

# ACTIVE VIBRATION SUPPRESSION OF BERNOULLI-EULER BEAM: EXPERIMENT AND NUMERICAL SIMULATION

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## Abstract

In order to design the most effective systems of vibration control of a distributed elastic object, it is necessary to have a model of this object, which would allow one to obtain the control results numerically without experiment. This gives an opportunity to compare the results of different control systems with each other and choose the most efficient ones. The paper is concerned with numerical simulation of the results of experimental study on suppression of forced vibrations of a cantilever metal beam with piezoelectric sensors and actuators by finite element method. The new designed control systems are based upon the results of numerical simulation and turn out to be more effective than those tested in the experiment. The numerical results previously received for modal control systems were significantly improved by using the optimization procedure, which allows one to select the optimal parameters of the filters used in the feedback loops of the designed control systems.

## Key words

Active vibration control, mechatronics, modal control, piezoelectric transducer, finite element model, frequency response design.

## 1 Introduction

The specific feature of control of distributed elastic systems is that these systems formally have an infinite number of degrees of freedom, therefore, they are not completely controllable and observable. In addition, these systems have an infinite number of resonances, and the resonance vibrations present the greatest danger to their operation.

Among the generally accepted approaches to the active vibration control of distributed systems there are two

well-known approaches: local and modal ones. A local or a decentralized method implies that the control action applied at some point of the system depends entirely on deformations or displacements of the system measured at this particular point, thus, local sensor-actuator connections are used. A modal approach is based on the separate control of different vibration modes of the elastic object, while the whole array of available sensors and actuators is used to control each mode. Article [Belyaev et al., 2018] gives an experimental comparison of these two approaches for the problem of suppression of forced bending vibrations of a cantilever metal beam. As a result of this study, the advantage of the modal method over the local one was demonstrated in cases where it is necessary to suppress vibrations of the object at several resonance frequencies.

The modal approach to vibration control of elastic systems was first formulated in [Gould and Murray-Lasso, 1966] and further developed in [Meirovitch, 1990; Meirovitch et al., 1983]. The theoretical foundations of the modal approach are also described in the author's works [Belyaev et al., 2017; Belyaev et al., 2018; Fedotov, 2019]. Either distributed sensors and actuators as modal filters [Lee and Murray-Lasso, 1990] or arrays of discrete control elements [Zenz et al., 2013] can be used for tracking and control of individual vibration modes of the elastic object in the modal control system. In the latter case, there arises a problem of identifying the control object related to determining the mutual influence of various control elements and different vibration modes of the object. This problem is usually solved either by the finite element modelling of the object [Braghin et al., 2012; Cenciello and Cavallo, 2017; Cinquemani et al., 2015] or analytically [Biglar et al., 2015; Song et al., 2018]. An experimental procedure for identifying the object in order to create a modal control

system was proposed by the author in [Belyaev et al., 2017]. For active vibration control of distributed elastic systems piezoelectric sensors and actuators are widely used as control elements due to their high operational characteristics.

The present paper continues the experimental study [Belyaev et al., 2018] and the numerical research [Fedotov, 2019]. The aim of this work is to numerically reproduce the main experimental results and to design new control systems that are more effective than systems previously obtained experimentally and numerically. For design of the control systems the optimization procedure for the transfer function parameters in the control loops is used, which allows one to compare the efficiency of control systems with different parameters and choose the best from these variants.

The first part of the article discusses the setting up of the experimental study on the control of forced bending vibrations of a metal beam and the results of operation of different local and modal control systems. Then the finite-element models of the control object are described and the comparison of the numerical and experimental results is presented. After that, the numerical procedure for calculating the control results using the frequency response functions of the object obtained by the finite-element method and known control laws is formulated, and the numerical results for previously designed control systems are given. In the final section of the article, new control systems are synthesized that are more effective than those tested in the experiment.

## 2 Experimental Setup

All stages of the experimental study are described in detail in [Belyaev et al., 2018].

