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COMPLEX MODES OF POPULATION NUMBER DYNAMICS WITH AGE AND SEX STRUCTURES

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Abstract

In this paper we research a three-component mathematical model of population number dynamics. It considers sex and age structure dynamics and density-dependent effects impact on survival rates of a younger age class. We investigate some scenarios of the stabilized number transition to nonlinear modes of dynamics dependent on system parameters values and the type of the survival function. The results of numerical simulations are discussed.

Key words

Population models, age and sex structure, stability, dynamic modes, chaos.

1 Introduction

In this paper we consider the nonlinear three-componential model showing age groups number dynamics in a limited population.

By the beginning of a regular reproduction season the model population is represented by two age classes: the first one including immature individuals, and the second one- mature female and male animals participating in reproduction.

The population number variation is determined by a younger class increase in number dependent on the female and male animals ratio in a population; transition of a younger class to a mature one; death rate in elder groups.

Increase in the population number is regulated by density-dependent limitation of a younger class survival.

2 Mathematical model

In our mathematical model, n - number of a reproduction season; z - the number of individuals in a younger age class; x and y - the number of female and male animals participating in reproduction; c - immature females quantity. Birth-rate r depends on the ratio between females and males in a population. According to the Bazikin model [Bazikin,1985] it is described by hyperbolic functional dependence

$$r = \frac{a \cdot y}{\rho \cdot x + y} \tag{1}$$

where a - produce of the at most possible number of embryos per one impregnated female and the quantity of pregnant females of all impregnated ones; ρ - females and males ratio in the population, when half of females are impregnated.

It is assumed that both immature females - w and males - v are most sensitive to population density parameters, their survival dependent on this factor.

A decrease of the survival rate is also caused by competition within a younger age class, as well as with individuals of elder age classes.

The considered model, then, can be represented as a system of three recurrent equations:

$$\left.\begin{array}{l} z_{n+1} = a \cdot x_n \frac{y_n}{\rho \cdot x_n + y_n} \\ x_{n+1} = c \cdot w(z_n, x_n, y_n) \cdot z_n + s \cdot x_n \\ y_{n+1} = (1-c) \cdot v(z_n, x_n, y_n) \cdot z_n + p \cdot y_n \end{array}\right\} \tag{2}$$

The younger age class dynamics is governed by the first equation. Other equations describe dynamics of elder age classes.

Because of the fact that density-dependent factors restrict population development, all functions of the survival rate monotonously decrease and tend to zero with infinite increase of the argument.

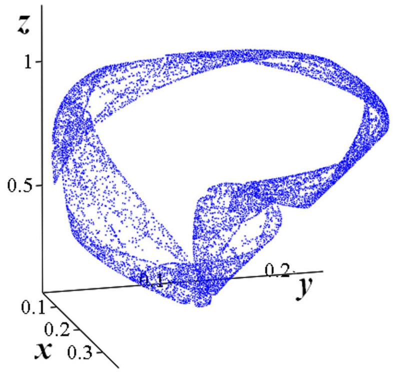

Figure 1. The phase portrait with the parameters $a = 3,6$, $c = 0,6, p = 0,7, s = 0,75, \alpha = 1, \rho = 0,05$

3 The model research

Analytical research of the system has shown that a loss of stability occurs in two ways.

In the first way the loss of stability occurs at the moment of the pair of complex- conjugate roots of the linearized system characteristic equation 2 passing through the unit circle ($|\lambda| = 1$).

As a result there appears a quasi-periodic motion acquiring a chaotic character under the variation of system parameters.

In the second way the loss of stability occurs when the root of the linearized system characteristic equation passes λ through -1 ($\lambda = -1$). Transition to chaos occurs through the cascade doubling of the period.

It is shown, that different types of density regulation in the population number increase correspond with completely different limited structures.

We have made some numerical simulations at the allowable (biologically substantial) parameter values of the system 2.

We have considered special cases of the basic model 2 under the survival function type provided that the survival rate of immature females is equal to the survival rate of immature males.

1. In the first case the survival rate of immature females and males depend on their own number increase. The survival rate function looks as follows $w = v = 1 - \alpha \cdot z$.

Instability of the system stationary point occurs when $|\lambda| = 1$.

Dynamics of the system becomes chaotic at that. There appears a limited invariant curve, which destroys creating a strange attractor under a further change of parameters in the system phase space (Figure 1).

Increase in birth rate (a) leads to the system‘s stationary solution instability and to oscillations emergence. At the same time stabilization of the system behavior takes place at increase the ρ parameter characterizing the gender ratio in a population.

Figures 2 and 3 illustrate bifurcation diagrams of the dynamic variable x (behavior of the dynamic variables y and z is similar) at increase of the coefficients a and ρ provided that the values of other parameters are fixed.

