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A ROBUST NONLINEAR CONTROL APPROACH FOR FLEXIBLE-LINK ROBOTS USING KALMAN FILTERING

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Abstract
A robust control approach for flexible-link robots is developed that comprises sliding-mode control theory and Kalman Filtering. There are two issues associated to the control of flexible structures, such as flexible-link robots: (i) simultaneous position control and suppression of the flexible structure vibrations. Assuming a known model of the robot dynamics, this can be succeeded with the use of robust model-based control schemes, such as sliding-mode control, (ii) obtaining measurements of the complete state vector of the vibrating structure, so as to implement state-feedback control. To solve the latter problem, in this paper, state estimation for the flexible-link robot is implemented with the use of Kalman Filtering. The fast recursion of the Kalman Filter provides real-time estimates of the robot's state vector through the processing of measurements coming from a limited number of sensors. The obtained state estimates are optimal with respect to the effects of measurement noise. The efficiency of the proposed state estimation-based sliding-mode controller is evaluated through simulation experiments in the case of a 2-link flexible manipulator.

Key words
Flexible-link robots, flexible structure vibrations, sliding-mode control, Kalman Filtering, state estimation-based control.

1 Introduction

Control for flexible-link robots is a non-trivial problem that has increased difficulty comparing to the control of rigid-link manipulators [Rigatos, 2011], [Wang and Gao, 2004]. This is because the dynamic model of the flexible-link robot contains the non-linear rigid link motion coupled with the distributed effects of the links' flexibility. This coupling depends on the inertia matrix of the flexible manipulator while the vibration characteristics are determined by structural properties of the links such as the damping and stiffness parameters. Moreover, in contrast to the dynamic model of rigid-link robots the dynamic model of flexible-link robots is an infinite dimensional one. As in the case of the rigid-link robot there is a certain number of mechanical degrees of freedom associated to the rotational motion of the robot's joints and there is also an infinite number of degrees of freedom associated to the vibration modes in which the deformation of the flexible link is decomposed [Rigatos, 2009]. The controller of a flexible manipulator must achieve the same motion objectives as in the case of a rigid manipulator, i. e. tracking of specific joints position and velocity setpoints. Additionally, it must also stabilize and asymptotically eliminate the vibrations of the flexible-links that are naturally excited by the joints rotational motion.

The inverse dynamics model-based control for flexible-link robots is based on modal analysis, i.e. on the assumption that the deformation of the flexible link can be written as a finite series expansion containing the elementary vibration modes. However, this inverse-dynamics model-based control may result into unsatisfactory performance when an accurate model is unavailable, due to parameters uncertainty or truncation of high order vibration modes in the model [Rigatos, 2009]. Moreover, based on the state space formulation, the sliding mode control, which belongs to the wider class of the variable structure control schemes, is a nonlinear robust controller suitable for flexible-link manipulators. Sliding-mode control can succeed simultaneous convergence of the flexible robot's joints angles and angular velocities to the desirable setpoints and efficient suppression of the flexible links vibrations. The inclusion of a switching control term in a

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sliding mode controller can provide robustness against parametric uncertainties and input disturbances [Etxebarria *et al.*, 2005], [Khaloub *et al.*, 2006], [Medina Martinez *et al*, 2008].

As mentioned, sliding-mode control is a state-feedback based controller and its implementation requires knowledge of the complete state vector of the controlled system [Sanz and Etxebarria, 2000], [Hui *et al.*, 2002]. However, there are certain elements in the state vector of the flexible-link robot which are difficult to measure, e.g. the vibration modes. Therefore, to apply sliding-mode control to the flexible manipulator it is necessary to use some kind of state estimator which can reconstruct the robot's state vector through the processing of measurements from a limited number of sensors, e.g. angles of the joints and the associated angular velocities [Bascetta and Rocco, 2006], [Nguyen and Egeland, 2005]. The Kalman Filter can provide real-time estimates of the state vector of the flexible link robot while assuring the optimality of estimation in the presence of measurement noises [Kamen and Su, 2009], [Rigatos and Tzafestas, 2007].

Indicative results about filtering-based control for flexible-link robots can be noted. In [Green and Sasiadek, 2005] and [Sasiadek and Green, 2001] state feedback control for a flexible-link robot is implemented with the use of a state vector that is estimated through Kalman Filtering. Using fuzzy rules, an online adaptation of the covariance matrix of the Kalman Filter is performed which aims at improving the vibration suppression capabilities of the filtering-based control. In [Lin and Lewis, 1993] a controller that follows the principles of singular perturbations theory is developed and the flexible-link robot model is decomposed into a fast and a slow dynamics subsystem. Then a two-time scale Kalman filter is designed for estimating the components of the robot's state vector associated both with the rigid (slow) and the flexible (fast) dynamics of the robot. The estimated state vector is used in the control loop. In [Nagarkatti *et al.*, 2001] an observer-based control scheme for flexible-link robots is developed where a fixed-gain state estimator processes measurements of the flexible-links' deformation. Lyapunov-like stability analysis is used to demonstrate the efficiency of the feedback control scheme. In [Post and Book, 2011] a method is proposed for improving the performance of flexible manipulators through the employment of robust state estimation techniques. The method is based on discrete-time Kalman filtering and sliding mode principles and is applied to the model of a 1-DOF flexible-link manipulator. Finally, in [Atashzar *et al.*, 2010] the Extended Kalman Filter is redesigned in the form of a disturbance observer to estimate the disturbance forces that are exerted on the end-effector of a single-link flexible robotic manipulator. The forces' estimates provided by the filter are used in the robot's feedback control loop.

In this paper it will be shown how a suitable formulation of the dynamic model of the flexible manipulator enables the application of the Kalman Filter recursion and provides accurate estimates of the robot's state vector which in turn can be used by a sliding-mode control loop. The paper extends and elaborates on the results of [Rigatos, 2012]. The performance of the proposed Kalman Filter-based sliding mode controller is also compared against a model-free method which is known as Energy-based control. In this method, instead of using an explicit dynamical model of the links, the main stability results are derived with the use of the total energy and the energy-work relationship for the robotic system [Wang and Gao, 2004], [Rigatos, 2011]. The evaluation of Kalman filter-based sliding-mode control against energy-based control derives useful results on the efficiency of this control approach.