The experimental setup is shown in Fig. 1. A control object is an aluminium beam (1) 70 cm long with a rectangular cross-section  $3 \times 35$  mm mounted in a vertical position at one point at the distance of 10 cm from the lower end. This beam experiences forced bending vibrations caused by the longitudinal vibration of the piezoelectric stack actuator (2), which is a part of the fixation connecting the beam with the stationary base (3). For suppression of the forced beam vibrations, rectangular piezoelectric sensors (4) and actuators (5) are used. Sensors and actuators are connected through a digital controller that converts the measured signals into control signals in accordance with the specified control algorithm.

The main purpose of the experimental setup is to implement experimentally and compare with each other the local and the modal approaches to active vibration control of the beam. Two sensor-actuator pairs are used for control purposes, this allows one to realize both local control with two feedback loops and modal control of two vibration modes of the beam. The objective of all the created control systems is to reduce the amplitude of forced bending vibrations of the beam in the frequency range, which includes the first and the second resonance

frequencies. The quality of control is determined by the vibration amplitude of the point at the upper end of the beam, which is measured by a laser vibrometer.

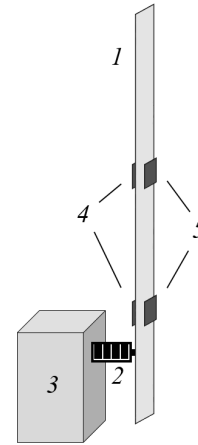


Figure 1. The experimental setup.

The actuators and sensors used are the rectangular patches of piezoelectric material with the dimensions  $50 \times 30 \times 0.5$  mm covered by electrodes on both sides. The operation of such elements in controlling the Bernoulli-Euler beam vibrations is described in [Preumont, 2006]. When an electric voltage is applied to the actuator electrodes, the piezoelectric layer stretches or contracts, leading to the bending deformation of the beam sector, which the actuator is attached to. Thus, the action of the actuator on the beam is equivalent to applying to the two sections of the beam (end sections of the actuator) a pair of bending moments opposite in sign. The operation of the sensor is similar: when the sector of the beam to which the sensor is attached is deformed, the sensor material is stretched or compressed in the longitudinal direction, which leads to the appearance of an electric voltage on the sensor electrodes measured as the sensor signal. For maximum efficiency of control of the first and the second bending modes of the beam, the sensors and the actuators are located on the most deformed regions of the beam when these modes are active. The coordinates of the first and the second sensor-actuator pairs, counted from the lower end of the beam:

$$\begin{aligned} 110.5 \text{ mm} &\leq x_1 \leq 160.5 \text{ mm}, \\ 377.5 \text{ mm} &\leq x_2 \leq 427.5 \text{ mm}. \end{aligned}$$

In order to design the control systems, it is necessary to measure frequency response of the control object. In total, nine frequency response functions (FRFs) were obtained in the frequency range from 1 Hz to 2 kHz: one FRF for each of the three external excitations (one of two patch actuators, the stack actuator) and each of the three measured signals (one of two sensors, the vibrometer). In addition, in order to create a modal control

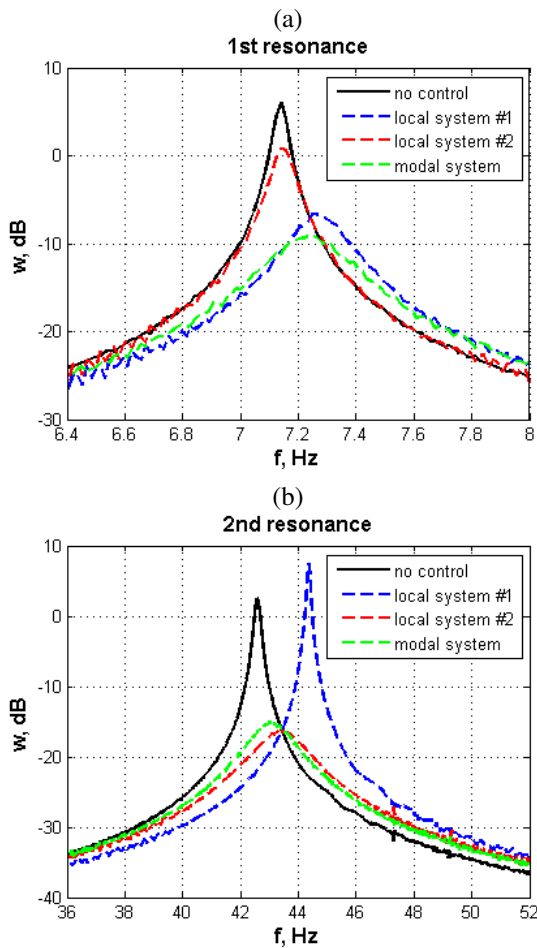


Figure 2. FRFs of the beam with and without control for different control systems at the first (a) and the second (b) resonances (experiment).

