MINIMIZATION OF POWER FLUX DRIVING EVAPORATION SYSTEMS PROPELLED BY THERMAL OR SOLAR ENERGY

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Abstract
In this paper we consider optimal control of drying systems which, by nature, require a large amount of thermal or solar energy. An optimization procedure searches for a minimum power consumed in various one-stage and multi-stage operations of fluidized drying. In these investigations, applications of static optimization and optimal control theory are essential. For steady one-stage systems, methods of differential calculus or Lagrange multipliers are usually sufficient to obtain the optimization solution. However, for power minimization in multi-stage drying systems (occasionally supported by heat pumps) optimal control methods are necessary. As opposed to abundant previous research on engines, we focus here on devices of heat pump type or separator type (energy consumers), each of them driven either by the radiative heat exchange or by the simultaneous transfer of energy and mass. We outline the dynamic programming procedure applied to these systems, and also point out a link between the present irreversible approach and the classical problem of minimum reversible work driving the system.

Key words
Optimization, heat pump dryers, solar energy, thermal efficiency, second law.

1 Introduction

Power data and limits on power yield are important indicators of systems' practical potential. They refer to various energy systems, in particular thermal, chemical and electrochemical engines and fuel cells. Thermal systems are easier to treat, yet, for chemical or electrochemical ones the search can involve extra, diverse aspects, such as: reaction invariants, reference components, process control, polarization data, kinetic efficiencies, stability properties, etc. Evaluation of limiting (maximum) power produced by various power generators has recently been the subject matter of the abundant, recent research, see, e.g., books and reviews [Szwast, 1990], [Berry et al., 2000], [Sieniutycz, 2003], [Sieniutycz, 2007], [Sieniutycz, 2012], [Sieniutycz and Jeżowski, 2009] which refer the reader to corresponding papers by many authors.

The sufficiency of physical considerations to quantify consumed power and irreversibility in practical processes is the fact well known for years; it enables one to develop irreversible thermoeconomics without a prior knowledge of standard economics. In particular, this sufficiency refers to mechanical energy limits which one evaluates with the help of classical (reversible) exergy or its irreversible extensions. In agreement with Gouy-Stodola law [Bosniakovic, 1965], all these extensions contain a minimum of the entropy generated within the system where minimizations are with respect to physical quantities, e. g. flows. Consequently, all irreversible extensions of exergy can be obtained without any prior considering of process economies.

Since the use of power and efficiency formulae derived from analysis of power generators can be extended to units which consume power, it is interesting to develop similar analysis for units such as thermal and solar heat pumps, drying separators and electrolysers. In this paper we focus on limits of power supplied to some thermal drying systems possibly supported by heat pumps.

While the general principles of energy consumption in drying and evaporation operations have been described in monographs, their application to drying, especially to drying driven by mechanical power, is still insufficient. Energy policy in drying has been reviewed; [Strumillo, 1983], [Strumillo and Lopez-Cacicedo, 1984], and [Strumillo and Kudra, 1981]. Exergy analyses of drying began [Bosniakovic, 1965] and [Opman, 1967]. Development of exergy costs and related drying optimizations can be found in papers [Szwast, 1990] and boks [Sieniutycz, 1991]. Processes supported by mechanical energy were initiated; [Zylla and Strumillo, 1981] have advocated use of heat pumps to decrease thermal energy consumption in drying.

A meaningful reduction of drying exergy could be achieved by the optimal control of many dryers; on the average, the potential for exergy reduction is more than 20%. Some industrial data are, however, still reported in terms of energy (enthalpy) rather than exergy efficiency, and suffer from the lack of a strong link with well established kinetic models.

Exact modeling of dried bodies naturally involves sets of coupled partial differential equations for temperature and moisture content. They are derived from the irreversible thermodynamic formalism using Onsager's theory [Luikov and Mikhailov, 1968]. Yet, solvable models of drying are most often lumped ones and deal with space averaged properties. Graphical and numerical solutions of the balance and kinetic equations are effective [Ciborowski, 1965].

The potential for exergy saving in drying processes through changes in design and operation is high. Awareness of limited energy supplies prompts significant effort in developing recovery processes. Possible operations and design modifications involve waste heat recovery from solids, energy recuperation from gases, application of heat pumps for waste energy upgrading, recycling of exhausted drying agent, combinations of mechanical and thermal drying, and use of solar energy.

Application of processes with consumption of mechanical energy and application of optimal control principles gives certain extra potential for improving the energy economy. Some of such processes use thermal separators and heat pumps. The heat pump is in principle the only device which would allow exploitation of the low-exergy sources commonly available in nature and industry. Heat pumps increase driving energy by adding low-quality energy taken from a low-exergy source to it to obtain energy of high quality economically. Analysis of power-assisted processes leads to optimization-determined bounds on the mechanical power input and exergy dissipation.

Power limits in practical processes with finite rates may be evaluated for power released from an engine system or power added to energy consuming system. Two basic models of units consuming power (considered here) may be analyzed. Steady-state models [Zylla and Strumillo, 1981], [De Vos, 1992], refer to situations when reservoirs are infinite. Whereas dynamical models of power-consuming units treat the case with a finite upper reservoir and gradually increasing chemical potential of the moisture [Sieniutycz and Jezowski, 2009].

In dynamical cases Lagrangian and Hamiltonian approaches to power functionals and optimization algorithms using canonical equations are effective. Finite-rate models incorporate minimum entropy production caused by irreversible diffusion phenomena. Modeling a power-assisted operation for the purpose of dynamic limits is a difficult task. Evaluation of dynamic limits requires calculation of sequential operations [Sieniutycz, 1999], [Sieniutycz, 2003], [Sieniutycz, 2004a], [Sieniutycz, 2006], [Sieniutycz and Jezowski, 2009], where total power yield is maximized at constraints which describe dynamics of energy and mass exchange. The dynamical model can be continuous or discrete; the latter are frequent for computational purposes. The results are limiting work functions in terms of end states, duration and (in discrete processes) number of stages [Sieniutycz, 1999]. A general theory of optimal discrete systems, nonlinear in time intervals, has recently be published in the present journal [Poświata, 2012]. Application of continuous theory of optimal control has been especially fruitful for chemical reactors with power production [Sieniutycz and Jezowski, 2009]. Another example is an optimal feedback control of traveling waves in a FitzHugh–Nagumo model [Takeuchi, Konishi and Hara, 2012].

