METASTABLE AND CHIMERA-LIKE STATES IN THE C.ELEGANS BRAIN NETWORK

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Abstract

We model the neuronal activity of the *C.elegans* network by coupling Hindmarsh-Rose oscillators through the adjacency matrix obtained from the corresponding brain connectivity. By means of numerical simulation, we produce the parameter spaces for quantities related to synchronization, metastability and chimera-like dynamics, identifying, thus, interesting complex patterns of collective behaviour.

Key words

Metastability, Chimera states, *C.elegans*, Complex networks

1 Introduction

The brain is an incredibly complex system where the different parts (cortical areas) work together but, at the same time, each part has its own special properties. Synchronization is important for cognitive functions and takes place at various levels, within and between cortical areas. The topological properties of a brain network and the local neuron dynamics affect synchronization. As a model organism we employ the *C.elegans* network and focus on the study of synchronization, chimera-like, and metastable states.

2 Community Detection

The communities of the *C.elegans* brain network are identified using the walktrap method [1] with six steps following Ref. [2]. The walktrap algorithm detects communities through a series of short random walks, based on the the idea that vertices encountered on any given random walk are more likely to lie within a community. The algorithm initially treats all nodes as communities of their own, then merges them into larger communities, and these into still larger ones and so on. Essentially, it tries to find densely connected subgraphs (i.e. communities) in a graph via random walks. The idea is that short random walks tend to stay in the same community. Following this procedure we have been able to identify six communities in the *C.elegans* brain network.

In Ref. [2] the authors computed various statistical quantities associated with the *C.elegans* brain network, such as the global clustering coefficient, the average of local clustering coefficients, the mean shortest path, the degree pdf of the network and the small-worldness measure. In the current study, our focus is on the dynamics that take place on the specific community-based network topology and how this is affected by the interplay of the coupling within and between communities.

3 System Dynamics

For the dynamics of each node we employ the Hindmarsh-Rose (HR) neural model [3; 4]. The resulting undirected brain network of N neurons connected simultaneously by electrical and chemical coupling is mathematically described by the following equations:

$$\dot{p}_i = q_i - \alpha p_i^3 + b p_i^2 - n_i + I_{\text{ext}}$$
$$+ g_l \sum_{j=1}^{N} G_{ij} H(p_j) - g_n(p_i - V_{\text{syn}}) \sum_{j=1}^{N} C_{ij} S(p_j),$$
$$\dot{q}_i = c - d p_i^2 - q_i,$$
$$\dot{n}_i = r[a(p_i - p_0) - n_i],$$
$$\dot{\phi}_i = \frac{\dot{q}_i p_i - \dot{p}_i q_i}{p_i^2 + q_i^2},$$

where p_i is the membrane potential of the i-th neuron, q_i is associated with the fast current Na^+ or K^+, n_i with the slow current Ca^{2+}, and ϕ_i is the phase of the i-th oscillator. Other parameters are chosen as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.005$, $p_0 = -1.6$ and $I_{\text{ext}} = 3.2$. Parameter r modulates the slow dynamics of the system and determines the number of spikes per burst, and we set it to 0.005 such that each neuron is chaotic.

The electrical coupling is given by a linear function $H(p) = p$ and the chemical coupling is given by a nonlinear one $S(p) = (1 + \exp[-\lambda(p - \Theta_{\text{syn}})])^{-1}$, where $\Theta_{\text{syn}} = -0.25$ and $\lambda = 10$. The parameter V_{syn} takes the value 2 for excitatory and -2 for inhibitory coupling. We consider the excitatory version of the network: If two neurons are connected under an excitatory synapse then, when the presynaptic neuron spikes, it induces the postsynaptic neuron to spike. $|p_i| < 2$, thus, $(p_i - V_{\text{syn}})$ is negative for excitatory coupling. $I_{\text{ext}} = 3.2$, for which the system exhibits a multi-scale chaotic behaviour characterized as spike bursting.

G_{ij} is a Laplacian matrix and describes electrical coupling within each community. C_{ij} is an adjacency matrix with diagonal elements equal to 0 and describes chemical coupling between the communities. Parameters g_n and g_l are the coupling strength associated to the chemical and electrical synapses, respectively.