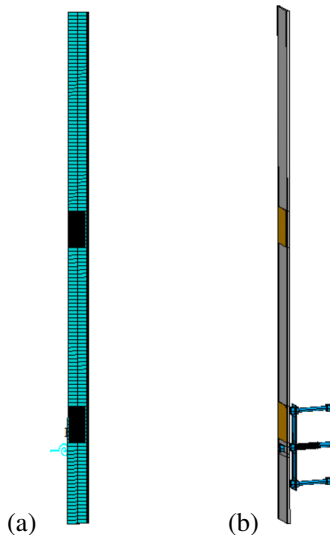


Figure 3. Finite element beam models: the first one with one-dimensional beam elements (a) and the second one with three-dimensional solid elements (b).

system, it is necessary to determine matrices that specify linear transformations of the measured and the control signals (mode analyzer and synthesizer, [Gould and Murray-Lasso, 1966]), providing separate control of the first and the second bending modes of the beam. These matrices were received from experiments in resonance regimes, carried out in accordance with the identification procedure described in [Belyaev et al., 2017]. After that, different control laws were synthesized using the frequency response design method [Dorf and Bishop, 2011; Franklin et al., 2006] based on the measured FRFs of the system in order to suppress forced bending vibrations of the beam.

As a result of experimental selection of the most effective control laws, two local and one modal system were obtained. Their transfer functions are too long to be given here; they are given in [Belyaev et al., 2018]. To measure the efficiency of the designed control systems, the frequency response of the beam was measured for excitation with the stack actuator and measuring the vibration amplitude of the point at the upper end of the beam using the vibrometer. The measured FRFs for different control systems in the vicinity of the first and the second resonances are shown in Fig. 2.

Local system #1 provides the reduction of the vibration amplitude at the first resonance by 12.7 dB, but it fails to suppress the second resonance (amplitude grows by 4.8 dB). Local system #2 is efficient at the second resonance, providing the decrease in the vibration amplitude by 18.9 dB, but it works worse at the first resonance (reduction by 5.2 dB). The modal system is effective at both resonances: the decrease in the vibration amplitude is 15.7 and 17.9 dB, respectively. Therefore, the advantage of the modal approach over the local one is demonstrated in cases where it is necessary to suppress forced vibrations of the object at several resonance frequencies. This result is explained by the fact that in the modal control system, in contrast to the local one, each control circuit is tuned to operate efficiently at specific resonance frequency and uses for this purpose all available sensors and actuators.

### 3 Finite Element Modeling

One of the objectives of this work is to reproduce numerically the results obtained in the experiment described in the previous section. To this end, the mechanical system under consideration was modeled in ANSYS finite element software.

In total, two finite element models of the system were created: the first one constructed of one-dimensional beam elements and the second one constructed of three-dimensional solid elements. The appearance of both models is shown in Fig. 3.

The first model is much simpler than the second one: it contains only 161 elements and 283 nodes, while the second has 3534 elements and 21088 nodes. The differences between these two models are the following: in the

first model the beam fixation is modeled by two springs (longitudinal and torsional), and the piezoelectric effect is not modeled directly. Instead of this, the actuator excitation is specified by an application of forces and moments, and the sensor signal is calculated from the longitudinal deformation of the piezoelectric material. In the second finite element model, the beam fixation is modeled entirely with the stack actuator and additional elements, and the piezoelectric effect is modeled directly by setting the properties of the piezoelectric material. In both models, the same damping coefficient of 0.002 is used for all vibration modes.

Using the finite element method, it is necessary to obtain the nine frequency response functions of the control object that were measured within the experiment. To do this, a harmonic analysis of the system is performed in the frequency range from 1 Hz to 2 kHz, where a harmonic excitation is applied either to the piezostack actuator (to the beam fixation point in the one-dimensional model), or to one of the piezopatch actuators. As the results for each of the simulations, the voltage at the electrodes of both sensors is measured, as well as the transverse displacement of the point at the upper end of the beam.

