Optimal estimation of the particle sources in Tore Supra tokamak

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Abstract

In this work, we consider the problem of particle source identification from distributed electron density measurements in fusion plasmas, such as the ones obtained in Tore Supra tokamak. Our aim is to propose a simplified diffusive transport model highlighting the coupling of the lower hybrid (LH) high frequency antennas with the plasma edge. Appropriate transport modeling in this region may be crucial for ITER, as it directly determines the efficiency of non-inductive heating. An efficient estimation strategy has to be developed to predict the behavior of the particle source term in the scrape-off layer. A simplified transport model, suitable for identification purposes, is proposed based on classical particle transport models. Two identification methods are proposed: a quasi-steady state approach and a linear approach. Tore Supra data is used to illustrate the different results with experimental measurements.

Keywords

Distributed parameters systems, Controlled thermonuclear fusion, Modeling and identification.

Physical background and results overview

Recent developments in control theory and controlled thermonuclear fusion research are naturally leading to research topics of common interest that are particularly challenging for both scientific communities. For example, new modeling and identification tools are needed for the understanding and analysis of complex physical phenomena. The representation (qualitative and quantitative) of particles transport at the plasma edge is an example of such topics.

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Tokamak experiments, such as *Tore Supra* or *JET*, are equipped with Lower Hybrid (LH) antennas to heat the plasma and create the toroidal current. The LH waves are recognized as the most efficient non-inductive current drive sources and their use is forecasted for ITER experiment. The efficiency of such antennas is strongly related to our ability to ensure an appropriate coupling between the waves and the plasma, which directly depends on the electron density in a region called the *scrape-off layer* (SOL), located between the last closed magnetic surface (the *separatrix*) and the wall. This region, as well as the key elements discussed in this work, are depicted in Fig. 1. The importance of local density control in the SOL is emphasized in [1, 2].

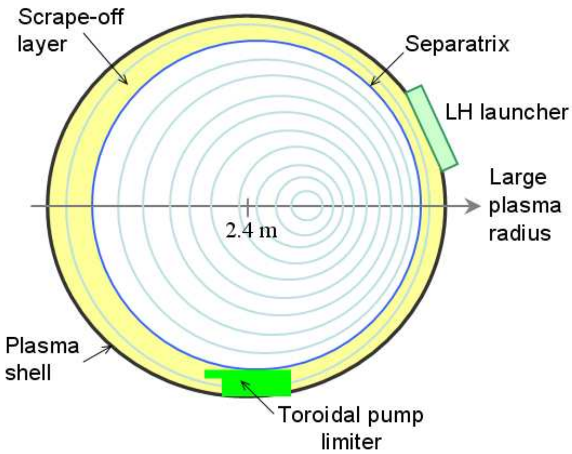

Figure 1: SOL, limiter and LH launcher in a tokamak cross section (i.e. Tore Supra).

The electron density in the SOL is directly influenced by the LH input power P_{LH} [2]. Indeed, a small but significant part of this power is absorbed in the plasma edge during the wave propagation to the core. A possible effect of this absorption is the gas ionization, which results in an increased electron density. This suggests that P_{LH} is a key parameter for the local control of the electron density, and consequently for the coupling efficiency. The development of new models based on experimental measurements that consider the impact of P_{LH} are consequently of prime interest. On Tore Supra, the electron density is measured by using a microwave reflectometer that has both a good spatial ($r \approx 1\ cm$) and temporal resolution ($t \approx 2\ ms$ for the data considered).

Our aim is to propose an identification method for determining the *source term*, i.e. the number of electrons created per unit time and volume, when the high frequency heating is switched on. To achieve this, we derive a simple particle transport model for the area of non-confined plasma (SOL) and develop an appropriate parametric identification method for distributed systems, based on the reflectometer measurements. The resulting algorithm then provides an efficient tool for analyzing the coupling phenomena associated with the electron density behavior in the SOL, even if the detailed physical relationships are unknown.

A simplified transport model

The particle transport model is derived from classical models that are used in thermonuclear fusion. Neglecting the convective effects in the scrape-off layer and considering some minor simplifications we get the following model:

$$\frac{\partial \tilde{n}}{\partial t} = D_{\perp}(r,t)\frac{\partial^2 \tilde{n}}{\partial r^2} - \frac{c_s(r,t)}{2L_c(r,t)}\tilde{n} + S(r,t) \quad (1)$$

where $D_{\perp}$ is the cross-field diffusion coefficient (typically $\sim 1\ m^2s^{-1}$), c_s is the sound speed, L_c is the connecting length along the flux tube to the flow stagnation point and $S = S_l + S_{LH}$ reflects the particle source induced by the limiter S_l and the LH antenna S_{LH}. L_c is deduced from the safety factor q (which typically varies by 10 % in the SOL) as $L_c = 2\pi rq$. The boundary conditions are of Neumann type on the center's side and of Dirichlet's type $\tilde{n}(L,t) = \tilde{n}_L(t)$ at the edge.

