MAGNETOELASTIC WAVE PROPAGATION IN A VORTEX ARRAY IN A SUPERCONDUCTING LAYER

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Abstract

The paper deals with the magnetoelastic transverse wave propagation along a “vortex” layer in a superconducting body. It occurred that the propagation is possible. The propagation conditions and dispersion of those waves have been considered.

Key words

superconductivity, magnetic vortices, vortex field stress, surface waves

1. Introduction

Magnetic flux can penetrate the type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes or fluxons) each carrying a quantum of magnetic flux. Since the vortices are formed by the applied magnetic field, around of each of them the supercurrent flows. Moreover, there also exist some Lorentz force interactions among them. Those interactions form an origin of an additional thermomechanical (stress) field occurring in the type-II superconductor. That field near the lower critical magnetic intensity limit value H_{c1} is of the elastic solid character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, there occurs a flow (creep or diffusion) of vortex lines in the superconducting body. The *fluidity* of the vortex array has been also observed when the applied magnetic field tends to its upper critical limit value H_{c2} [Blatter, Feigelman, Geshkenbein, Larkin and Vinokur; Brandt; Cyrot and Pavuna].

In a pure crystal the arrangement of the vortex cores is such that they form a parallel line structure (Figure 1a). However, the imperfections in a real crystal cause that the vortex lines may be curved and even tangled [Brandt] (Figure 1b).

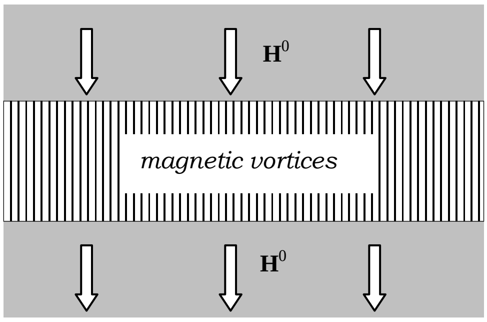

Figure 1a. Vortices in pure crystal

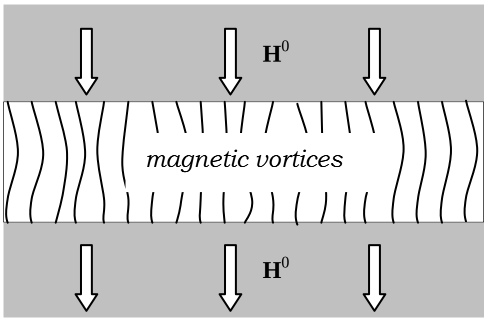

Figure 1b. Vortices in real crystal

Since the interactions among vortices occur with the help of the Lorenz force, the vortex field dynamics may be observed if that field has been perturbated. So, a transmission of energetic signals along that field is possible. The paper deals with investigations of magnetoelastic wave propagation in an environment consisting of a vortex array normal to the middle surface of a superconducting layer (see for instance [Achenbach; Maruszewski and van de Ven]) (Figure 2). As we mentioned before, the vortex field can exist in two phases: solid and fluid ones. Even, the coexistence in the interval $H_{c1} < H < H_{c2}$ is possible. In the paper we confine only to the wave propagation in the solid phase. Because of the above peculiar feature of the vortex field, there occurred a need to define a new stress tensor reflecting coexistence of those phases. The presented research concerns an analysis of propagation condition and dispersion of the magnetoelastic waves in a vortex array. Please note, that if outside the layer the magnetic field distribution is commonly continuous, then within the layer we observe a discrete arrangement of magnetic vortices. Since our description has been done entirely within continuous manner, the vortex field in a layer has been averaged with respect to the layer thickness. That averaging procedure has ensured the description within the continuous model.

2 Magnetothermoviscoelastic stress within the mixed state

The general stress tensor σ_{ij} concerning the coexistence of the ordered (lattice) and disordered (fluid) states of vortices has the form [Maruszewski (1998), Maruszewski (2007)]:

$$\sigma_{ij} = \left[\left(\frac{1}{3}\alpha K - \frac{2}{3}\alpha G\right)\varepsilon_{kk} - \frac{2}{3}\alpha\eta\dot{\varepsilon}_{kk} - \lambda^T\theta - \beta p\right]\delta_{ij} + 2\alpha G\varepsilon_{ij} + 2(\alpha+\beta)\eta\dot{\varepsilon}_{ij}, \quad (1)$$

where the elastic bulk (K) and shear (G) module are as follows:

$$3K = 2\mu + 3\lambda, \quad G = \mu, \quad (2)$$

λ, μ are the Lamè constants of the *lattice* and λ^T is the thermoelastic constant of the *lattice*, ε_{ij} is the strain tensor of the vortex structure, η is a viscous coefficient [Cyrot and Pavuna], θ is the relative temperature in the form

$$\theta = \frac{T - T_c}{T_c}, \quad \theta < 0,$$

T_c is the critical superconducting phase temperature and p is the pressure of the vortex field.