Modeling of power generation processes is consistent with general philosophy of optimization [Szwast, 1988], [Berry et al., 2000], [Sieniutycz, 2006a], [Sieniutycz and Farkas, 2005], [Sieniutycz and Jezowski, 2009]. Finite-rate, endoreversible models include irreducible losses of classical exergy caused by resistances. Constraints take into account dynamics of mass transport and rates of moisture flux. Optimal performance functions, which describe dynamical extrema of power and incorporate a residual minimum entropy production, are determined in terms of end states, duration and (in discrete processes) number of stages [Sieniutycz, 2003a], [Sieniutycz, 2004a], [Poświata, 2003], [Poświata, 2004], [Poświata, 2005]. Processes and systems at the frontiers of cybernetics and physics which can be benefited from the control theory can be discussed and classified [Fradkov, 2012].

Similarly like in heat systems, in drying systems enhanced limits follow from constrained optimization of total supplied power. Optimization analysis leads to the dryer's efficiency and limiting power [Berry et al., 2000], [Sieniutycz, 2003b], [Sieniutycz and Jezowski, 2009]. Classical reversible theory is capable of determining energy limits in terms of exergy changes [Bosniakovic, 1965], [Opman, 1967], [Berry *et al.*, 2000]. Unfortunately, reversible limits are too distant from reality (real energy consumption is much higher than the reversible lower bound and/or real energy production is much lower than the reversible upper bound).

Yet, by introducing rate dependent factors, irreversible thermodynamics offers more realistic limits. Consequently, irreversible cycles need to be considered, Fig. 1).

Irreversible thermodynamics of finite rates (used here and in many other works which deal with processes of finite durations or those in equipments of finite size [Sieniutycz, 2003a], implies enhanced limits on the work consumption which are stronger (higher) than those predicted by the classical work of thermodynamics. These limits are identical or closer to those used in engineering design. In this paper we focus on limits evaluated when work is supplied to a heat pump which

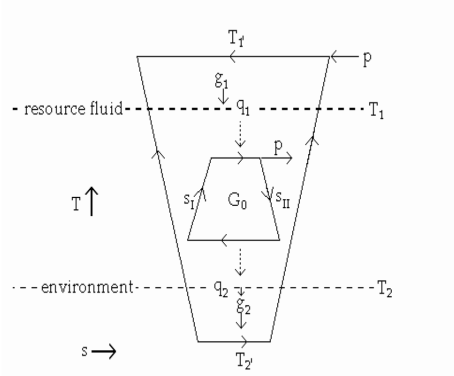

Figure 1. Designations and comparison of two basic cycles with power production or consumption subject to internal and external dissipation: power production in an engine and power consumption in a heat pump.

heats a drying gas. They lead to estimates of minimum work supplied to a heat pump.

Calculations of the minimum power show that the data differ for power generated and consumed, and depend on parameters of the system, e. g., flux densities, number of mass transfer units, polarizations, electrode surface area, average chemical rate, etc.. These data provide bounds for dryers as energy consumers, which are more exact and informative than classical reversible bounds.

Nomenclature

A, a — cumulative and local heat exchange area, respectively

A — finite time availability, generalized exergy

B, b — classical specific exergy of controlled and controlling phase

c — specific heat at the constant pressure

e — specific consumption of work, or power supply per unit mass flux

e — economic value of exergy unit

F — cross-sectional area of the controlled system

G — mass flux (ΔG^n is the mass flux through stage *n*)

g_1, g_2 — partial conductances (g is overall conductance)

HTU — height of transfer unit

I — specific enthalpy of solid phase

i — specific enthalpy of gaseous phase

l — length coordinate

M — molar mass

N, n — total number of stages and current stage number

P, p — cumulative power output and stage power output

Q, q — cumulative heat flux and heat flux to a stage

R — universal gas constant

R — performance criterion of cost type

r — specific heat of evaporation

S, s — specific entropy of controlled and controlling phase, respectively

S_σ — intensity of entropy production per unit mass flux

T — temperature of controlled phase (fluid or solid)

T^e — constant temperature of thermal reservoir or environment

$T\prime$ — Carnot temperature as an effective inlet temperature of controlling phase

T_1, T_2 — temperatures of upper and lower reservoirs (usually $T_2 = T^e$ and $T_1 \equiv T$)

$T_{1\prime}, T_{2\prime}$ — upper and lower temperature of circulating fluid

t — physical time, contact time

$u = \Delta T/\Delta\tau$ — rate of temperature change as control variable

V — volume of physical system

v — linear velocity of controlled phase

$W \equiv P/G$ — total specific work output, total power per unit mass flux

Ŵ — absolute moisture content in solid

X -absolute humidity of controlling phase

x — transfer area coordinate

α' — overall heat transfer coefficient

β — relative humidity

γ_1, γ_2 — coordinates of partial conductances (γ is an overall conductance)

$\eta = p/q_1$ — first-law efficiency

θ — interval of an independent variable or time at a stage

μ — chemical potential

μ' — coefficient of gas utilization

ρ — mass density

σ — entropy production

τ — non-dimensional time, number of the heat transfer units (x/HTU)

Φ — factor of internal irreversibility

Subscripts

a — adiabatic

C — Carnot point

d — dry state

f — fluid

g — gas, inert gas

f — fluid at flow

i — i-th state variable

k — k-th component

m — at maximum power

n — stage number

l — liquid

p — vapor

s — solid, dry solid, equilibrium with solid

v — per unit volume

w — active component, moisture

σ — dissipative quantity

1, 2 — first fluid (driving fluid) and second fluid (reservoir fluid)