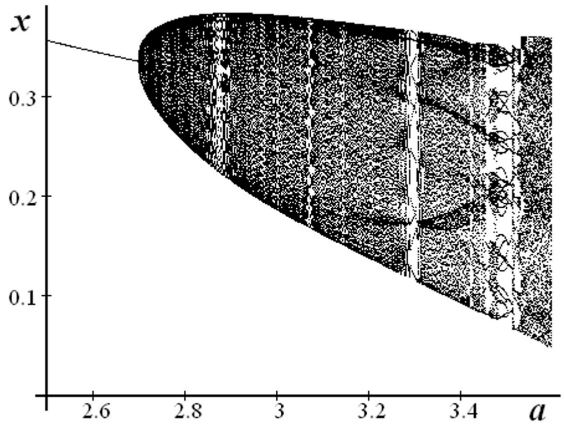

Figure 2. Bifurcation diagrams for a and fixed parameters $c = 0,6, p = 0,7, s = 0,75, \alpha = 1, \rho = 0,05$

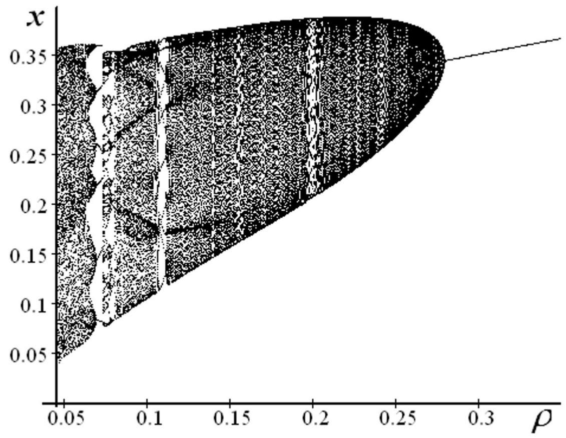

Figure 3. Bifurcation diagrams for ρ and fixed parameters $a = 3,6, c = 0,6, p = 0,7, s = 0,75, \alpha = 1$

2. The immature females or males survival depend on the mature females or males number. The survival rate function looks as follows $w = v = e^{-\alpha \cdot x}$ or $w = v = e^{-\beta \cdot y}$.

In this case we select the exponential dependence to demonstrate the dynamic modes connected with nonlinearity of the system.

The loss of stability in the system occurs when $\lambda = -1$.

We have made numerical simulation when $w = v = e^{-\alpha \cdot x}$.

Figure 4 shows the strange attractor of the system as a result of the period doubling.

If the functions of survival depend on the number of mature females, their decrease leads to destabilization of the system.

Figure 5 shows the dynamic modes map on the (s, a) parameters plane.

The dynamic modes map shows bifurcation lines of the doubling period, accumulating to the chaos border. This corresponds to the cycle of length 2 on the phase plane of the system.

There is a cycle of length 3 at some parameter values

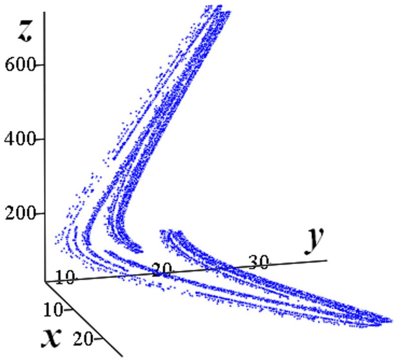

Figure 4. The phase portrait with the parameters $a = 30$, $c = 0,5, p = 0,7, s = 0,2, \alpha = 1, \rho = 0,005$

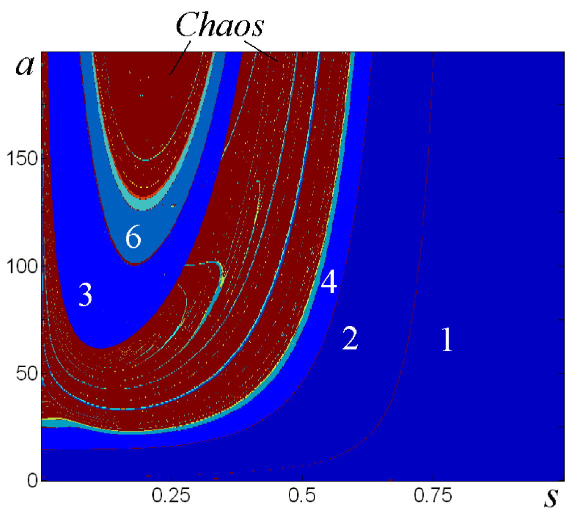

Figure 5. The dynamic modes map on the (s, a) parameters plane at $c = 0,5, p = 0,7, \alpha = 1, \rho = 0,005$. The periods of oscillations are numerated

s and a.