The structure of the paper is as follows: In Section 2 the dynamic model of the flexible-link robot is studied and inverse dynamics control for flexible-link robots is introduced. In Section 3 sliding-mode control for the flexible-link robot is analyzed. In Section 4 the Kalman Filter is proposed for estimating the non-measurable elements of the state vector of the flexible-link robot through the processing of measurements that come from a limited number of sensors. In Section 5 energy-based control for flexible-link robots is explained. In Section 6 simulation experiments are carried out to evaluate the performance of Kalman Filter-based sliding-mode control in comparison to inverse dynamics control and energy-based control, when applied to the model of a 2-link flexible manipulator. Finally, in Section 7 concluding remarks are stated.

2 Model-Based Control for Flexible-Link Robots

2.1 The Euler-Bernoulli Model of Flexible-Link Robots

A common approach in modelling of flexible-link robots is based on the Euler-Bernoulli model [Wang and Gao, 2004], [Rigatos, 2009], [Rigatos, 2011]. This model consists of nonlinear partial differential equations, which are obtained after using some approximation or simplification. In case of a single-link flexible manipulator the basic variables of this model are $w(x,t)$ which is the deformation of the flexible link, and $\theta(t)$ which is the joint's angle

$$E{\cdot}I{\cdot}w^{''''}(x,t) + \rho\ddot{w}(x,t) + \rho x\ddot{\theta}(t) = 0, \quad (1)$$

$$I_t\ddot{\theta}(t) + \rho\int_0^L x\ddot{w}(x,t)dx = T(t). \quad (2)$$

In Eq. (1) and (2), $w^{''''}(x,t) = \frac{\partial^4 w(x,t)}{\partial x^4}$, $\ddot{w}(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2}$, while I_t is the moment of inertia of a rigid link of length L, ρ denotes the uniform mass density and EI is the uniform flexural rigidity with units $N{\cdot}m^2$. To calculate $w(x,t)$, instead of solving analytically the above partial differential equations, modal analysis can be used which assumes that $w(x,t)$ can be approximated by a weighted sum of orthogonal basis functions

$$w(x,t) = \sum_{i=1}^{n_e} \phi_i(x) v_i(t) \tag{3}$$

where index $i = [1, 2, \cdots, n_e]$ denotes the normal modes of vibration of the flexible link. Using modal analysis a dynamical model of finite-dimensions is derived for the flexible link robot. In the general case a n-link flexible robot can be assumed while it can be considered that only the first two vibration modes of each link are significant ($n_e = 2$). Fig. 1 shows a 2-link flexible robot and the associated reference frames. Σ_1 is a point on the first link with reference to which the deformation vector is measured. Similarly, Σ_2 is a point on the second link with reference to which the associated deformation vector is measured. In that case the dynamic model of the robot becomes [Wang and Gao, 2004]:

$$\begin{pmatrix} M_{11}(z) & M_{12}(z) \\ M_{21}(z) & M_{22}(z) \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{v} \end{pmatrix} + \begin{pmatrix} 0_{n\times n} & 0_{n\times 2n} \\ 0_{2n\times n} & D(z) \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{v} \end{pmatrix} + \begin{pmatrix} 0_{n\times n} & 0_{n\times 2n} \\ 0_{2n\times n} & K(z) \end{pmatrix} \begin{pmatrix} \theta \\ v \end{pmatrix} + \begin{pmatrix} F_1(z,\dot{z}) \\ F_2(z,\dot{z}) \end{pmatrix} + \begin{pmatrix} G_1(z,\dot{z}) \\ G_2(z,\dot{z}) \end{pmatrix} = = \begin{pmatrix} T(t) \\ 0_{2n\times 1} \end{pmatrix} \tag{4}$$

where $z = [\theta, v]^T$, with $\theta = [\theta_1, \theta_2, \cdots, \theta_n]^T$, $v = [v_{11}, v_{12}, v_{21}, v_{22}, \cdots, v_{n1}, v_{n2}]^T$ (vibration modes for links 1 and 2), $[F_1(z,\dot{z}), F_2(z,\dot{z})]^T \in [R^{n\times 1}, R^{2n\times 1}]^T$ (centrifugal and Coriolis forces) and $[G_1(z,\dot{z}), G_2(z,\dot{z})]^T \in [R^{n\times 1}, R^{2n\times 1}]^T$ (torque developed due to gravitational forces). The elements of the inertia matrix are: $M_{11} \in R^{n\times n}$, $M_{12} \in R^{n\times 2n}$, $M_{21} \in R^{2n\times n}$, $M_{22} \in R^{2n\times 2n}$. The damping and stiffness matrices of the aforementioned model are $D \in R^{2n\times 2n}$ and $K \in R^{2n\times 2n}$. Moreover the vector of the control torques is $T(t) = [T_1(t), T_2(t), \cdots, T_n(t)]^T$.

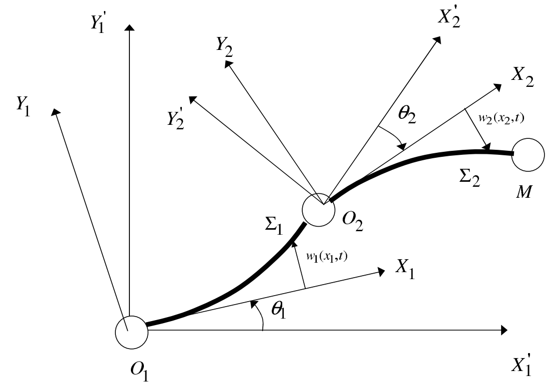

Figure 1. A 2-link flexible robot and the associated reference frames.