* — modified quantity

Superscripts

e — equilibrium with environment

f — final state

i — initial state

n or *N* — from *n*-th stage or from *N*-th (last) stage

' — driving state or inlet state of controlling phase

2 Imperfect Performance Coefficients of Power Consumption

Usually, performance coefficients of power consumtion units, such as heat pumps, dryers, separators, *etc.*, are reciprocals of efficiencies power-production units (engines) modulo to a sign. By evaluating entropy production in an infinitesimal cycle σ_s (the sum of external and internal parts) as the difference between the outlet and inlet entropy fluxes we find in terms of the first-law efficiency η

$$S_\sigma = \frac{dQ_1(1-\eta)}{T_2} - \frac{dQ_1}{T_1} = \frac{dQ_1}{T_2}\left(1-\eta-\frac{T_2}{T_1}\right) \quad (1)$$

where T_1 and T_2 are temperatures of the two reservoirs, respectively, Fig. 1. Equation (1) states that the deviation of engine's efficiency from the Carnot efficiency is related to the entropy production. This property leads us to an important analytical formula for the real efficiency, suitable in process optimization. To derive this efficiency formula we note that the thermal efficiency of a real thermal engine or heat pump can always by written in the form

$$\eta = 1 - \frac{dQ_2}{dQ_1}. \quad (2)$$

In terms of the factor of internal irreversibilities

$$\Phi \equiv 1 + T_{1'}\frac{d\sigma_s^{int}}{dQ_1}$$

the entropy balance of working medium takes a form

$$\Phi\frac{dQ_1}{T_{1'}} = \frac{dQ_2}{T_{2'}}. \quad (3)$$

The factor Φ can be found from the internal entropy production within the machine. As Φ is often a complicated function of the operating variables, an averaged Φ over the cycle is used, treated as the process constant. In terms of unknown temperatures of circulating fluid $T_{1'}$ and $T_{2'}$ and internal irreversibility Φ real efficiency η follows as

$$\eta = 1 - \frac{dQ_2}{dQ_1} = 1 - \Phi\frac{T_{2'}}{T_{1'}} \quad (4)$$

This equation simplifies, of course, to the Carnot formula in terms of both primed T when internal entropy source vanishes (the so-called endoreversible operation). We stress that no special assumptions were made to derive Eq. (1) and the first equality of Eq. (4).

Now, following many earlier works [Berry *et al.*, 2000], [Sieniutycz, 2003a], [Sieniutycz, 2003b], [Sieniutycz and Jezowski, 2009], a quantity called Carnot temperature T' is introduced that satisfies the thermodynamic relation

$$T' \equiv T_2\frac{T_{1'}}{T_{2'}}. \quad (5)$$

The name Carnot temperature is used for the quantity T' simply because the efficiency of an internally reversible engine expressed in terms of T' and T_2 satisfies the Carnot formula, see Eq. (6) below. Note that, in agreement with Eq. (5), temperature T' characterizes the effect of the temperature ratio for circulating fluid, $T_{1'}/T_{2'}$, in the thermal machine. After using Eq. (5) in Eq. (4) thermal efficiency η in terms of temperature T' assumes a simple, pseudo-Carnot form [Sieniutycz, 2003a]

$$\eta = 1 - \Phi\frac{T_2}{T'}. \quad (6)$$

Equation (6) is suitable to evaluate fluxes of power production or consumption in various steady and unsteady systems. Yet, to get a heat flux one must apply a definite model of heat exchange [Berry *et al.*, 2000].

For power consumption systems, Eq. (6) is usually applied in the form of its negative reciprocity which describes the so-called coefficient of performance, a quantity applied to assessment of heat pumps, refrigerators and other power consumers.

3 Thermal and Mechanical Power

In this paper we analyze the so-called active systems, i.e. those capable of power production or consumption. Comparison of mathematical models of thermal engines with those of other active systems leads to conclusions about a formal link between models of various active energy systems.

To enhance modeling generality we use here a description of the thermal unit with conductances g_1 and g_2 that are functions of T_1 and T_2. This allows us to describe active nonlinear processes, in particular those with radiation. Variable coefficients of heat transfer are $\alpha_1(T_1)$ and $\alpha_2(T_2)$. The entropy balance for the internal part of the machine

$$g_2(T_2)(T_{2'}-T_2)T_{2'}^{-1} - \Phi g_1(T_1)(T_1-T_{1'})T_{1'}^{-1} = 0 \quad (7)$$

proves that T' is a single unconstrained control. We use Eq. (5) to insert $T_{2'} = T_{1'}T_2/T'$ into Eq. (7). We then obtain $T_{1'}$ in terms of T'

$$T_{1'} = (\Phi g_1 T_1 + g_2 T')(\Phi g_1 + g_2)^{-1} \quad (8)$$

and the corresponding equation for $T_{2'}$, which is

$$T_{2'} = T_2\frac{T_{1'}}{T'} \quad (9)$$

The driving energy flux follows as

$$q_1 = g_1(T_1 - T_{1'}) = g\prime(T_1 - T') \tag{10}$$

and the second heat flux is

$$q_2 = q_1(1 - \eta) \tag{11}$$

where η is defined by the pseudo-Carnot expression (6).

In Eq. (10) an overall conductance, g', appears, defined as

$$g'(\Phi, T_1, T_2) \equiv g_1 g_2 (\Phi g_1 + g_2)^{-1} = (g_1^{-1} + \Phi g_2^{-1})^{-1}. \tag{12}$$

This is, in fact, a suitably modified overall conductance of an inactive heat transfer which uses given partial conductances, g_1 and g_2.

From Eqs. (6) and (9) propelling mechanical power $p = \eta q_1$ follows

$$p = q_1 \eta = g\prime(\Phi, T_1, T_2)(T_1 - T')(1 - \Phi\frac{T_2}{T'}) \tag{13}$$

where the overall effective conductance $g\prime$ is defined by Eq. (12). Optimal control $T\prime$ ensuring the upper bound for power production in engine modes is

$$T'_{opt} = (\Phi T_1 T_2)^{1/2}. \tag{14}$$

As an interesting digression, we point out that T'_{opt} is also the optimal temperature of a solar collector maximizing its exergy output when T_1 and T_2 are, respectively, the collector's stagnation temperature and ambient temperature, and $\Phi = 1$.