Figure 5 shows stabilization of the population number dynamics when the survival rate of mature females increases.

Figure 6 shows a bifurcation diagram of the dynamic variable at the birth rate variation, under the condition that $s = 0,2$.

The dynamic modes diversity takes place generally in that area of the parameters values where birth rate accepts the values not characteristic for natural populations. Therefore their dynamic modes are of more theoretical interest.

It is shown that the trajectories behavior is similar to the case $w = v = e^{-\alpha \cdot x}$ when the survival rate of immature individuals is dependent on only the male number ($w = v = e^{-\beta \cdot y}$).

3. In the third case survival rates of immature individuals depend on the younger and mature female number.

The survival rate function looks as follows $w = v = 1 - \alpha \cdot z - \beta \cdot x$.

Loss of stability in the system occurs in the two ways described above.

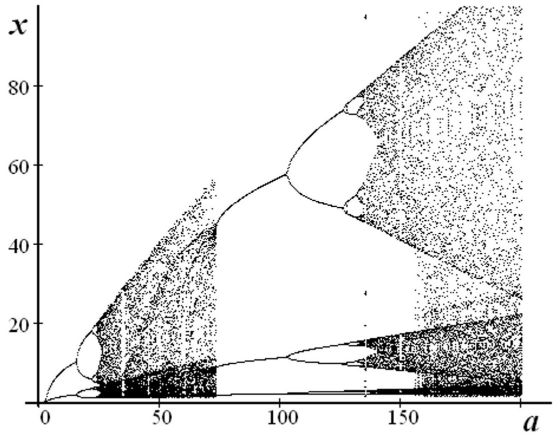

Figure 6. Bifurcation diagram for a and fixed parameters $c = 0,5, p = 0,7, s = 0,2, \alpha = 1, \rho = 0,005$

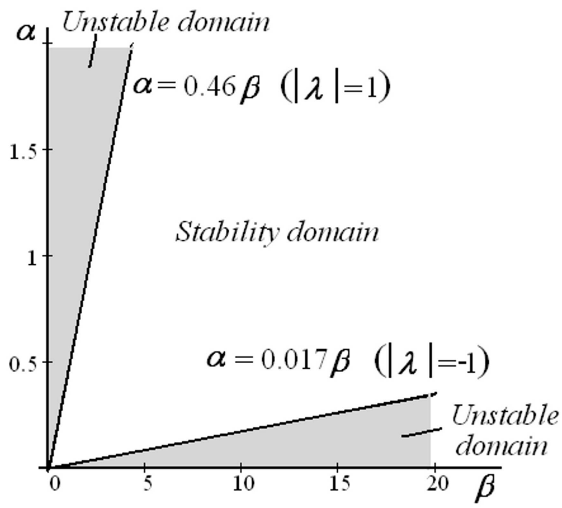

Figure 7. Areas of the system‘s various dynamic behavior on the (β, α) parameters plane at $a = 4,1, c = 0,5, p = 0,8, s = 0,8, \rho = 0,005$

In this case it is assumed that the male number increase does not considerably influence the survival rate of a younger class.

Actually, if the population is polygamous it includes a small number of mature males participating in the process of reproduction.

Therefore, their contribution to competition is minimal.

The population dynamics type (cycles, invariant curves, etc.) depends on a ratio of the parameters α, β describing elder age groups intensity of competitive pressure upon the survival of younger individuals (figure 7).

Figure 7 shows that transition to chaos through the cascade of the doubling period is possible only in the case when loss of young individuals is mainly caused by competition with adult females.

The area of stability decreases at the parameter a increase.

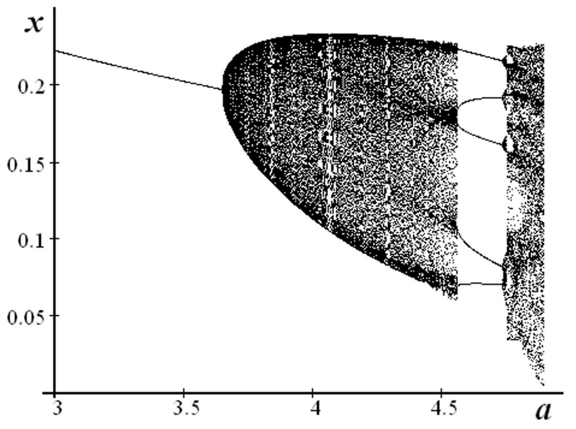

Figure 8. Bifurcation diagram for a and fixed parameters $\rho = 0,045$, $c = 0,5$, $p = 0,8$, $s = 0,8$, $\alpha = 1$, $\beta = 1$