This gives the FRFs similar to those measured experimentally. Fig. 4 shows one of these functions, corresponding to the excitation of the beam vibrations using the first actuator and measuring the signal using the second sensor. Different lines correspond to the two finite element models and the experimental data. It can be seen that the numerical results are in good agreement with the experiment. The model with three-dimensional elements gives a slightly more accurate result than the one with one-dimensional elements, therefore it is used in the further study.

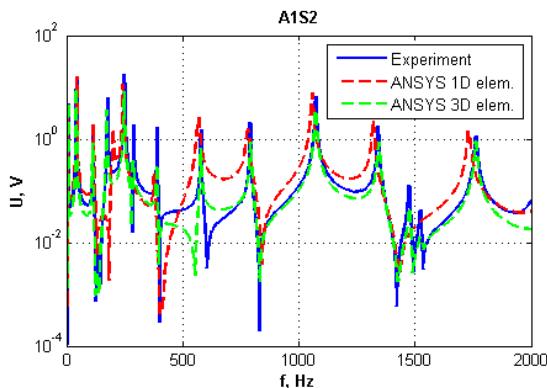


Figure 4. FRFs of the beam corresponding to the 1st actuator and the 2nd sensor (experiment and simulation).

#### 4 Calculation of Control Results

Based on the results of finite element modeling, it is necessary to obtain numerically the FRFs of the beam

corresponding to different control systems tested in the experiment. These functions are derived from the existing FRFs of the beam without control in accordance with the mathematical procedure described below.

Let three excitation sources act simultaneously on the beam, namely, the electric voltage  $U_d$  applied to the stack actuator,  $U_1$  applied to the first patch actuator, and  $U_2$  applied to the second patch actuator. Three quantities are measured: the transverse displacement of the point at the upper end of the beam  $w$ , the voltage at the electrodes of the first sensor  $Y_1$  and of the second sensor  $Y_2$ . The measured values are expressed through the applied excitations using the transfer functions  $H_d, H_d^{(1)}, H_d^{(2)}, H_a^{(1)}, H_a^{(2)}, H_{as}^{(11)}, H_{as}^{(12)}, H_{as}^{(21)}, H_{as}^{(22)}$ :

$$\begin{aligned} w &= H_d U_d + H_a^{(1)} U_1 + H_a^{(2)} U_2, \\ Y_1 &= H_d^{(1)} U_d + H_{as}^{(11)} U_1 + H_{as}^{(21)} U_2, \\ Y_2 &= H_d^{(2)} U_d + H_{as}^{(12)} U_1 + H_{as}^{(22)} U_2. \end{aligned}$$

Let the control actions  $U_1$  and  $U_2$  depend on the measured signals  $Y_1$  and  $Y_2$  of the sensors as follows:

$$\begin{aligned} U_1 &= -R_{11} Y_1 - R_{12} Y_2, \\ U_2 &= -R_{21} Y_1 - R_{22} Y_2. \end{aligned}$$

In this case, using simple mathematical transformations, we can express the displacement of the point at the upper end of the beam  $w$  in terms of the voltage applied to the stack actuator  $U_d$ :

$$\begin{aligned} w &= H_d U_d + H_a^{(1)} U_1 + H_a^{(2)} U_2, \\ U_1 &= U_d \left( -R_{21} H_d^{(1)} - R_{22} H_d^{(2)} + (R_{11} R_{22} - R_{12} R_{21}) (H_{as}^{(12)} H_d^{(1)} - H_{as}^{(11)} H_d^{(2)}) \right) / \left( 1 + R_{11} H_{as}^{(11)} + R_{12} H_{as}^{(12)} + R_{21} H_{as}^{(21)} + R_{22} H_{as}^{(22)} + (R_{11} R_{22} - R_{12} R_{21}) (H_{as}^{(11)} H_{as}^{(22)} - H_{as}^{(12)} H_{as}^{(21)}) \right), \\ U_2 &= U_d \left( -R_{11} H_d^{(1)} - R_{12} H_d^{(2)} - (R_{11} R_{22} - R_{12} R_{21}) (H_{as}^{(22)} H_d^{(1)} - H_{as}^{(21)} H_d^{(2)}) \right) / \left( 1 + R_{11} H_{as}^{(11)} + R_{12} H_{as}^{(12)} + R_{21} H_{as}^{(21)} + R_{22} H_{as}^{(22)} + (R_{11} R_{22} - R_{12} R_{21}) (H_{as}^{(11)} H_{as}^{(22)} - H_{as}^{(12)} H_{as}^{(21)}) \right). \end{aligned}$$