Equation (13) also serves to minimize the work supply in a process with drying agent heated by heat pumps, as described in Section 5. In that case we need to determine a lower bound for power consumption in heat pump modes. Note, however, that Eq. (14) is invalid and useless for heat pumps.

4 Dryers with Optimally Controlled State of Inlet Gas

In this section we introduce optimal operations of drying with controlled inlet gas state. An example of such traditionally controlled drying (with no heat pumps) is multistage fluidized drying schematized in Fig. 3. In this operation each single stage can be modeled in two ways (pseudo-homogeneous and non-homogeneous), as shown in Fig. 2 below.

Dealing with multi-stage application, we consider a cascade composed of fluidized dryers, in which each stage may be governed by either of two models in Fig. 2. It is assumed that – macroscopically – a pulverized solid contacts cross-currently with a gas, as in Fig. 3. It may also be assumed that drying process occurs either in the first or in the second drying period.

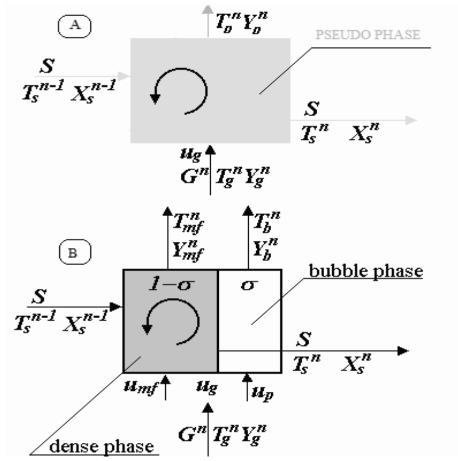

Figure 2. Two ways of modeling of fluidized drying. Case A: [Szwast, 1990] and [Sieniutycz, 1991], case B: [Poświata, 2005]. The scheme allows simultaneous treatment of the batch fluidization and the fluidization in a horizontal exchanger as the continuous limit of the cascade.

The virtue of cascade models is that any gas or solid parameter or any stage property may explicitly change with the stage number of the cascade. Therefore, it is unnecessary to assume that, e. g., the evaporation enthalpy, thermal coefficient, *etc.* is the same for each stage of the dryer cascade. This property is especially useful when dealing with difficult models which admit only numerical solving.

The multistage idea means, of course, the repetition of the single-stage operation in next stages. The gas leaving a stage is released to the atmosphere; similarly, the outlet solid from a stage flows to the dryer of the next stage. Such process may be continued.

A typical question is: how many stages are there in the system? The answer to this question is possible by making the economical analysis. In practice it suffices to limit cascade calculations to several stages and accept as the last computational stage the one beyond of which the sum of the operational cost and investment cost stops to decrease. As the investment cost of the cascade grows linearly with its total number of stages, the optimal number of stages is usually limited to a few stages.

Yet, there are also cascades created for purely computational tasks of continuous systems (dryers, reactors, *etc.*) They are composed of stages which are small computational units rather than real apparatuses. In those cases there is no cost optimization versus total number of stages. Suitable number of stages is then found by division of the systems holdup time by the holdup time interval at one stage.

For relatively efficient stages, we may assume that the outlet solid and the outlet gas are in the equilibrium due to a large specific solid area.

Cascade in Fig. 3 refers to operation in which every drying agent is heated before the dryer by a medium

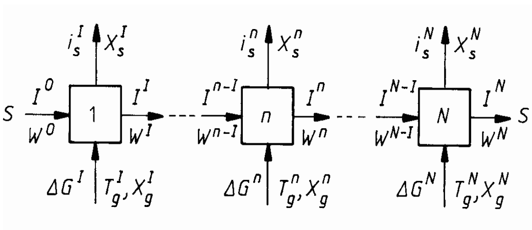

Figure 3. A crosscurrent cascade of fluidized dryers.

with purposely adjusted state, in order to achieve sufficiently high drying temperatures within the stages. These cascades imply optimization problems in which process controls are inlet parameters of heating gas (temperatures and humidities) and gas flows across each stage. In these problems explicit input fluxes of mechanical power are not considered as possible controls. A discrete optimization algorithm with a constant Hamiltonian is a frequent optimization tool in this case [Szwast, 1988], [Berry at al, 2000],[Poświata, 2005]. Optimization criteria for cascades in Fig. 3 involve exergy costs or economic costs necessary to drive drying operation [Szwast, 1990], [Sieniutycz, 1991], [Poświata, 2003], [Poświata, 2004].

The block scheme of Fig. 3 may also constitute a multistage representation of a dynamic process with power production. It applies then to batch fluidized drying in which the controlled entity is a load of solid, whereas the controlling inputs are gas streams with variable inlet states and flows.

In the both cases considered above, the set $X = \{I, W, \tau\})$ describes an enlarged (time containing) state vector of the dried solid, and u refers to a set of controls, i. e. gas temperatures, gas humidities and gas flows.

Methods of dynamic programming and maximum principle can be applied to accomplish multistage optimization of the above drying processes [Sieniutycz, 1991], [Sieniutycz, 2006b], [Sieniutycz and Jezowski, 2009]. While an original continuous problem (if exists) is governed by the Hamilton-Jacobi-Bellman (HJB) theory, its discrete counterpart is described by dynamic programming, (DP), [Berry *at al.*, 2000]. Dynamic programming offers description in terms of wave-fronts which, in the drying case, are surfaces of constant exergy input per unit mass of gas. On the other hand, the method of maximum principle (or a similar method of variational calculus) constitutes the description in terms of process trajectories. For a multistage drying process, its discrete model describes multistage evolution of drying, and leads to the optimal function of driving exergy. This function can be determined either in terms of the solid state and the number of transfer units τ, or in terms of Hamiltonian h. The latter quantity, the same for each point of an optimal path, is a common measure of the optimal process intensity. While the number τ measures the residence time of the solid phase, the quantity h quantifies the minimal irreversibility in the system.