Since just the magnetic field intensity decides in which state the vortices are, we propose two parameters α and β which determine their actual state (cf. [Maruszewski (2007)]). Hence their dimensionless forms are the following:

$$\alpha = \left(\frac{H_{c2} - H}{H_{c2} - H_{c1}}\right)^2, \quad \alpha = \begin{cases} 0 \text{ if } H = H_{c2} \\ 1 \text{ if } H = H_{c1} \end{cases} \quad (3)$$

$$\beta = \left(\frac{H - H_{c1}}{H_{c2} - H_{c1}}\right)^2, \quad \beta = \begin{cases} 0 \text{ if } H = H_{c1} \\ 1 \text{ if } H = H_{c2} \end{cases} \quad (4)$$

$$\alpha + \beta = \begin{cases} = 1 & \text{if } H = H_{c1} \text{ or } H = H_{c2} \\ = f(H) & \text{if } H_{c1} < H < H_{c2}\,. \end{cases} \quad (5)$$

3 Formulation of the problem

Let us now consider a particular problem which deals with the dynamics of the previously defined vortex field. Following [Maruszewski (2007)] the basing equations governing dynamical interactions in the vortices read:

$$\begin{aligned} &\mu u_{i,jj} + \eta\dot{u}_{i,jj} + (\lambda+\mu)u_{j,ij} + \frac{1}{3}\eta\dot{u}_{j,ij} \\ &- \mu_0\left(h_{r,i} - h_{i,r}\right)H_r^0 - \rho\ddot{u}_i = 0 \\ &\lambda_0^2 h_{i,kk} - h_i + u_{i,k}H_k^0 - u_{k,k}H_i^0 = 0 \end{aligned} \quad (6)$$

For the sake of simplicity we have related to the lattice like state of the vortex field ($\alpha = 1, \beta = 0$ in (1)) as well as we have omitted the temperature influence on the vortex field in (1).

This way we have confined only to interactions among viscoelastic and magnetic fields. The linear form of equations (6) results from

$$H_k = H_k^0 + h_k, \quad |h_k| << \left|H_k^0\right|$$

where H_k^0 denotes a constant external magnetic field.

Using equations (6) we consider now the problem of wave propagation in a superconducting layer.

The geometry of the problem is presented in Figure 2.

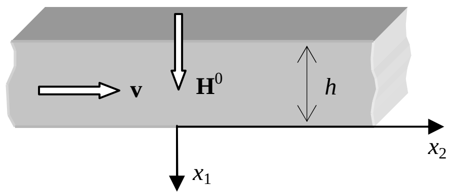

Figure 2. Geometry of the problem.

The solutions of equations (6) are looked for in the following form:

$$f(x_1,x_2,t)=\tilde{f}(x_1)\exp[i(\omega t-kx_2)], \qquad (7)$$

where $\tilde{f}(x_1)$ stands for each function in (6), i.e.

$$\tilde{f}(x_1)=\{u_3,h_3\}, \qquad (8)$$

because

$$\mathbf{u}=[0,0,u_3], \quad \mathbf{h}=[0,0,h_3].$$

Now we transfer all the necessary variables and relations to their dimensionless forms, as follows

$$x_1=hx, x_2=hy, x_3=hz, t=T\tau, T=\frac{h}{v_T},$$

$$H_1^0=H_{c1}H^0, h_3=H_{c1}h_z, u_3=hu_z, \Omega=\omega T,$$

$$V=\frac{v}{v_T}, k=\frac{\omega}{v}=\frac{\Omega}{vT}=\frac{\Omega}{Vh}, \tilde{\rho}=\frac{\rho h^2}{T^2\mu}=1, \qquad (9)$$

$$\tilde{\eta}=\frac{\eta}{T\mu}, \tilde{\lambda}=\frac{\lambda}{\mu}, \tilde{\mu}=1, \tilde{\mu}_0=\mu_0\frac{H_{c1}^2}{\mu},$$

$$\tilde{\lambda}_0^2=\frac{\lambda_0^2}{h^2}, \tilde{c}=\frac{c}{v_T}, v_T=\sqrt{\frac{\mu}{\rho}}, h_3^0=H_{c1}\mathrm{H}_z$$

where v denotes the phase velocity, c is the speed of light then v_T denotes the transverse elastic mode velocity in the layer.