3.1 Parameter Spaces

In order to quantify the synchronization level of the neural activity in the whole network we use the global order parameter ρ_g which originates from the theory of coupled phase oscillators of the Kuramoto type [5] and, can be computed by a complex number defined as:

$$\rho_g(t)e^{i\Phi(t)} = \frac{1}{N}\sum_{j=1}^{N} e^{i\phi_j(t)},$$

where N is the number of oscillators. By taking the modulus of this quantity, one can measure the phase coherence in a certain population of N neurons. $\Phi(t)$ is the average phase in the respective population of oscillators and ϕ_i is the phase variable of the i-th neuron of the HR system.

We have calculated the converging value of ρ_g in the parameter space of the two coupling strengths g_l and g_n. The result is shown in the left panel of Fig. 1, where we observe that high syncrhonization levels for the entire network can be achieved only for low chemical coupling strengths.

One should have in mind that complex systems (like the brain) do not converge to stable synchronized states but, instead, exhibit metastability. This means that temporarily they may be found in the vicinity of one stable state before spontaneously leaving away from it toward another. A second feature of many complex systems is competition. In the context of synchronization, this is demonstrated as chimera states [5; 6; 7; 8; 9; 10; 11], where one community of oscillators synchronizes while other communities are desynchronized

In order to quantify how metastable and chimera-like the observed dynamics is, we employ the two measures first introduced by M. Shanahan in [12]. The level of metastability can be calculated from the so-called metastability index [12], given but the expression:

$$\lambda = \langle\sigma_{\text{met}}\rangle_C, \tag{2}$$

where

$$\sigma_{\text{met}}(c) = \frac{1}{T-1}\sum_{t\leq T}(\rho_c(t) - \langle\rho_c\rangle_T)^2. \tag{3}$$

In the above equation, C is the set of all M communities. The order parameter of each community, $\rho_c(t)$, is sampled at discrete times $t \in 1..T$. For a given community, the variance $\sigma_{\text{met}}(c)$ of $\rho_c(t)$ over all time points, gives us an indication of how much the synchrony in this community fluctuates in time. Averaging over all communities in C gives us an index of the metastability in the entire network.

Similarly, the so-called chimera-like index [12] is given by:

$$\chi = \langle\sigma_{\text{chi}}\rangle_T, \tag{4}$$

where

$$\sigma_{\text{chi}}(t) = \frac{1}{M-1}\sum_{c\in C}(\rho_c(t) - \langle\rho(t)\rangle_C)^2. \tag{5}$$

In the above expression, $\sigma_{\text{chi}}(t)$ is an instantaneous quantity that gives the variance of $\phi_c(t)$ over all communites in C at a given time t. The average of this quantity in time indicates how chimera-like a certain state is. The middle and right panel of Fig. 1 show how the metastability and chimera-like index varies in the parameter space (g_l, g_n), respectively.

3.2 Complex patterns and dynamical states

We select 4 points of interest on the (g_l, g_n) parameter space (marked by letter A-D) in order to highlight some interesting patterns among the melange of dynamical regimes that our system exhibits. These points are chosen such that the following four cases are covered: (A) both λ and χ are low-valued, (B) metastability prevails i. e. $\lambda \gg \chi$ (when normalized to 1), (C) "chimera-likeness" prevails over metastability i. e. $\chi \gg \lambda$ (when normalized to 1), and (D) λ and χ are equal and middle-valued (around 0.5 when normalized to 1).

Figure 2 shows the space-time plots of the p-variable for points A-D. The nodes in the communities are relabeled such that each community is placed next to each other in space (community 1 (far left),... community 6 (far right)).