To perform dynamic programming optimization (DP optimization) of the original exergy cost

$$R^N = \sum b^n \theta^n \tag{15}$$

an augmented cost is defined (n=1...N)

$$R'^N = \sum (b^n + h)\theta^n \tag{16}$$

in which the term $h\theta^n$ plays the role of the investment cost at stage n. The basic function of the dynamic programming algorithm, i. e. optimal performance function $R^{*N}(I^n, W^n, h)$, is defined as

$$R^{*N}(I^n, W^n, h) \equiv \min R'^N \equiv \min \sum (b^n + h)\theta^n \tag{17}$$

This function is evaluated by Bellman's recurrence equation of dynamic programming (DP), subject to the discrete state equations

$$I^{n-1} = \Upsilon_I^n(I^n, W^n, \theta^n), \tag{18}$$

$$W^{n-1} = \Upsilon_W^n(I^n, W^n, \theta^n) \tag{19}$$

as process constraints. The transformation functions Υ_I^n and Υ_W^n in state equations (18) and (19) are derived by combining energy and mass balances with appropriate kinetic relationships at stage n [Sieniutycz, 1991].

Forward DP algorithm is applied, [Berry at al, 2000]. For the minimum exergy cost defined as $R^{*N}(I^n, W^n, h) \equiv \min R'^N$, Bellman's recurrence equation has the following form

$$R^{e*n}(I^n, W^n, h) \equiv$$

$$\equiv \min\{(b^n + h)\theta^n + R^{*n-1}(I^{n-1}, W^{n-1}, h)\} \tag{20}$$

where the inlet coordinates (I^{n-1}, W^{n-1}) are expressed in terms of the outlet state coordinates (I^n, W^n) with the help of state equations (18) and (19).

In the dynamic programming procedure a computer generates tables of optimal controls and optimal costs by solving Bellman's recurrence equation for the optimal performance function modified by the presence of Lagrange multiplier, h. The presence of the multiplier h reduces the problem dimesionality, i. e. eliminates time τ from the state vector, the effect which improves the computational accuracy.

The references cited provide details regarding optimal control parameters and optimal exergy criteria. Here are main summarizing results referred to the "exergy optimization" in which the total input of gas exergy is the optimization criterion to be minimized. Multistage fluidized drying in the first drying period proceeds mainly at the first stage. Fixed cost defined by the number of stages in the cascade decides about the advisability of applying of cascade in practical drying. When the Lewis factor or the ratio of resistance coefficients of mass and heat transfer equals 1, optimal driving forces are virtually independent of hydrodynamics and kinetics. Only optimal gas flow-rate strongly depends on hydrodynamics and kinetics in the bed.

Optimal driving forces of transfers involved depend on the parameter ζ which describes the ratio of prices of thermal and chemical exergies, whereas the optimal gas flow rate is independent on that parameter. For low values of ζ driving forces increase with stage number n because the optimal inlet gas humidity decreases with n. For large ζ driving forces decrease with n because lowered values of equilibrium gas humidity caused by decreasing solid temperatures T_s are not compensated by a very slow decrease of the inlet gas humidity. Optimal gas flow rates θ_g rapidly decrease with a coefficient of investment costs. One may observe that parameter ζ in practice does not influence gas flow rate, and only influences driving forces of mass and heat transport processes [Szwast, 1990], [Sieniutycz, 1991], [Poświata, 2003], [Poświata, 2004], [Poświata, 2005].

5 Power Consumtion for Drying with Heat Pumps

Let us now consider a power-assisted operation in which a drying agent is heated before each dryer by a heat pump, in order to achieve a sufficiently high temperature. In this case the gas leaving the previous stage enters the heat pump and dryer of the next stage, Fig. 4. The properties of the heat pump as the heating device are important in this analysis, the better COP results in more efficient heating. This example again refers to a non-ideal fluidized drying in which one stage can be modeled in two ways (pseudo-homogeneous and non-homogeneous), Fig. 2.

As shown in Fig. 4 a stage of this complex multi-step operation often comprises not a single dryer but rather an appropriate group of various units which is repeated when the process proceeds from one step to another. This is just the case of a multi-stage drying operation, in which gas at each stage is heated with a heat pump and then is directed to a dryer (note that only one stage of that operation is shown in Fig. 4). In the considered case a continuous drying process occurs in a co-current dryer. The purpose is to minimize the work consumption in, say, the two-stage operation by a suitable choice of the intermediate moisture content between the first and the second stage.

One may ask: how many heat pumps (and stages) are there in the optimal system? Again, as in the example of the previous section, the answer is possible by making the economical analysis, and terminate cascade calculations when sum of operational and investment

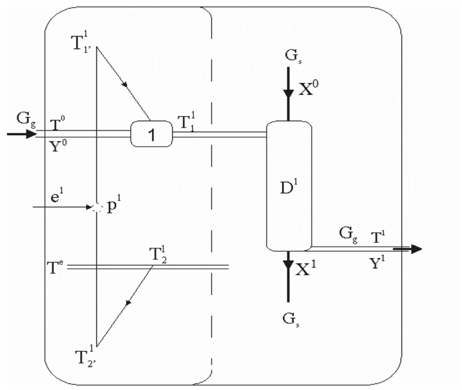

Figure 4. A scheme of one-stage drying operation with a heat pump 1 and a continuous co-current dryer D^1 with falling particles. The multistage idea means, of course, repetition of this single-stage set in the next stages.

cost stops to decrease. As the investment cost grows remarkably with number of stages, the optimal number of stages is usually limited to a few stages.

The balances of mass and heat yield

$$\frac{rG_s}{cG_g}(W^0 - W^1) = (T_1^1 - T^1), \tag{21}$$

$$T_1^1 - T^1 = -\frac{r}{c}(X_1^1 - X_s(T^1)), \tag{22}$$

whereas power consumed at a single stage per unit flow of gas, e^1, is described by an expression

$$e^1 = -\frac{p}{G_g} = c\left(1 - \frac{T^e}{T_1^1 + u^1}\right) u^1 \theta^1 \tag{23}$$

where $u^1 = -q^1/g > 0$ is a measure of the energy supply in the temperature units, g is the overall conductance, and q^1 is the energy supply to the drying gas in the condenser of the heat pump.