2.2 Design of an Inverse Dynamics Controller

The principle of inverse dynamics control is to transform the nonlinear system of Eq. (4) into a linear one, so that linear control techniques can be applied. From Eq. (4) it holds that:

$$M_{11}\ddot{\theta} + M_{12}\ddot{v} + F_1(z,\dot{z}) + G_1(z,\dot{z}) = T(t) \tag{5}$$

$$M_{21}\ddot{\theta} + M_{22}\ddot{v} + F_2(z,\dot{z}) + G_2(z,\dot{z}) + D\dot{v} + Kv = 0 \tag{6}$$

Eq. (6) is solved with respect to $\ddot{v}$

$$\ddot{v} = -M_{22}^{-1}M_{21}\ddot{\theta} - M_{22}^{-1}(F_2(z,\dot{z}) + G_2(z,\dot{z})) - -M_{22}^{-1}D\dot{v} - M_{22}^{-1}Kv. \tag{7}$$

Eq. (7) is substituted in Eq. (5) which results into:

$$(M_{11} - M_{12}M_{22}^{-1}M_{21})\ddot{\theta} - M_{12}M_{22}^{-1}(F_2(z,\dot{z}) + +G_2(z,\dot{z})) - M_{12}M_{22}^{-1}(D\dot{v} + Kv) + +F_1(z,\dot{z}) + G_1(z,\dot{z}) = T(t). \tag{8}$$

The following control law is now introduced [Wang and Gao, 2004], [Rigatos, 2009], [Rigatos, 2011]:

$$T(t) = -M_{12}M_{22}^{-1}(F_2(z,\dot{z}) + G_2(z,\dot{z})) -M_{12}M_{22}^{-1}(D\dot{v} + Kv) + F_1(z,\dot{z}) + G_1(z,\dot{z}) +(M_{11} - M_{12}M_{22}^{-1}M_{21})u_0 \tag{9}$$

$$u_0 = \ddot{\theta}_d - K_d(\dot{\theta} - \dot{\theta}_d) - K_p(\theta - \theta_d) \tag{10}$$

By replacing Eq. (9) in Eq. (8) one finally gets

$$\ddot{\theta} = u_0 \tag{11}$$

Eq. (11) implies that linearisation and decoupling of the robotic model has been achieved. Substituting Eq. (10) into Eq. (11) gives:

$$\ddot{e}(t) + K_d \dot{e}(t) + K_p e(t) = 0 \tag{12}$$

Gain matrices K_p and K_d are selected, so as to assure that the poles of Eq. (12) are in the left semiplane. This results into $lim_{t\to\infty} e(t) = 0 \Rightarrow lim_{t\to\infty}\theta(t) = \theta_d(t)$. Consequently, for $\theta_d(t)$ =constant it holds $lim_{t\to\infty}\ddot{\theta}(t) = 0$. Then Eq. (7) gives

$$\ddot{v} = -M_{22}^{-1}(F_2(z,\dot{z})+G_2(z,\dot{z}))-M_{22}^{-1}D\dot{v}-M_{22}^{-1}Kv \tag{13}$$

and for $F_2(z,\dot{z}) = 0_{4\times1}$ and $G_2(z,\dot{z}) = 0_{4\times1}$ results into

$$\ddot{v} + M_{22}^{-1}D\dot{v} + M_{22}^{-1}Kv = 0 \tag{14}$$

which is the differential equation of the free damped oscillator. Suitable selection of the damping matrix D and the elasticity matrix K assures that $lim_{t\to\infty} v(t) = 0$.

3 Design of a Sliding-Mode Controller

Sliding-mode control for flexible-link robots has been studied in several papers [Etxebarria *et al.*, 2005],[Sanz and Etxebarria, 2000]. In the sequel and for simplifying the presentation of the control scheme a 2-link flexible manipulator will be assumed, i.e. $n = 2$. The flexible-link robot model of Eq. (4) can be written as

$$\begin{pmatrix}\ddot{\theta}\\ \ddot{v}\end{pmatrix} = -\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}^{-1}\{\begin{pmatrix}0 & 0\\ 0 & D\end{pmatrix}\begin{pmatrix}\dot{\theta}\\ \dot{v}\end{pmatrix} + \begin{pmatrix}0 & 0\\ 0 & K\end{pmatrix}\begin{pmatrix}\theta\\ v\end{pmatrix} + \\ + \begin{pmatrix}F_1\\ F_2\end{pmatrix} + \begin{pmatrix}G_1\\ G_2\end{pmatrix} - \begin{pmatrix}T\\ 0\end{pmatrix}\} \tag{15}$$

The model of the flexible-link robot dynamics is written in state-space form after defining the following state vector:

$$x = [\theta_1, \theta_2, v_{11}, v_{12}, v_{21}, v_{22}, \dot{\theta}_1, \dot{\theta}_2, \dot{v}_{11}, \dot{v}_{12}, \dot{v}_{21}, \dot{v}_{22}]^T \tag{16}$$

The following notation is used for the inverse of the inertia matrix of the flexible-link robot

$$\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}^{-1} = \begin{pmatrix}N_{11} & N_{12}\\ N_{21} & N_{22}\end{pmatrix} \tag{17}$$

where $N_{11}{\in}R^{2\times2}$, $N_{12}{\in}R^{2\times4}$, $N_{21}{\in}R^{4\times2}$ and $N_{22}{\in}R^{4\times4}$. It holds that

$$\ddot{\theta} = -N_{12}Kv - N_{12}D\dot{v} - N_{11}F_1 - N_{11}G_1 + N_{11}T \tag{18}$$

The elements of the damping matrix $D{\in}R^{4\times4}$ are denoted as $D(i,j)$, where $D(i,j){\neq}0$ for $i = j$, while the elements of the stiffness matrix $K{\in}R^{4\times4}$ are denoted as $K(i,j)$, where $K(i,j){\neq}0$ for $i = j$. Additionally the terms of the Coriolis and the gravitational vectors are $F = \left(F_1 \in R^{2\times1}, F_2 \in R^{4\times1}\right)^T$ and $G = \left(G_1 \in R^{2\times1}, G_2 \in R^{4\times1}\right)^T$. To obtain a more compact mathematical description in the design of the controller, and without loss of generality, in the rest of this section it will be considered that $F_2 = 0_{4\times1}$ and $G_2 = 0_{4\times1}$.