This method allows one to calculate the transfer functions of the system with control based on the known transfer functions of the system without control and selected control laws. The FRFs stack actuator-vibrometer obtained in this way nearby the first and the second resonances for different control systems synthesized within the experiment is shown in Fig. 5.

The result of the local system #1 is the decrease in the vibration amplitude at the first resonance by 15.1 dB and an increase in the amplitude at the second resonance by 1.4 dB. The result of the local system #2 is the decrease in the vibration amplitude at the first and the second resonances by 7 and 19.5 dB, respectively, while for the

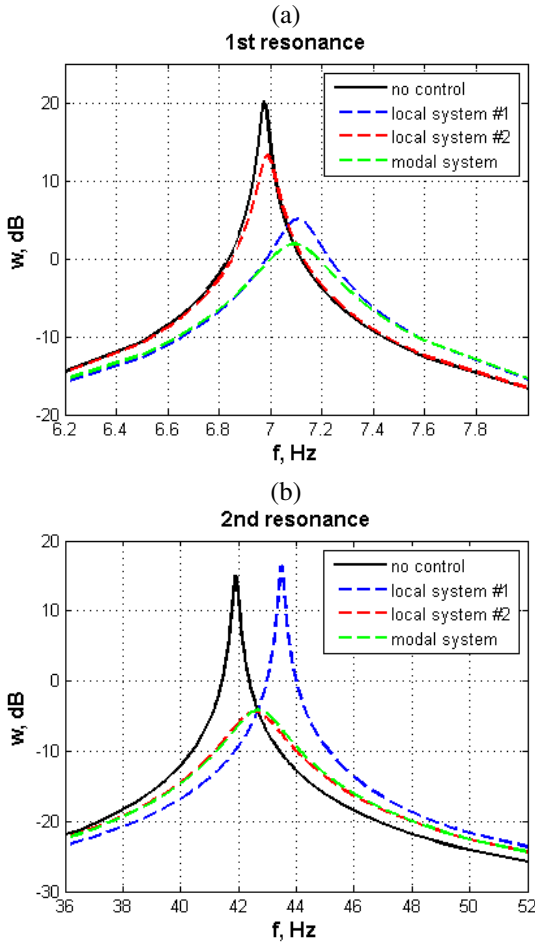


Figure 5. FRFs of the beam with and without control for different control systems at the first (a) and the second (b) resonances (simulation).

modal system, the reduction is 18.3 and 19.3 dB, respectively. It should be noted that both the given numerical values and the obtained curves are close to the experimental results.

## 5 Design of New Modal Control Systems

The next stage of the work is devoted to the following task: it is necessary to synthesize a new, most effective control system, based on the FRFs of the object obtained numerically. This system should be modal, since in the problem under consideration the modal approach gives better results than the local one, as shown earlier in the experimental study. In contrast to the experimental study, during the synthesis of the control laws, the effectiveness of the created systems is defined not experimentally, but numerically, which is much simpler and faster. For this reason, it becomes possible to test a large number of different variants of the transfer functions and choose the most efficient one, that is, to take the transfer function ensuring the greatest decrease in the amplitude of forced vibrations of the beam at the first and the sec-

ond resonances.

The synthesis of optimal control laws for active vibration control of distributed elastic systems is a complex and creative task. This issue is addressed in [Cazzulani et al., 2012; Kim et al., 2011] considering fundamental control strategies such as PPF (positive position feedback), NDF (negative derivative feedback) and some others. In the present study, more complex control laws are used in order to achieve greater efficiency.