In an introductory analysis we may assume absence of internal irreversibilities within heat pump. If this assumption cannot be omitted, the performance coefficient of the support heat pump contains imperfection factor Φ, as shown below. For the first stage

$$COP^1 =$$

$$-\left(1 - \Phi^1 \frac{T^e}{T'^1}\right)^{-1} = -\left(1 - \Phi^1 \frac{T^e}{T_1^1 + u^1}\right)^{-1} \tag{24}$$

and for the stage n

$$COP^n =$$

$$-\left(1-\Phi^n\frac{T^e}{T'^n}\right)^{-1}=-\left(1-\Phi^n\frac{T^e}{T_1^n+u^n}\right)^{-1}. \quad (25)$$

Substituting into Eq. (23) the temperature following from Eq. (22)

$$T_1^1=T^1+\frac{r}{c}(X_s(T^1)-X_1^1), \quad (26)$$

and taking into account that $X_1^1=X^0$ [also $X_1^n=X^{n-1}=X_s(T^{n-1})$, for $n=2,\dots,N$] we find the mechanical energy consumption at the first stage

$$e^1=c\left(1-\frac{\Phi T^e}{T^1+rc^{-1}(X_s(T^1)-X^0)+u^1}\right)u^1\theta^1. \quad (27)$$

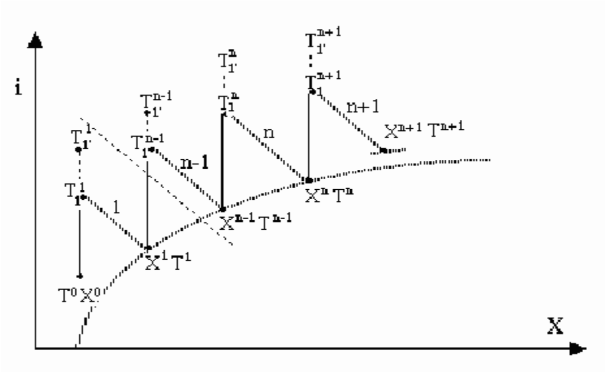

Figure 5. Changes of gas states in a multistage work-assisted drying operation. Primed states refer to temperatures of circulating fluids in heat pumps which heat gases supplied to dryers $1,2,\dots,n$.

Equation (27) is transformed further in view of the link between u^1 and θ^1 (consider difference constraint describing $\Delta T^n=u^n\theta^n$ for $n=1$) with $u^1\theta^1=T^1+rc^{-1}(X_s(T^1)-X^0)-T^0$.

In terms of the "adiabatic temperature function"

$$T^a(T^1)\equiv T^1+rc^{-1}X_s(T^1) \quad (28)$$

the work expression takes the final form

$$e^1=c\left(1-\frac{\Phi T^e}{T^{a1}-rc^{-1}X^0+\frac{T^{a1}-rc^{-1}X^0-T^0}{\theta^1}}\right)\times$$

$$\times\left(T^{a1}-rc^{-1}X^0-T^0\right) \quad (29)$$

where $T^{a1}\equiv T^a(T^1)$. An analogous function, but with the shifted superscripts, is valid for the second stage

$$e^2=c\left(1-\frac{\Phi T^e}{T^{a2}-rc^{-1}X_s^1+\frac{T^{a2}-rc^{-1}X_s^1-T^1}{\theta^2}}\right)\times$$

$$\times\left(T^{a2}-rc^{-1}X_s^1-T^1\right) \quad (30)$$

where $X_s^1\equiv X_s(T^1)$ and $T^{a2}\equiv T^a(T^2)$. The constraint $X^1=X_s(T^1)$ resulting from the phase equilibrium at the outlet of first dryer is incorporated in the second work expression. In terms of the adiabatic temperature at the second stage

$$e^2=c\left(1-\frac{\Phi T^e}{T^{a2}-rc^{-1}X_s^1+\frac{T^{a2}-T^{a1}}{\theta^2}}\right)\left(T^{a2}-T^{a1}\right) \quad (31)$$

where $T^{a1}\equiv T^a(T^1)$and $T^{a2}\equiv T^a(T^2)$.

The sum of both specific works, $R^2=\sum e^n=e^1+e^2$, represents total power per mass unit of gas flow, or total specific work consumed in the two stage process considered. In the associated optimization problem, R^2 is the thermodynamic cost of the two stage system that should be minimized. For a fixed holdup time $\tau^2=\tau^1+\tau^2$ there are two free controls: θ^1 and T^1. The procedure searches for an optimal interstage temperature T^1 and an optimal heat transfer area of the first heat pump a^1 present in the control variable θ^1. The requirement of a sufficiently low final moisture content in solid defines amount of the evaporated moisture per unit time. Thus power minimization is easy for two stage system. The optimization can be generalized to N-stage cascade system, as outlined below.

Dynamic programming or discrete maximum principle are applied to accomplish multistage optimization of the present cascade system in which each stage may contain a dryer and its support heat pump [Sieniutycz, 1991], [Sieniutycz, 2006b], [Sieniutycz and Jeżowski, 2009]. Optimal work can be determined either in terms of end process state and number of transfer units τ or in terms of this state and the Hamiltonian h. The latter quantity, is a measure of the optimal process intensity. While the nondimenional number of transfer units τ measures the residence time of drying gas, the quantity h quantifies a minimal irreversibility in the system.

For a cost or exergy performance function, the enlarged cost, modified by presence of Lagrange multiplier h, and defined as the sum

$$R'^N\equiv\sum_n(e^n+h\theta^n) \quad (32)$$

is minimized, where e^n is intensity of original costs at stage n, h is Lagrange multiplier or time penalty, and θ^n is time interval at the n-th stage.

