage the plant is considered as rigid and it is possible to use any known method for regulator synthesis.

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