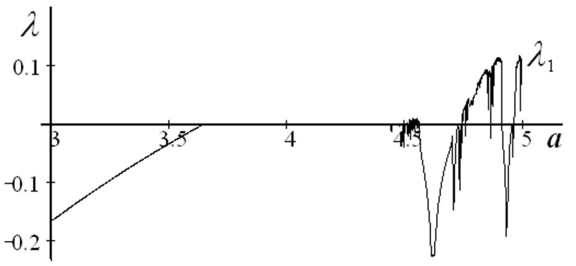

Figure 9. Dependence graph of the maximal Liapunov exponent at increase of the parameter value a, $\rho = 0,045$, $c = 0,5$, $p = 0,8$, $s = 0,8$, $\alpha = 1$, $\beta = 1$

If $\alpha < \beta$, the model population number dynamics stabilizes at the mature females survival rate increase.

Subject to $\alpha \geq \beta$, the model population number dynamics stabilizes against a background of the females survival rate decrease s.

It is shown, that the numbers of different age groups may display either regular or chaotic behavior. It takes place when the birth rate value a and value ρ characterizing the male and female ratio, pass through their critical values.

Figures 8 and 9 illustrate the bifurcation diagram of the dynamic variable and the Liapunov's exponent diagram dependence on the birth rate a under the specific values of other parameters.

We have calculated the Liapunov exponents with the Benettin algorithm [Nejmark and Landa,1987].

The maximal Liapunov exponent is negative if there is a stable equilibrium in the system.

At the moment of the stationary decision loss of equilibrium, and appearance and realization of a limit cycle, the maximal Liapunov exponent is equal to zero.

Figure 9 shows there is a great number of the parameter a values at which the maximal Liapunov exponent is positive.

The strange attractor exists in the model at these values of the parameter a (figure 10).

The diagram of the maximal Liapunov exponent contains fallings into the area of negative values, corresponding to the so-called periodicity windows (figure 9).

In the case when $\alpha < \beta$ the number dynamics can be presented by a cycle with the period of 2, 4, etc. years.

The number of cycles accumulates to the chaos border, gradually forming a uniform figure - the strange attractor (figure 11).

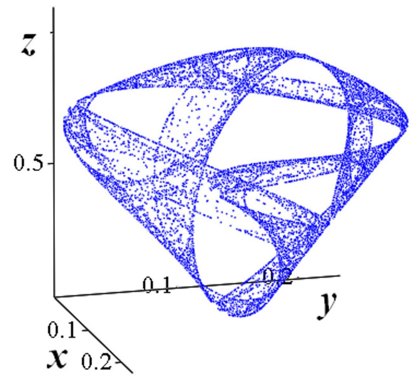

Figure 10. The phase portrait with the parameters $a = 4,9$, $\rho = 0,045$, $c = 0,5$, $p = 0,8$, $s = 0,8$, $\alpha = 1$, $\beta = 1$

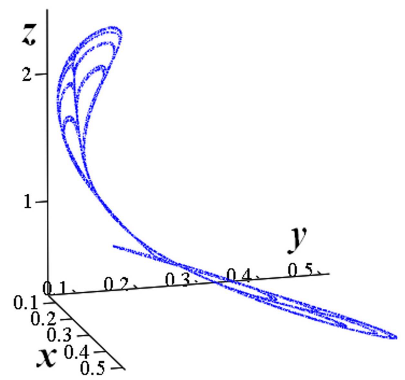

Figure 11. The phase portrait with the parameters $a = 4,9$, $\rho = 0,005$, $c = 0,5$, $p = 0,8$, $s = 0,8$, $\alpha = 0,1$, $\beta = 5$

4 Conclusion

Research of the three-component model, when the density-dependent factors regulate the growth of the younger class number, is carried out. The equilibrium points of the system are found. The conditions of existence and stability for every one are determined.

It is shown, that the population number dynamics character is determined by birth rate a, and the value ρ characterizing sex ratio in the population. Namely, the parameter a value increase and the parameter ρ value decrease may cause loss of stationary decision stability.

It is shown that loss of stability of the equilibrium population age distribution may occur in two ways.

Consequently, the number oscillations and the chaotic dynamic behavior of a population are possible in the system. Therefore the model may have limit stationary or complex chaotic fluctuations.

The occurrence of 2-cycles is possible when competition pressure on the side of adult individuals is stronger than self- limitation of the immature age class.

Thus, the type of population dynamics depends on a ratio of the parameters characterizing competitive pressure of all age classes upon immature individuals survival rates.

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References

Nejmark, Ju.I. and Landa, P.S. (1987). Stochastic and chaotic fluctuations. Moscow. 424. (In Russian).

Bazikin, A.D. (1985). Mathematical biophysics of interacting populations. Moscow. 182. (In Russian).