When total power is the original optimization criterion, a computer generates tables of optimal controls and optimal costs by solving Bellman's recurrence equation for the optimal performance function $R^{*N}(T^n,X^n,h)\equiv\min R'^N$ or

$$R^{*N}(T^n,X^n,h)\equiv\min\sum_n(e^n+h\theta^n) \quad (33)$$

Bellman's recurrence equation has the following form

$$R^{*n}(T^n, X^n, h) \equiv \min(e^n + h\theta^n + R^{*n-1}(T^{n-1}, X^{n-1}, h)) \tag{34}$$

where the outlet state variables from stage n are not arbitrary but are expressed in terms of inlet state variables in the way defined by the state transformations.

$$T^{n-1} = \Upsilon_T'^n(T^n, X^n, \theta^n), \tag{35}$$

$$X^{n-1} = \Upsilon_X'^n(T^n, X^n, \theta^n) \tag{36}$$

which are designated with primes since they are not identical with those of the previous section. The gas state vector $Y^n = (T^n, X^n)$ in these formulas comprises the temperature and concentration, whereas controls u^n and v^n are rates of change for state variables in the considered system.

The presence of the Lagrange multiplier h is associated with absence of time variable τ^n within the set of state coordinates (dimensionality reduction). Some of end coordinates (T^0, X^0) and (T^N, X^N) may be fixed, but total duration, τ^N, must be free, consistent with the dimensionality reduction. For an assumed h an optimal duration follows as a function of fixed end values of state Y^n and total number of stages, N. Accuracy of DP results is much better after the state variable τ^n is eliminated, i. e. when the problem is described by only two state variables, T^n and X^n. The recurrence equation (34) also serves to generate numerical generalizations of function R^{*n} when both transfer coefficients and heat capacities vary along the process path, and an analytical solution cannot be obtained.

6 Power Consumption Limits in General Thermochemical Systems

Heat pumps and HP-supported dryers are typical representatives of systems driven by the consumed power. A general thermokinetic theory of power consumers can be developed based on systems' thermodynamic and kinetic properties, as outlined below.

Consider power consumption in a linear thermochemical system with possible electric fluxes (case of electrolysers). Assume that a two reservoir arrangement is sufficient to accomplish a separation process or heat pump heating. Both these operations need for their running some instantaneous power supplied to the system. In terms of total resistances of upper and lower parts of two-reservoir system

$$R_s = R_{1s} + R_{2s}, \;\; R_n = R_{1n} + R_{2n}, \quad R_{el} = R_{1e} + R_{2e}, \tag{37}$$

and after considering coupled transfer of heat, mass, and electricity, power expression reads

$$\begin{aligned} p = {} & (T_{1'} - T_{2'})I_s + (\mu_{1'} - \mu_{2'})I_n + (\phi_{1'} - \phi_{2'})I_e = \\ & (T_1 - T_2)I_s + (\mu_1 - \mu_2)I_n + (\phi_1 - \phi_2)I_e \\ & + R_{ss}I_s^2 + R_{nn}I_n^2 + R_{ee}I_e^2 + R_{sn}I_sI_n + R_{se}I_sI_e + R_{ne}I_nI_e \end{aligned} \tag{38}$$

where non-primed quantities refer to bulk states and primed ones to the active, power-consuming part of the system. As shown by this equation, general thermodynamic framework allows for at least rough assessment of power consumption limits in thermo-electro-chemical systems of the simplest, standardized topology (with no counterflows). This topology corresponds to power consumption unit (e. g. heat pump) immersed between two reservoirs, one with high potentials and one with low ones, as described in a number of publications. The reason why this power assessment can only be rough is explicit in Eq. (38) or a like which has ignored information about the topological structure of many various flows in the system. Let us also add that possible electrolysers are here described by the formalism of inert components rather than by the ionic description.

For simplicity, Eq. (38) assumes that active (power consuming, primed) driving forces involve only: one temperature difference, trivial chemical affinity and the operating voltage as the difference of the electric potentials ϕ. Total power consumption (38) is the sum of thermal, substantial and electric components.

Equation (38) constitutes the simplest account of thermo-electro-chemical separators and heat pumps; indeed it does not contain any topology parameter. Complex configurations of flows contacting such as countercurrent contacting that may exist in fuel cell electrolyzers are not taken into account in Eq. (38). Linear systems described by this equation are those with constant (current independent or flux independent) resistances or conductances. They satisfy Ohm type or Onsager type laws linking thermodynamic fluxes and thermodynamic forces (dissipative driving forces which are represented by products $R_{ik}I_k$ in Eq. (38)). While many thermal separation systems and fuel cell electrolyzers are nonlinear, i. e. possess current dependent resistances, the dependence is often weak, so a linear model can be a good approximation. Below, by applying Eq. (38) we shall attempt to develop a simple evaluation of power limits for heat pumps and separation systems under the specified assumptions.

After introducing the enlarged flux vector $\mathbf{I} = (I_s, I_n, I_e)$, the enlarged vector of potentials $\mu = (T, \mu, \phi)$ and the related resistance tensor $\mathbf{R}$, Eq. (38) can be written in a simple form

$$p = (\mu_1 - \mu_2)\mathbf{I} + \mathbf{R} : \mathbf{II}. \tag{39}$$

The bulk driving forces $\mu_1 - \mu_2$ are given constants, whence, in systems with constant resistances, we are confronted with a simple minimization problem for a quadratic power consumption function p. While the dimensionality of the potential vector μ will often be quite large in real systems, the structure of Eq. (39) will be preserved whenever the power expression will be considered in the above matrix-vector notation.

Minimum power corresponds with vanishing partial derivatives of function p. The optimal (power-minimizing) vector of currents at the minimum power point of the system can be written in the form

$$I_{mp} = -\frac{1}{2}\mathbf{R}^{-1}(\mu_1 - \mu_2) \equiv -\frac{1}{2}\mathbf{I}_F. \tag{40}$$

This result means that power-minimizing current vector in separators and heat pumps is equal to the *negative of* one half of the purely dissipative current at the Fourier-Onsager point. The latter point refers to the system's state at which no power production occurs.

Consistently, Eqs. (39) and (40) yield the following result for the minimum power input to heat pump or thermal separation system

$$p_{mp} = -\frac{1}{4}(\mu_1 - \mu_2)\mathbf{R}^{-1}(\mu_1 - \mu_2) \tag{41}$$

(negative power supply follows because the engine convention is always used, in which power released is positive).

In terms of the purely dissipative flux vector at the Fourier-Onsager point, where neither power production nor power consumption occurs, the above limit of minimum power is represented by an equation

$$p_{mp} = -\frac{1}{4}\mathbf{R} : \mathbf{I}_F I_F. \tag{42}$$

The minus sign corresponds again with the engine convention which requires that power supplied to the system is negative.