Therefore, one can write the dynamics of the joints of the flexible-link robot in a matrix form:

$$\begin{array}{c}\ddot{x}_1 = f_1(x) + g_1(x)u\\ \ddot{x}_2 = f_2(x) + g_2(x)u\end{array} \tag{19}$$

where $u = \left(T_1\ T_2\right)^T$, $g_1(x) = \left(N_{11}(1,1)\ N_{11}(1,2)\right)$, $g_2(x) = \left(N_{11}(2,1)\ N_{11}(2,2)\right)$, while functions $f_1(x)$ and $f_2(x)$ are defined as

$$\begin{array}{rr} f_1(x) = & -N_{12}(1,1)K(1,1)x_3 - N_{12}(1,2)K(2,2)x_4\\ & -N_{12}(1,3)K(3,3)x_5 - N_{12}(1,4)K(4,4)x_6\\ & -N_{12}(1,1)D(1,1)x_9 - N_{12}(1,2)D(2,2)x_{10}\\ & -N_{12}(1,3)D(3,3)x_{11} - N_{12}(1,4)D(4,4)x_{12}\\ & -N_{11}(1,1)F_1(1,1) - N_{11}(1,2)F_1(2,1)\\ & -N_{11}(1,1)G_1(1,1) - N_{11}(1,2)G_1(2,1)\end{array} \tag{20}$$

$$\begin{array}{rr} f_2(x) = & -N_{12}(2,1)K(1,1)x_3 - N_{12}(2,2)K(2,2)x_4\\ & -N_{12}(2,3)K(3,3)x_5 - N_{12}(2,4)K(4,4)x_6\\ & -N_{12}(2,1)D(1,1)x_9 - N_{12}(2,2)D(2,2)x_{10}\\ & -N_{12}(2,3)D(3,3)x_{11} - N_{12}(2,4)D(4,4)x_{12}\\ & -N_{11}(2,1)F_1(1,1) - N_{11}(2,2)F_1(2,1)\\ & -N_{11}(2,1)G_1(1,1) - N_{11}(2,2)G_1(2,1)\end{array} \tag{21}$$

In the equations describing the joint dynamics the terms $g_1(x)$ and $g_2(x)$ depend on the elements of the inertia matrix of the flexible-link robot and are considered to be known. On the other hand, the terms $f_1(x)$ and $f_2(x)$ are considered to vary in uncertainty ranges, given by

$$|f_1 - \hat{f}_1|{\leq}\Delta F_1, \quad |f_2 - \hat{f}_2|{\leq}\Delta F_2 \tag{22}$$

The following tracking error for the joints angles is defined:

$$e_1 = x_1 - x_1^d, \quad e_2 = x_2 - x_2^d \tag{23}$$

Moreover, the sliding surface vector $s = [s_1, s_2]^T$ is defined with elements

$$s_1 = \dot{e}_1 + \lambda_1 e_1, \quad s_2 = \dot{e}_2 + \lambda_2 e_2 \tag{24}$$

To succeed convergence of the tracking error to zero for the i-th element of the state vector the following conditions should hold:

$$\frac{1}{2}\frac{d}{dt}s_i^2 \leq -\eta_i|s_i|, \ \ \eta_i > 0, \ \ i = 1,2 \tag{25}$$

The sliding-mode control law is finally given by

$$u = \begin{pmatrix} g_1(x) \\ g_2(x) \end{pmatrix}^{-1} \cdot \\ \cdot \begin{pmatrix} \ddot{x}_1^d - \hat{f}_1(x) - \lambda_1(\dot{x}_1 - \dot{x}_1^d) - k_1 sgn(s_1) \\ \ddot{x}_2^d - \hat{f}_2(x) - \lambda_2(\dot{x}_2 - \dot{x}_2^d) - k_2 sgn(s_2) \end{pmatrix} \tag{26}$$

To define the permissible values for the switching gains $k_i \ i = 1,2$ the following conditions are used

$$\frac{1}{2}\frac{d}{dt}s_i^2 \leq -\eta_i|s_i| \Rightarrow s_i\dot{s}_i \leq -\eta_i|s_i| \Rightarrow \\ [f_i(x) + g_i(x)u - \ddot{x}_i^d + \lambda_i(\dot{x}_i - \dot{x}_i^d)]s_i \leq -\eta_i|s_i| \tag{27}$$

The conditions given in Eq. (27) can be written as follows

$$\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} \{ \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} + \begin{pmatrix} g_1(x) \\ g_2(x) \end{pmatrix} u - \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_2^d \end{pmatrix} + \\ + \begin{pmatrix} \lambda_1(\dot{x}_1 - \dot{x}_1^d) \\ \lambda_2(\dot{x}_2 - \dot{x}_2^d) \end{pmatrix} \} \leq \begin{pmatrix} -\eta_1|s_1| \\ -\eta_2|s_2| \end{pmatrix} \tag{28}$$

Substituting in Eq. (28) the control law u that was calculated in Eq. (26), one obtains

$$\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} \begin{pmatrix} f_1(x) - \hat{f}_1(x) - k_1 sgn(s_1) \\ f_2(x) - \hat{f}_2(x) - k_2 sgn(s_2) \end{pmatrix} \leq \begin{pmatrix} -\eta_1|s_1| \\ -\eta_2|s_2| \end{pmatrix} \tag{29}$$

or equivalently

$$(f_1(x) - \hat{f}_1(x) - k_1 sgn(s_1))s_1 \leq -\eta_1|s_1| \\ (f_2(x) - \hat{f}_2(x) - k_2 sgn(s_2))s_2 \leq -\eta_2|s_2| \tag{30}$$

and using Eq. (22) one has

$$\Delta F_1 s_1 - k_1 sgn(s_1)s_1 \leq -\eta_1|s_1| \\ \Delta F_2 s_2 - k_2 sgn(s_2)s_2 \leq -\eta_2|s_2| \tag{31}$$

or equivalently

$$\Delta F_1 s_1 - k_1|s_1| \leq -\eta_1|s_1| \\ \Delta F_2 s_2 - k_2|s_2| \leq -\eta_2|s_2| \tag{32}$$

The switching control gains are chosen to satisfy

$$k_1 = \Delta F_1 + \eta_1, \quad k_2 = \Delta F_2 + \eta_2 \tag{33}$$

Substituting Eq. (33) into Eq. (32) gives

$$\Delta F_1 s_1 - \Delta F_1|s_1| - \eta_1|s_1| \leq -\eta_1|s_1| \\ \Delta F_2 s_2 - \Delta F_2|s_2| - \eta_2|s_2| \leq -\eta_2|s_2| \tag{34}$$

or equivalently

$$\Delta F_1 s_1 \leq \Delta F_1|s_1| \\ \Delta F_2 s_2 \leq \Delta F_2|s_2| \tag{35}$$

This assures that $lim_{t\to\infty} s_i = 0, \ i = 1,2$ and consequently the asymptotic elimination of the tracking error for the joints' angle and rotation speed.