A: This point corresponds to low metastability and low chimera-like index. This means that the network as a whole does not switch to different synchronization patters frequently in time, and simultaneously, the
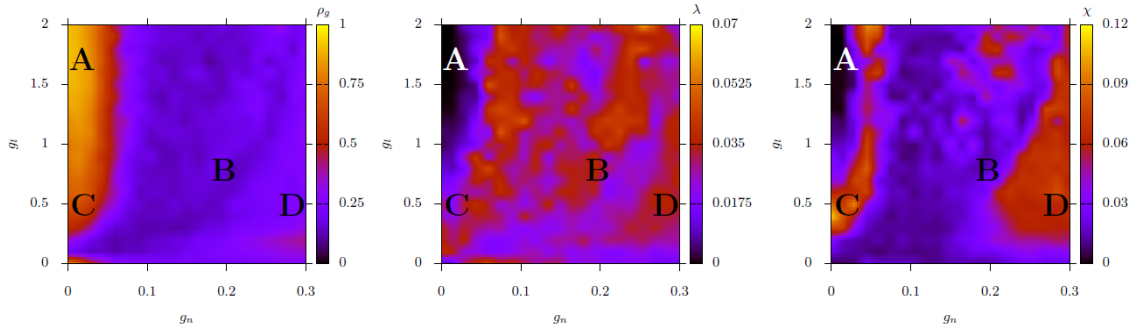
Figure 1. Parameter spaces in the (g_n, g_l) parameter space for the global order parameter (left), the metastability index (middle) and the chimer-like index (right).
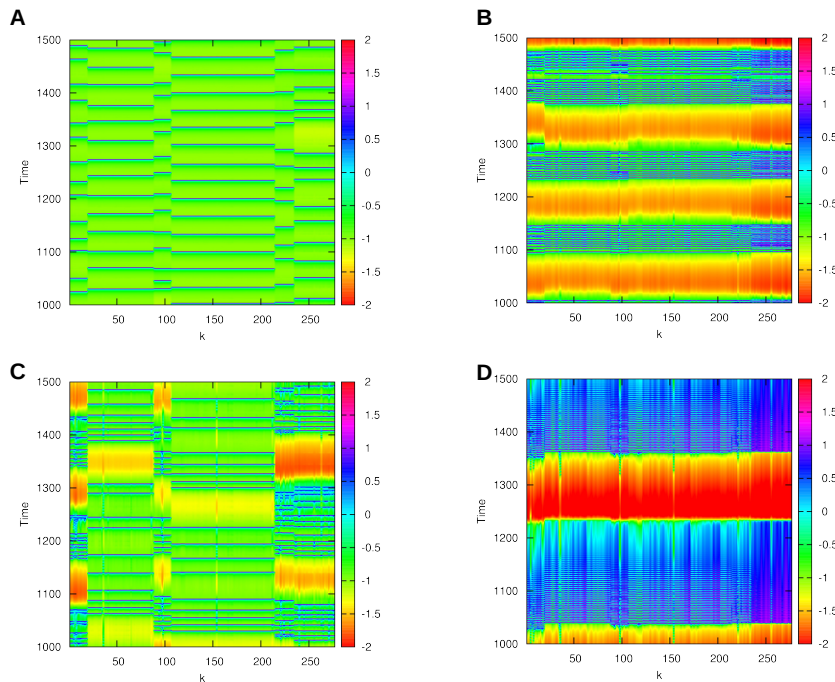

Figure 2. Space-time plots of the whole network for the 4 points marked in the parameter spaces of Fig. 1. **A**: λ and χ very low, **B**: λ high, χ low, **C**: χ high, λ low, **D**: χ and λ of same medium value. Node indices have been reordered.

6 communities are to a large extent in syncrhony with each other. This is expected for such a combination of electrical and chemical couplings and is in agreement with the high value of the global order parameter (Fig. 1(left)).

B: This point shows the effect of metastability when the chimera-like index is low-valued. This is illustrated by the rather regular pattern in space (due to low χ) which in time switches between slow quiescent periods (yellow-red) to fast spiking intervals (blue-green) that correspond to synchronous and incoherent regimes, respectively.

C: This point corresponds to a chimera-like state, where metastability is less prominent. Communities 1,

5 and 6 seem to be the ones that are more incoherent while communities 2 and 4 show long intervals of coherence. If one takes a snapshot in time, one may find coexsting synchronized and unsynchronized communities, a state reminiscent of the well-known chimera states reported in many systems both theoretically and experimentally.

D: This point refers to the case where both λ and χ are prominent in the system.

4 Conclusion

We have identified complex patterns of collective behaviour in a community-organized network based on the *C.elegans* connectivity matrix. In particular,

metastable and chimera-like states are observed as a result of the interaction between topology and local dynamics.

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