On using now (7) – (9) in (6) we arrive at the following set of ordinary differential equations which governs the propagation of waves in the considered layer (we omit superimposed tildes in the sequel):

$$\begin{gathered}(\mu+i\Omega\eta)\frac{d^2u_z}{dx^2}+\frac{\Omega^2}{V^2}\left(V^2\rho-\mu-i\Omega\eta\right)u_z \\ +\mu_0H^0\frac{dh_z}{dx}=0 \\ \lambda_0^2\frac{d^2h_z}{dx^2}-\left(\lambda_0^2\frac{\Omega^2}{V^2}+1\right)h_z+H^0\frac{du_z}{dx}=0\end{gathered} \qquad (10)$$

4 Solution of the problem

On using (7,8) in (10) the characteristic equation of (10) has the form:

$$\begin{gathered}\lambda_0^2A(\Omega)p^4+\left[\lambda_0^2B(\Omega,V)-F(\Omega,V)A(\Omega)\right. \\ \left.-\mu_0H^{02}\right]p^2-F(\Omega,V)B(\Omega,V)=0\end{gathered}. \qquad (11)$$

In this way we obtained the solutions of set (10) in the following form:

$$\begin{gathered}u_z=S_1e^{p_1x}+S_2e^{-p_1x}+S_3e^{p_2x}+S_4e^{-p_2x} \\ h_z=-M(p_1,\Omega,V)S_1e^{p_1x}+M(p_1,\Omega,V)S_2e^{-p_1x}+ \\ -M(p_2,\Omega,V)S_3e^{p_2x}+M(p_2,\Omega,V)S_4e^{-p_2x}\end{gathered}, \qquad (12)$$

where $\pm p_1,\pm p_2$ are roots of (11) and

$$\begin{gathered}A(\Omega)=1+i\Omega\eta, \\ B(\Omega,V)=\frac{\Omega^2}{V^2}\left(V^2-1-i\Omega\eta\right), \\ F(\Omega,V)=\lambda_0^2\frac{\Omega^2}{V^2}+1, \\ M(p_k,\Omega,V)=\frac{p_k}{\mu_0H^0}+\frac{\frac{\Omega^2}{V^2}\left(V^2-1\right)}{\mu_0H^0p_k}, k=1,2.\end{gathered} \qquad (13)$$

From now on, we assume that viscous influence on the mechanical interactions among vortices is negligible.

Propagation of the wave is possible only if squares of both roots of the characteristic polynomial are positive, i.e.

$$B(\Omega,V)>0. \qquad (14)$$

As a consequence of that restriction we obtain

$$V<1 \qquad (15)$$

if $\tilde{f}=\tilde{f}(x_1)$ in (7).

Finally, we have to determine dispersion of the waves in the vortex magnetic structure. That should be derived from the boundary and continuity conditions for the solutions (12).

Those conditions at $x=-1$ and $x=0$ read:

$h_z=0$ - continuity of the tangent component of the magnetic field intensity

$\sigma_{31}=0$ - the surfaces are free of loadings.

The specification of the above conditions for the surface $x=-1$ is as follows

$$\begin{gathered}-M(p_1,\Omega,V)S_1e^{-p_1}+M(p_1,\Omega,V)S_2e^{p_1}- \\ M(p_2,\Omega,V)S_3e^{-p_2}+M(p_2,\Omega,V)e^{p_2}=0,\end{gathered} \qquad (16)$$

$$\begin{gathered}S_1p_1e^{-p_1}-S_2p_1e^{p_1}+S_3p_2e^{-p_2} \\ +S_4p_2e^{p_2}=0\end{gathered}. \qquad (17)$$

Similarly, at $x=0$ one obtains

$$-M(p_1,\Omega,V)S_1 + M(p_1,\Omega,V)S_2 - M(p_2,\Omega,V)S_3 + M(p_2,\Omega,V) = 0 \quad (18)$$

$$S_1\, p_1 - S_2\, p_1 + S_3\, p_2 - S_3 p_2 = 0 \; . \quad (19)$$

Hence the looked for dispersion relation reads

$$\det \mathbf{W} = 0\,, \quad (20)$$

where $\mathbf{W} = (W_{ij})$ stands for the matrix of coefficients in set of equations (16-19). The only solution of (20) exists solely if V satisfies relation

$$V = 1\,. \quad (21)$$

That means that the wave propagates with the determined velocity and the above fact indicates that $\tilde{f}$ is independent of x_1 (see (7),(15)). So the propagating wave (7) is non dispersive.

Conclusions

After all the considerations that have been done, it occurred that in the vortex layer in a superconductor transverse magnetoelastic wave propagation is possible. The velocity of the wave satisfies inequality (15), i.e. it is less than the velocity of the bulk transverse mode only if the wave amplitude might be dependent on x_1 (on the thickness of the layer). However, the detailed analysis of (20) indicates that the amplitude is independent of x_1 and the wave propagates only if velocity V satisfies (21).
Since the main aim of the paper was to check if wave propagation is possible in such a defined layer, at all, we have linearized fundamental equations (6). The original nonlinear nonlinear character of the magnetoelastic waves in the vortex array will be investigated in the near future.

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