4 Estimation of the Non-Measurable State Variables

Knowing that certain elements of the state vector of the flexible-link robot are not directly measurable, e.g. vibration modes, it becomes necessary to estimate these variables with the use of a state observer or filter. Indicative research results on state estimation-based control for flexible-link robots have been given in [Hui *et al.*, 2002], [Bascetta and Rocco, 2006], [Nguyen and Egeland, 2005]. To obtain a state estimation-based control scheme for the flexible manipulator, in this paper the state-space description of the flexible-link robot dynamics in the form of Eq. (36) is used:

$$\dot{x} = Ax + Bu_a \\ y = Cx \tag{36}$$

where $x \in R^{12\times1}$ is the previously defined state vector, $u_a = [T_1 - F_1 - G_1, T_2 - F_1 - G_1]^T$, while matrices A and B are defined as

$$A = \begin{pmatrix} 0_{6\times6} & I_{6\times6} \\ [0_{2\times2}, -N_{12}K] & [0_{2\times2}, -N_{12}D] \\ [0_{4\times2}, -N_{22}K] & [0_{4\times2}, -N_{22}D] \end{pmatrix} \quad B = \begin{pmatrix} 0_{6\times2} \\ N_{12} \\ N_{22} \end{pmatrix} \tag{37}$$

$$C = \begin{pmatrix} 1 & 0 & 0_{1\times10} \\ 0 & 1 & 0_{1\times10} \\ 0_{1\times6} & 1 & 0_{1\times5} \\ 0_{1\times7} & 1 & 0_{1\times4} \end{pmatrix} \tag{38}$$

Thus, it is considered that the measurable elements of the robot's state vector are the joints' angles and the joints' angular velocities. After applying common discretization methods the linear continuous-time model of the flexible-link robot of Eq. (36) is turned into a discrete-time linear model, which makes use of the discrete-time equivalents of matrices A, B and C defined in Eq. (37) and Eq. (38).

For the latter discrete-time model the application of the recursion of the discrete-time Kalman Filter is possible. The discrete-time Kalman filter can be decomposed into two parts: i) time update (prediction stage), and ii) measurement update (correction stage) [Kamen and Su, 2009]. The first part employs an estimate of the state vector $x(k)$ made before the output measurement $y(k)$ is available (a priori estimate). The second part estimates $x(k)$ after $y(k)$ has become available (a posteriori estimate). The covariance matrices associated with $\hat{x}^-(k)$ and $\hat{x}(k)$ are defined as: $P^-(k) = Cov[e^-(k)] = E[e^-(k)e^-(k)^T]$ and $P(k) = Cov[e(k)] = E[e(k)e^T(k)]$.

The recursion of the discrete-time Kalman Filter is formulated as:

measurement update:

$$\begin{array}{l} K(k) = P^-(k)C(k)^T[C(k){\cdot}P^-(k)C(k)^T + R]^{-1} \\ \hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C(k)\hat{x}^-(k)] \\ P(k) = P^-(k) - K(k)C(k)P^-(k) \end{array} \quad (39)$$

time update:

$$\begin{array}{l} P^-(k+1) = A(k)P(k)A^T(k) + Q(k) \\ \hat{x}^-(k+1) = A(k)\hat{x}(k) + B(k)u(k) \end{array} \quad (40)$$

Remark: The stages in modeling of the robot's dynamics, that assure consistency between the initial continuous-time description of the flexible link's robot dynamics and the final implementation of discrete-time state estimation-based control scheme, are summarized as follows: The nonlinear dynamic model of the flexible-link robot is a continuous-time one and is described by the set of partial differential equations given in Eq. (1) and (2). After the decomposition of the flexible-link's deformation into vibration modes, as described in Eq. (3), this finally takes the form of a continuous-time state-space model described in Eq. (4). By rearranging the elements of the state-space model, as explained in Eq. (18) to (21), one can write the robot's model in the form a continuous-time linear state-space model that is described in Eq. (36). Matrices A, B and C of the linear state-space model are defined in Eq. (37) and (38). Next, by applying common discretization methods (e. g. Tustin transform) the continuous-time linear model of the robot's dynamics is transformed into a linear discrete-time model where matrices A, B, and C are substituted by their discrete-time transformed equivalents. For this latter model, the application of the standard discrete-time Kalman Filter recursion is possible according to Eq. (39) and (40).

5 Energy-Based Control of the Flexible-Link Robot

Energy-based control can be also used for the control of flexible-link robots. Energy-based control of flexible-link robots assures closed-loop system stability in the case of constant set-points (point-to-point control). The kinetic energy E_{kin} of a n-link flexible robot is given by [Wang and Gao, 2004], [Rigatos, 2011]:

$$E_{kin} = \sum_{i=1}^{n} \frac{1}{2}\rho \int_0^{L_i} [\dot{p}_{x_i}^2 + \dot{p}_{y_i}^2]dx \quad (41)$$

where $[p_{x_i}, p_{y_i}]$ are coordinates the elementary part of the i-th flexible link, as shown in Fig. 1. On the other hand the potential energy E_p of a planar n-link flexible robot is due to the links deformation and is given by

$$E_p = \sum_{i=1}^{n} \frac{1}{2} EI \int_0^{L_i} [\frac{\partial^2}{\partial x^2} w_i(x,t)]^2 dx \quad (42)$$

Thus to estimate the robot's potential energy, measurement of the flexible links strain $\frac{\partial^2 w_i(x,t)}{\partial x^2}$ is needed. The potential energy includes only the energy due to strain, while the gravitational effect as well as longitudinal and torsional deformations are neglected.