The transfer functions for each loop of the designed control systems is constructed using three different filters. The first one is a low-pass filter, which reduces the signal amplitude at high frequencies and thus increases the stability of the closed-loop system. The second one is an inverse notch filter, which raises to the desired value the phase of the control signal in the working frequency domain so that at the required resonance frequency the control action has the opposite phase with the external excitation and can effectively compensate it. The third one is a notch filter, which reduces the amplitude of the signal at the resonance frequency where the risk of instability is the highest, and thus allows one to raise the overall gain value.

In order to create the most effective control systems, a special algorithm in Matlab language is used, which finds the optimal parameters of the filters used in each feedback loop. This algorithm calculates and compares with each other the control results for different sets of parameters of these filters. At first, it is necessary to specify the ranges for each of the varied parameter. Then, for each set of the parameters within the specified range, the algorithm finds the optimal gain value in the control loop, which provides the most effective vibration suppression and at the same time does not cause instability in the closed-loop system. The control results are calculated from the FRFs of the beam using the procedure formulated in the previous section, and the stability of the system is analyzed using the Nyquist criterion. Finally, the algorithm compares the control results for the systems under consideration and selects the most efficient from them. This method allows one to obtain the best control laws within the considered class of functions; however, it does not guarantee that more efficient laws cannot be received by using more complicated filters.

Previously, based on numerical results without using the specified optimization procedure, a modal control system #2 was obtained; this result is presented in [Fedotov, 2019]. Transfer functions of the first and second control loops of this system are the following:

$$R_1^{(2)}(s) = (403s^4 + 2.9 \cdot 10^4 s^3 + 9.7 \cdot 10^8 s^2 + 8.6 \cdot 10^9 s + 1.9 \cdot 10^{12}) / (s^5 + 1.1 \cdot 10^3 s^4 + 2.7 \cdot 10^6 s^3 + 1.5 \cdot 10^9 s^2 + 7.8 \cdot 10^9 s + 3.5 \cdot 10^{12}),$$

$$R_2^{(2)}(s) = (2.2 \cdot 10^5 s^4 + 8 \cdot 10^6 s^3 + 2.7 \cdot 10^{11} s^2 + 4.2 \cdot 10^{12} s + 1.7 \cdot 10^{16}) / (s^6 + 372s^5 + 1.5 \cdot 10^6 s^4 + 4.1 \cdot 10^8 s^3 + 4.2 \cdot 10^{11} s^2 + 3.4 \cdot 10^{13} s + 2.6 \cdot 10^{16}).$$

As a result of the above optimization procedure, a new control system is proposed with the following transfer functions:

$$R_1^{(3)}(s) = (172s^4 + 1.5 \cdot 10^4 s^3 + 4.2 \cdot 10^8 s^2 + 1 \cdot 10^{10} s + 3.9 \cdot 10^{11}) / (s^5 + 1.1 \cdot 10^3 s^4 + 2.7 \cdot 10^6 s^3 + 1.5 \cdot 10^9 s^2 + 1.3 \cdot 10^{10} s + 5.4 \cdot 10^{12}),$$

$$R_2^{(3)}(s) = (1.1 \cdot 10^5 s^4 + 2.4 \cdot 10^6 s^3 + 1.3 \cdot 10^{11} s^2 + 1.3 \cdot 10^{10} s + 4.7 \cdot 10^{13}) / (s^6 + 374s^5 + 1.7 \cdot 10^6 s^4 + 4.6 \cdot 10^8 s^3 + 6 \cdot 10^{11} s^2 + 8 \cdot 10^{13} s + 6.2 \cdot 10^{16}).$$

The results of both designed systems in comparison with the modal control system obtained in the experiment are presented in Fig. 6. As before, to determine the quality of control, the frequency response of the beam is analyzed at the first and the second resonances. In the figure, system #1 was obtained within the experiment, system #2 was designed within the numerical study without using the optimization procedure, and system #3 was synthesized using this procedure.