On the other hand, power dissipated at the Fourier-Onsager point is

$$p_F = \mathbf{R} : \mathbf{I}_F I_F. \tag{43}$$

Comparison of Eqs. (42) and (43) proves that, in linear thermo-electro-chemical separators with power support, at least 25% of power dissipated in the natural transfer process must be supplied as power surplus in order to run a power-consuming system. Yet, this general result cannot, probably, be exact in arbitrary systems of complex topology and with nonlinearities, where significant deviations may be expected.

In fact, the present result describes the largest (most disadvantageous) power surplus that can be approximately applied to real heat pumps, dryers, electrolysers, and other separation systems. For these systems significant deviations from Eqs. (40)–(42) are nonetheless expected depending on nature of nonlinearities and topology variations, and also on topology improvements to include countercurrent contacting. Despite of limitation of the result (42) to linear cases, its value is significant because it shows the order of magnitude of thermodynamic limitations in power consumption systems.

The analysis presented here proves that a link exists between the mathematics of heat pumps, separators and electrolysers, and also that, possibly, the theory of electrolysers can be unified with the theory of thermal and chemical separators and heat pumps. All these systems are power consumers. However, serious topological differences between these systems may occasionally render them quite dissimilar.

Explanation of some physical effects is in order. While the power ratios involving Eqs. (42) and (43) can be regarded as efficiency measures, they should not be confused with commonly used, popular performance coefficients, especially first law-based coefficients. There are several definitions of performance coefficients, based on first or second laws, proposed for measuring and comparing performance of separation processes. Only second-law-based performance coefficients are entirely correct measures which show how close the process approaches a reversible process.

7 Final Remarks

The example presented shows how to optimize a drying operation with gas heated by a sequence of heat pumps. The optimization criterion is the power consumption, the optimal solution should assure the minimum of consumed power. The optimal transfer areas are close in value, the optimal temperatures constitute an increasing sequence. The optimal work supplied to the two-stage system decreases distinctly with the total transfer area.

While the present modelling and particular numerical results are limited to multistage operations with heat pumps and continuous co-current dryers with falling particles, after suitable modifications, our approach can be extended to different and more complex configurations.

The practical application of concrete energy sources for drying technology is supported following the theoretical recommendations by introducing the irreversible thermodynamic analysis of the cycles [Mansoori and Patel, 1979], [Berry *at al.*, 2000]. Upper and lower limits for the coefficient of performance of solar absorption cooling cycles have been derived from the first and second laws [Mansoori and Patel, 1979]. These limits depend not only on the environmental temperatures of the cycle components but also on the thermodynamic properties of refrigerants, absorbents and mixtures thereof. Quantitative comparative studies of different refrigerant-absorbent combinations are now possible.

The approach of the finite time thermodynamics (FTT) applied in the context of heat pumps has raised an interesting discussion in which its followers and opponents may find lots in common regarding some issues, but also disagree about the others. For example, a renewed and eloquent criticism of the FTT approach was published [Gyftopoulos, 1999], in addition to a presence of a stream of publications regarding FTT. Many of the latter papers contain in-depth derivations of the power expression and the Curzon–Ahlborn-Novikov efficiency (CAN efficiency), including thorough explanations of connection between the CAN efficiencies and typical efficiencies of real heat engines [Chen et al, 2001] and heat pumps [Li et al, 2010]. In fact, both types of publications quote some experimental data to support their own conclusions and final stand-point [De Vos, 1992], [Berry *et al.*, 2000], [Chen *et al.*, 2001].

The theoretical estimation of minimum work supplied to a heat pump used in drying technology ought to be compared with the experimental energy consumption criteria. Szwast's exergy optimizations in a class of drying systems with granular solids [Szwast, 1990],[Berry *et al.*, 2000] show an agreement of ca 25% between the calculated and experimental consumptions of the propelling exergy. Perhaps the most careful use of the experimental data is given by De Vos who illustrated the practical usefulness of CAN theory for engines on the example of the quantitative description of the nuclear power plant Doel 4 in Belgium and explained the difference between the predicted value of engine's optimal efficiency ($\eta_{CAN} = 0.293$) and the experimental thermal efficiency ($\eta = 0.350$). By simple economical considerations De Vos also explained why the actual efficiency of the engine is larger than its CAN efficiency [De Vos, 1992].

The analysis presented here proves that a common thermodynamic ground exists for thermal engines and thermal separators, and also that, to some extent, the theory of thermal separators can benefit from the theory of thermal engines. Yet the topological differences of both systems may occasionally render both of them quite dissimilar, which, of course, imposes limits for the exploitation of the theory of thermal engines in separation problems.

A digression can be made to engine systems, which are, in principle, beyond the subject of the present investigation, yet they use a power equation similar to Eq. (38), but with a negative sign of dissipative (resistance containing) terms. In the vast literature of engines (power generators) abundant information can be found regarding their efficiency coefficients based on the first law, such as theoretical reversible efficiency or popular fuel-to-electricity efficiency. These efficiencies can sometimes generate numerical values greater than 100%, depending on whether the change in entropy of overall chemical reaction involved is positive or negative. See, for example, a paper [Rao *et al.*, 2004] on various efficiency definitions for FC engines. The fuel cell efficiencies, $\eta = \Delta G/\Delta H$ or $-W/\Delta H$, which are often applied to fuel cell engines, can easily achieve numerical values much higher than 0.25 [power ratio of Eqs. (42) and (43)]. They are first-law efficiencies defined in a different way than the power ratios P_{mp}/P_F derived from Eqs. (42) and (43). For engines, these power ratios represent specific, second-law power yield efficiencies of the simplest standardized thermo-electro-chemical process. They are not equivalent, neither theoretically nor numerically, with the most common fuel-to-electricity efficiencies of engines. Other second law efficiencies can also be defined for engines. One of the most correct and practical definition of efficiency for a fuel cell engine operating near ambient temperature is the ratio of the actual voltage to the reversible voltage, [Li, 2006].

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