Moreover, the energy provided to the flexible-link robot by the i-th motor is given by

$$W_i = \int_0^t T_i(\tau)\dot{\theta}(\tau)d\tau. \quad (43)$$

Consequently, the power of the i-th motor is $P_i(t) = T_i(t)\dot{\theta}_i(t)$, where $T_i(t)$ is the torque of the i-th motor and $\dot{\theta}_i(t)$ is the motor's angular velocity. Thus, the aggregate motors energy is given by

$$W = \sum_{i=1}^{n} \int_0^t T_i(\tau)\dot{\theta}_i(\tau)d\tau \quad (44)$$

The energy that is provided to the flexible-link robot by its motors takes the form of: (i) potential energy (due to the deformation of the flexible links) and (ii) kinetic energy. This energy flow is described by

$$[E_{kin}(t) + E_p(t)] - [E_{kin}(0) + E_p(0)] = W. \quad (45)$$

Energy-based control of flexible-link robots considers that the torque of the i-th motor (control output) is based on a state feedback controller of the form [Wang and Gao, 2004], [Ge *et al.*, 1996]:

$$T_i(t) = -K_{p_i}[\theta_i(t) - \theta_{d_i}(t)] - K_{d_i}\dot{\theta}_i(t) -$$

$$-K_i w_i^{''}(x,t) \int_0^t \dot{\theta}_i(s) w_i^{''}(x,s)ds, \quad (46)$$

where $i = 1, 2, \cdots, n$, K_{p_i} is the i-th P control gain, K_{d_i} is the i-th D control gain, θ_{d_i}, is the desirable angle of the i-th link, K_i is also a positive (constant) gain, and $w_i(x,t)$ is the deformation of the i-th link. The proposed control law of Eq. (46) assures the asymptotic stability of the closed-loop system in case of constant set-points (point to point control). The following Lyapunov (energy) function is considered [Ge *et al.*, 1996], [Wang and Gao, 2004], [Rigatos, 2011]:

$$V = E_{kin} + E_p + \frac{1}{2}\sum_{i=1}^{N} K_{p_i}[\theta_i(t) - \theta_{d_i}(t)]^2 +$$

$$+\frac{1}{2}\sum_{i=1}^{n} K_i[\int_0^t \dot{\theta}_i(s) w_i^{''}(s,t)ds]^2 \quad (47)$$

where E_{kin} denotes the kinetic energy of the robot's links, while E_p and denotes the potential energy of the robot's links due to deformation. It holds that $V(t) > 0$ because $E_{kin} > 0$, $E_p > 0$, and

$$\begin{array}{c} \frac{1}{2}\sum_{i=1}^{n} K_{p_i}[\theta_i(t) - \theta_{d_i}(t)]^2 > 0, \\ \frac{1}{2}\sum_{i=1}^{n} K_i[\int_0^t \dot{\theta}_i(t) w_i(s,t)]^2 > 0 \end{array} \tag{48}$$

Moreover, it holds that

$$\begin{array}{l} \dot{V}(t) = \dot{E}_{kin} + \dot{E}_p + \sum_{i=1}^{n} K_{p_i}[\theta_i(t) - \theta_{d_i}(t)]\dot{\theta}_i(t) \\ +\frac{1}{2}\sum_{i=1}^{n} 2K_i[\int_0^t \dot{\theta}_i(s,t) w_i^{''}(s,t)ds][\dot{\theta}_i(t) w_i^{''}(x,t)] \end{array} \tag{49}$$

while the derivative of the robot's energy is found to be

$$\dot{E}_{kin}(t) + \dot{E}_p(t) = \sum_{i=1}^{n} T_i(t)\dot{\theta}_i(t) \tag{50}$$

where the torque generated by the i-th motor is given by Eq. (46). Using the above one gets

$$\begin{array}{l} \dot{V}(t) = \\ -\sum_{i=1}^{n} K_{p_i}[\theta_i(t) - \theta_{d_i}(t)]\dot{\theta}_i(t) - \sum_{i=1}^{n} K_{d_i}\dot{\theta}_i^2(t) \\ -\sum_{i=1}^{n}[K_i w_i^{''}(x,t)\int_0^t \dot{\theta}_i(s) w_i^{''}(s,t)ds]\dot{\theta}_i(t) \\ +\sum_{i=1}^{n} K_{p_i}[\theta_i(t) - \theta_{d_i}(t)]\dot{\theta}_i(t) \\ +\sum_{i=1}^{n}[K_i w_i^{''}(x,t)\int_0^t \dot{\theta}_i(s) w_i^{''}(s,t)ds]\dot{\theta}_i(t) \end{array} \tag{51}$$

which finally results into, $\dot{V}(t) = -\sum_{i=1}^{n} K_{d_i}\dot{\theta}_i^2$. Obviously, from $\dot{V}(t)$ it holds that $\dot{V}(t) \leq 0$, which implies stability of the closed-loop system, but not asymptotic stability. Asymptotic stability can be proven as follows [Wang and Gao, 2004]: If the i-th link did not converge to the desirable angle, i.e. $lim_{t\to\infty}[\theta_i(t) - \theta_{d_i}(t)] = lim_{t\to\infty} e_i(t) = a_i$ then the torque of the i-th motor would become equal to a small positive constant. This is easy to prove from Eq.(46) where the terms $K_{d_i}\dot{\theta}_i(t) = 0$, $K_i w_i(x,t)\int_0^t \dot{\theta}_i(s) w_i^{''}(s,t)ds = 0$, while the term $K_{p_i}[\theta_i(t) - \theta_{d_i}(t)] = K_{p_i} a_i$ becomes equal to a positive constant.

However, if $T_i(t)$ = constant $\neq 0$ then the i-th link should continue to rotate. This means that $e_i(t) \neq a_i$, which contradicts the initial assumption $\lim_{t\to\infty} e_i(t) = a_i$. Therefore, it must hold $\lim_{t\to\infty} T_i(t) = 0$ and $\lim_{t\to\infty} \theta_i(t) = \theta_{d_i}(t)$. Consequently, $\lim_{t\to\infty} V(t) = 0$.

It is noted that the model-free concept that is followed by energy-based control gives some advantages over model-based techniques that implement modal analysis. The latter control methods may result into unsatisfactory control performance due to model uncertainty or truncation of high order vibration modes.

6 Simulation Tests

6.1 Inverse dynamics control for a 2-link FLR

The 2-link flexible robot of Fig. 1 is considered. The robot consists of two flexible links of length $L_1 = 0.45m$ and $L_2 = 0.45m$, respectively. The dynamic model of the robot is given by Eq. (4). The elements of the inertia matrix M are:

$$\begin{array}{c} M_{11} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \ M_{22} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ M_{12} = M_{21}^T = \begin{pmatrix} 1 & 1 & 0.2 & 0.3 \\ 0.5 & 0.1 & 2 & 0.7 \end{pmatrix} \end{array} \tag{52}$$

The damping matrix is $D = diag\{0.04, 0.08, 0.03, 0.06\}$ while the stiffness matrix is $K = diag\{0.02, 0.04, 0.03, 0.06\}$. The inverse dynamics control law given in subsection 2.2 was employed. The selection of the gain matrices K_p and K_d determines the transient response of the closed loop system. The following controller gains have been considered: $K_p = diag\{10.5, 15.5\}$ and $K_d = diag\{10.9, 15.0\}$. The desirable joints' positions were $\theta_{d_1} = 1\ rad$ and $\theta_{d_2} = 1.4\ rad$. It was considered that an additive disturbance torque $d_i(t) = 0.3cos(t)$ affected each joint.