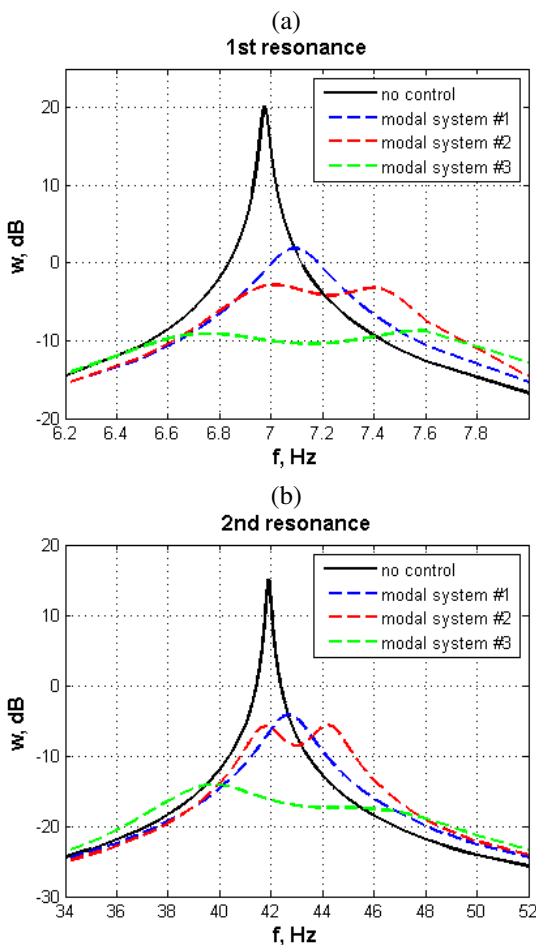


Figure 6. FRFs of the beam with and without control for different modal control systems at the first (a) and the second (b) resonances (simulation).

As can be seen from Fig. 6, system #2 gives a rather small advantage in comparison with system #1: the decrease in the vibration amplitude at the first and the second resonances is 23 and 20.7 dB, respectively, against 18.3 and 19.3 dB. At the same time, system #3 is much more efficient, providing the amplitude reduction at the first and the second resonances by 28.9 and 29.2 dB. This means that the proposed method for the synthesis of control systems using the optimization procedure gives good results.

All the numerical results for different control systems obtained in the present study are summarized in Table 1. For each control system, the difference between the resonance amplitudes of vibration of the upper endpoint of the beam with and without control at the first and the second resonances is presented. For local systems #1 and #2 as well as for modal system #1 both experimental and numerical data are listed, while for modal systems #2 and #3 only simulation results are given since these two systems were not tested experimentally.

Table 1. Change in the level of endpoint vibrations of the beam for different control systems (experiment and simulation)

Control system		$\Delta w_1$ , dB	$\Delta w_2$ , dB
Local #1	exp.	-12.7	4.8
	sim.	-15.1	1.4
Local #2	exp.	-5.2	-18.9
	sim.	-7	-19.5
Modal #1	exp.	-15.7	-17.9
	sim.	-18.3	-19.3
Modal #2	sim.	-23	-20.7
Modal #3	sim.	-28.9	-29.2

## 6 Conclusion

In the framework of the present study, a numerical simulation of a cantilevered metal beam with piezoelectric sensors and actuators was performed. An experimental study of this system is described in [Belyaev et al., 2018]. Two finite element models of the system are created: a simplified model with one-dimensional beam elements and a more complex one with three-dimensional solid elements. The frequency response functions obtained as a result of the analysis of these models for different variants of excitation and measurement are close to experimental, while the result received for the second model turned out to be more accurate.

Based on the frequency response of the control object obtained numerically, solutions to the problem of beam

vibrations in the presence of control were derived for different control systems tested in the experiment. As a result of using each of the control systems, a decrease in the vibration amplitude of the point at the upper end of the beam at the first and the second resonances of bending vibrations of the beam was analyzed. The results of using these systems obtained numerically turned out to be slightly more efficient than the experimental ones, but in total, they are in good agreement with each other.

Numerical modeling of the control object made it possible to design new modal control systems that are more efficient than the systems synthesized within the experiment. The efficiency of modal system #2 described in [Fedotov, 2019] is slightly higher than the efficiency of experimental modal system #1, while system #3, synthesized in the framework of the present study, gives a significantly better result. Namely, system #3 provides the level of beam vibrations approximately 10 dB lower than system #1 at both resonances. The result obtained is explained by the fact that system #3 was created using the optimization procedure for the parameters of the transfer functions in the control loops. Therefore, the proposed method allows one to create active control systems that effectively reduce the amplitude of the forced vibrations of a distributed elastic object.

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