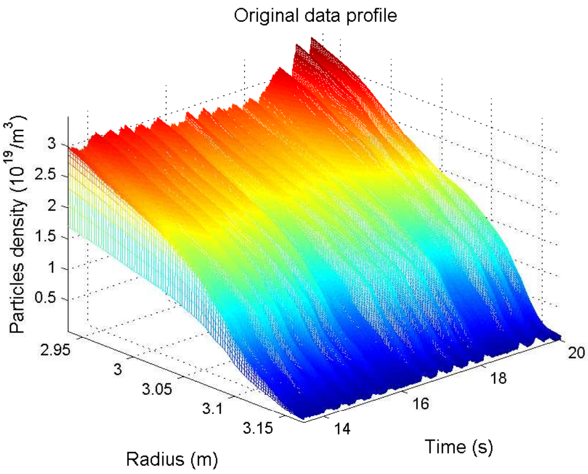

Figure 2: Time and space dependency of the original data (TS 38953).

The data set considered in this paper is presented in Fig.2 (Tore Supra shot 38953): it is characterized by repeated LH impulses, which we can clearly see on the figure as the density peaks.

Identification of the source term

The proposed identification method is based on the infinite dimensional model (1) and the experimental data $\tilde{n}_m(r,t)$ given by Tore Supra shots. To simplify and determine the source term from the given signals of $\tilde{n}_m(r,t)$ we will focus on two different methods: a quasi-steady state approach and a linear approach, detailed in this section.

a) Quasi-steady state (QSS) approach

To elaborate the QSS method, we consider slow variations in the source term and transport coefficients, expressed in the time-scale t' ($\tilde{n}$ has faster variations than the inputs modulation). The density is then supposed to have reached its equilibrium value corresponding to $S(x,t')$ at the time of the measurement ($\partial\tilde{n}/\partial t = 0$). The QSS behavior is then given by:

$$\tilde{n}_{qss}(x,t') = \frac{\tilde{n}_L - \mathcal{C}(S)}{\cosh\lambda}\cosh\lambda x + \frac{1}{\sqrt{\alpha\gamma}}\int_0^x \sinh[\lambda(\eta - x)]S(\eta,t')d\eta,$$

where $\alpha = D_\perp/L^2$ is the diffusion term, $\gamma = \overline{c_s}/(2\overline{L_C}) > 0$ is the sink term, $\mathcal{C}(S) \doteq \frac{1}{\sqrt{\alpha\gamma}}\int_0^1 \sinh[\lambda(\eta-1)]S(\eta,t')d\eta$ and $\lambda = \sqrt{\gamma/\alpha}$.

For the quasi-steady state appproach we establish a parametric identification method to determine the parameter set from experimental data and the QSS model. As an approximation, the source term is considered as the sum of two gaussians (limiter and LH sources) determined from the QSS description. Our analysis is then restricted to the function described as:

$$S(x,t') \approx \sum_{i=l,\,LH} \vartheta_i(t')e^{\beta(x,\,\mu_i(t'),\,\sigma_i(t'))},$$

where l and LH denote the limiter and the LH source, respectively, ϑ_i sets the amplitude, $\beta(\cdot)$ is the dilatation function, σ_i a dilatation coefficient and μ_i the translation. The set of parameters $\theta(t') \doteq \{\vartheta_l,\, \mu_l,\, \sigma_l,\, \vartheta_{LH},\, \mu_{LH},\, \sigma_{LH}\}$ is obtained by solving the least squares problem:

$$\min_\theta \left\{ J(\theta,\, t') = \frac{1}{2}\int_0^1 (\tilde{n}_m(x,t') - \tilde{n}_{qss}(x,\theta,t'))^2 dx \right\}$$

for each sampling instant t', using the experimental measurements $\tilde{n}_m$ and with boundary constraints on θ given by the physical properties of the system. We observed that this approach can provide for satisfying results but is limited by the inherent nonlinearities implied in the resolution of the optimisation problem.

b) Linear approach

The estimation problem can also be considered in a linear framework, thus allowing for more robust optimization algorithms, at the price of a possibly increased estimation error. For the linear approach we approximate our model with a space discretization (central difference scheme) to approximate (1) as the state space model:

$$\frac{d}{dt}\tilde{n}_l = A\tilde{n}_l + BS_l(t)$$

where the l subscript denotes the linear approach. The previous model permits us to use classical methods for observation and estimation, such as the Kalman Filter. This recursive estimator is a powerful method to predict the source term as a time-dependent vector and is capable to reject noise disturbances. To implement the filter, we define our state space model as:

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} \tilde{n}_l \\ S_l \end{bmatrix} &= \begin{bmatrix} A & B \\ 0 & -\tau_i^{-1}I \end{bmatrix}\begin{bmatrix} \tilde{n}_l \\ S_l \end{bmatrix} + W(t) \\ y &= \begin{bmatrix} I & 0 \end{bmatrix}\begin{bmatrix} \tilde{n}_l \\ S_l \end{bmatrix} + V(t) \end{aligned}$$

with $W(t)$ a stochastic process vector, $V(t)$ the measurement white noise and τ_i an arbitrary time constant associated with the source. The source term S_l is the estimated state variable, determined thanks to a Kalman filter that is set with the covariance matrices associated with W and V.

Conclusions

Based on existing physical models, we proposed a simplified model that implies diffusive transport and a sink term, additionally to the source term. This infinite dimensional model is a simple but coherent method to modelise the particle transport in the scrape-off layer. Two different identification methods are proposed to estimate the source term based on this model and the experimental measurements.

References

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