In simulation diagrams about angular position and velocity setpoint tracking, the horizontal axis represents time in sec, while since the robot's control takes place in the configuration space the vertical axis represents angle in rad and angular velocity in rad/sec. Moreover, as shown in Eq. (3), the vibration modes variables $v_i(t)$ are functions of time and are associated with the deformation of the flexible links $w(x,t)$.

The performance of the model-based controller of the flexible-link robot in the presence of disturbance is depicted in Fig. 2 and Fig. 3. It can be seen that vibrations around the desirable joint positions cannot be eliminated.

6.2 Sliding-Mode Control for a 2-link FLR

The sliding-mode control scheme proposed in section 3 was tested on the 2-link flexible robotic manipulator model. It was assumed that the complete state vector of the robot was not directly measurable. Thus, it was considered that only the joints' angles $\theta_i,\ i = 1,2$ and the associated angular velocities $\dot{\theta}_i,\ i = 1,2$ could be obtained through sensor measurements, whereas the vibration modes of the links $v_{11}, v_{12}, v_{21}, v_{22}$ were not measurable and had to be reconstructed with the use of the Kalman Filter.

The obtained results are depicted in Fig. 4 where convergence of the joints' angles and velocities to the desirable setpoints is shown. In Fig. 5 the evolution in time of the vibration modes of the flexible links is presented. Fig. 6 presents the estimation of the flexible-links' vibration modes, provided by the Kalman Filter.

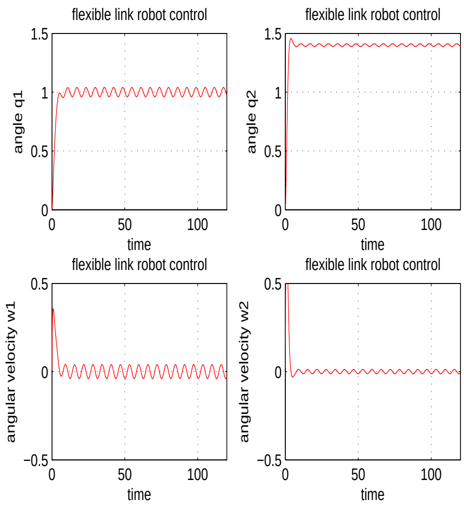

Figure 2. Inverse dynamics control of a 2-link flexible robot under additive motor-torques disturbances: joints' angles (rad) and joints' angular velocity (rad/sec).

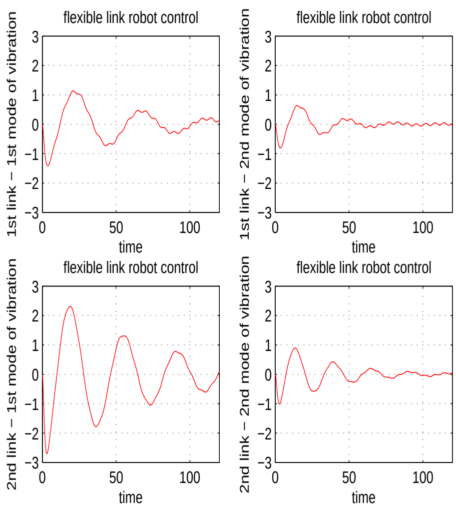

Figure 3. Inverse dynamics control of a 2-link flexible robot under additive motor-torques disturbances: the first two vibration modes for each link.

It can be noticed that the Kalman Filter state estimates track with satisfactory accuracy the real values of the non-measurable state vector elements. Finally, Fig. 7 depicts the control inputs (torques) applied to the joints of the flexible-link robot.

6.3 Energy-Based Control for Flexible Robotic Manipulators

The same robotic model as in the case of Kalman filter-based sliding mode control was used to simulate the variation of the manipulator's joints with respect to time. Energy-based control of flexible-link robots is

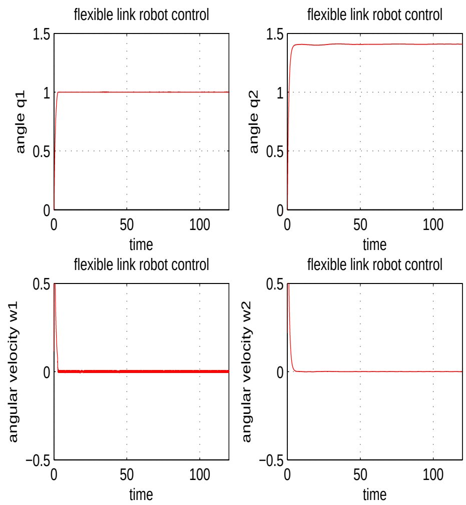

Figure 4. Sliding-mode control of a 2-link flexible robot under additive motor-torques disturbances: joints' angles (rad) and joints' angular velocity (rad/sec) for each link.

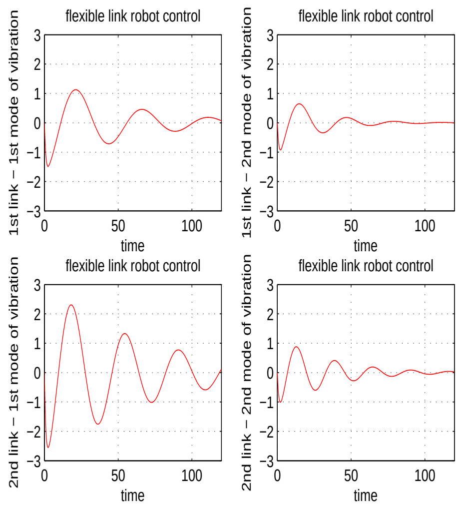

Figure 5. Sliding-mode control of a 2-link flexible robot under additive motor-torques disturbances: the first two vibration modes for each link.

based on Eq. (46). The following controller gains have been chosen: $K_p = 1.9$, $K_d = 7.2$ and $K_i = 0.1$. The desirable joint positions were again $\theta_{d_1} = 1.0\, rad$ and $\theta_{d_2} = 1.4\, rad$. To derive the control signal of Eq. (46) the strains at the base of each link were used, i. e. $w_i^{''}(0,t)$.

The variation of the angle of the link and of the angular velocity of the link with respect to time are given in Fig. 8. The the first two vibration modes which evolved in time as shown in Fig. 9. The vibrations attenuated as the link approached to the desirable final position.

From the simulation experiments it can be noticed that as the Kalman Filter-based sliding-mode controller, the

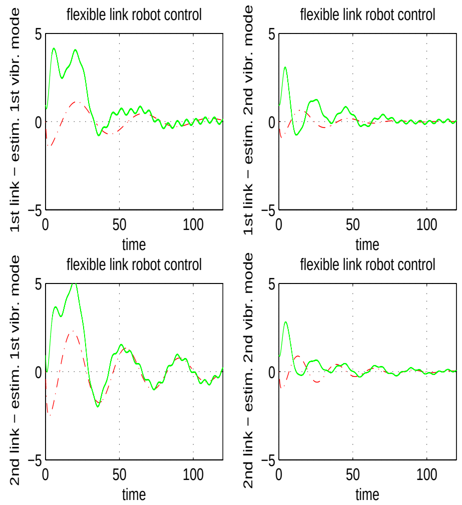

Figure 6. Estimates (continuous lines) of the non-measurable state vector elements of the flexible-link robot (vibration modes), provided by the Kalman Filter.

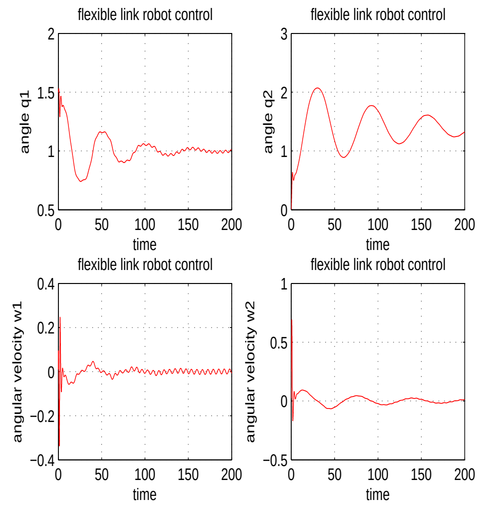

Figure 8. Energy-based control of a 2-link flexible robot under additive motor-torques disturbances: joints' angles (rad) and joints' angular velocity (rad/sec) for each link.

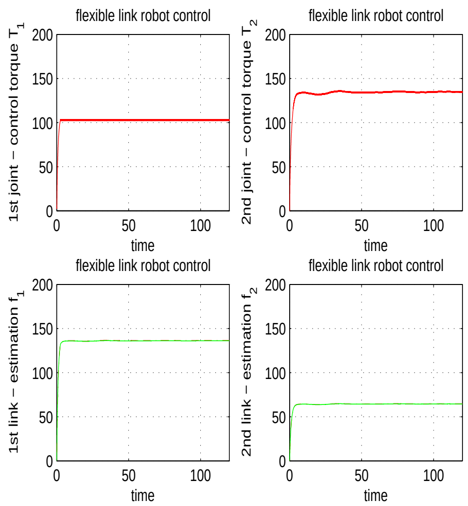

Figure 7. Top row: Control inputs (torques) $T_i,\ i = 1, 2$ applied to the joints of the flexible-link robot, Bottom row: estimation of function $f_i,\ i = 1, 2$ of the flexible-link dynamics.

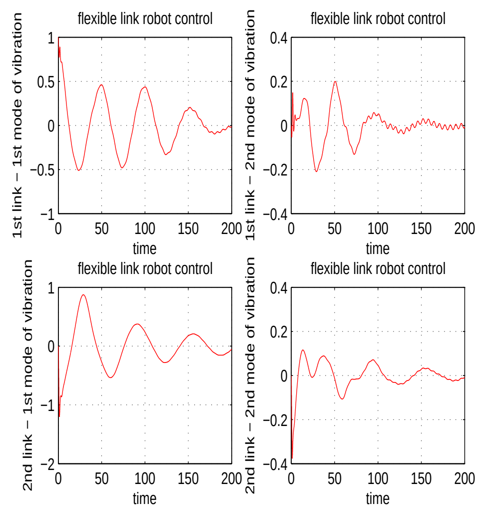

Figure 9. Energy-based control of a 2-link flexible robot under additive motor-torques disturbances: the first two vibration modes for each link.

energy-based controller is also efficient in controlling the position and in suppressing vibrations of the flexible links. However, an advantage of the Kalman Filter-based sliding mode control is that it succeeds accurate tracking for any type of joint angle and velocity set-point whereas the convergence of the energy-based control is assured only in the case of constant set-points.

7 Conclusions

In this paper, the implementation of Kalman Filter-based sliding-mode control for flexible-link manipulators has been examined. Sliding-mode control is a state feedback-based control approach which enables the flexible manipulator joints to track accurately the desirable position and angular velocity setpoints and at the same time results in asymptotic elimination of the effects of the flexible-links vibrations. The inclusion of a switching control term in the sliding-mode controller provides the control loop with robustnesss to modeling uncertainties and external disturbances. However, a difficulty in the implementation of sliding-mode control for flexible-link manipulators is that certain elements of the robot's state vector, e.g. the vibration modes of the links, are difficult to measure. In this paper it has been shown that a suitable formulation of the dynamic model of a flexible-link robot in the state-space form enables the application of the Kalman Filter recursion and provides real-time estimates of the robot's state vector. In turn, the state estimates can be used by the sliding-mode control scheme. The filtering approach is based on the processing of measurements coming from a limited number of sensors, such as measurements of the joints' angles and rotation speed. The efficiency of the proposed Kalman Filter-based sliding mode control scheme has been tested through simulation experiments in the case of a two-link flexible manipulator. The superior performance of the proposed control scheme in comparison to inverse dynamics control has been shown. Moreover, the proposed Kalman Filter-based sliding mode controller was tested against energy-based control.

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