"Channel" and "jokers" revealing at reconstruction Of the Chaotic systems.

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In the work it is shown, that during the reconstruction by a method of two sliding windows the heterogeneity of phase space is being displayed. By the size of time-dependent quality coefficient of the reconstructed model we succeeded to allocate the areas of small dimensional behavior ("channels") and areas ("jokers"), where the system behavior becomes unpredictable. It was established, that local lypunov parameter increases on border and as consequence the time of predictability decreases. It is shown also, that under the influence of noise the sizes of the "jokers" areas increase.

1. Introduction. Channels and jokers

Many works [1] are devoted to the opportunity of reconstruction of adequate mathematical model at the observable time series but the interest to this problem does not decrease. Numerous researches [2, 3] however have shown at examples of model systems, that the received results of chaotic systems reconstruction are often considerably lower at the prognostic ability then the limiting time of predicted behavior [1]:

$$\tau = \frac{1}{2\lambda} \ln \frac{\sigma_x^2}{\sigma_\xi^2} \quad (1).$$

Here λ - the greatest lyapunov parameter, σ_ξ^2 - dispersions of noise, inaccuracy of model and so forth, σ_x^2 - the dispersion of observable value (the size of attractor).

There are several reasons for it [1, 3 and 4]: the errors of reconstruction; the errors of numerical differentiation and others. One more reason causing serious difficulties at the construction of nonlinear processes models, by the of authors opinion, is the heterogeneity of their phase space – the presence of the "channels" and "jokers" areas which have been introduced by G.G.Malinetskii[5].

According to [5] at the nonlinear systems the well predicted steady movement of a phase trajectory can become periodically, smoothly or sharply, unstable or likelihood. It complicates the opportunity of the forecast for such systems and the description of their dynamics essentially. At [5] there have been introduced the new class of mathematical models - dynamic systems with jokers for the description of such systems. At them the jokers are the regions at the phase space where the system dynamics becomes predicted badly, it simply changes, becomes complicated or even likelihood and casual. As opposed to the jokers – the channels are introducing as the areas of steady, small modes and well predicted movement.

Let's note two possible reasons of channels occurrence in the phase space: the channels can be the areas of local small mode movements, at which the good forecast is being provided with the help of simple of model function; and the second, the channels can be the areas of local stability of chaotic system.

2. Channel and jokers displays during the reconstruction.

At work [5] for the revealing of the area of unstable movement trajectories (jokers) at the phase space the method named "test for a linear prediction" have been offered. The essence of the method is following: we take area around of the phase space point x_i (x_i must be excluded of the area) and construct according to the data of the area the linear forecast for the time Δt. Further according to the constructed forecast the point x_i is being extrapolated on to the time Δt forward and the mistake of extrapolation $\varepsilon(t)$ is being estimated. Changing the base point x_i and analyzing a kind of dependence $\varepsilon(t)$, we find in the phase space the areas of badly predicted movement - jokers. The results of this test for the Rossler system are shown on fig.1.

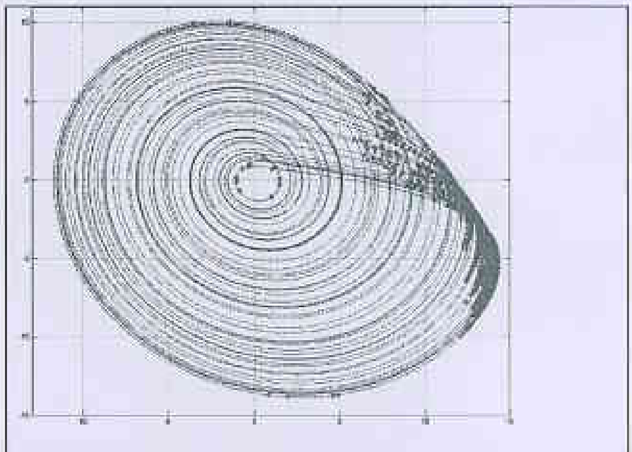

Fig.1. The two-dimensional phase portrait of the Rossler system. Points designate the area of jokers received by means of the test for a linear prediction

Naturally the following question arises: whether is It possible to define the areas of Jokers by the means of global reconstruction methods? In several of works we have presented a method, developed for the reconstruction of non-stationary dynamic systems [1-3] specially; it showed high sensitivity to the changes of the operating parameters.

The brief essence of the method is following [1]. Let we observe the time process $y(t)$ is being observed (generally vector $\vec{y}(t)$), being generated by some nonlinear dynamic system which submits to the equation $\vec{d}[\vec{y}(t),\vec{A}]=0$. Here $\vec{A}$ - is the vector of the system parameters. For the reconstruction of the dynamic model (identification) of the system, being guided by aprioristic reasons, we choose the model $\vec{d}_{\text{mod}}[\vec{y}(t),\vec{a}]=0$ to which the system presumably is being submitted.

$$d_{\text{mod}}(y,t)=\frac{dy_i}{dt}-\sum_{k=1}^{p-1}a_k F_k(\vec{y}(t))=0, \qquad i=1...n \qquad (2)$$

The parameterization of model (2) i.e. identification of the define value of the vector parameters $\vec{a}$ is carried out on the basis of available data $\vec{y}(t)$. The reconstruction of the dynamic system model is carried out in two adjacent sliding time windows T_1 and T_2, and the parameters of the model $\vec{a}$ are being chosen on the basis of the criterion similar to Fisher's criterion. The developed algorithm unites the procedure of parameterizations with the procedure of check on the being renewed the sequence. Two-window procedure of identification, unlike one-window algorithms, has appeared, being more sensitive to non-stationary properties of the observable process model. Alongside with the identification of a kind and parameters of the model the offered algorithm allows to reveal effectively the sites of modulation and non-stationary (within the limits of the chosen model) behavior.

Using of the additive models of a kind (2) where the symbol F_k designates any nonlinear functions, for example, degree functions in the case of polynomial models, and p - the quantity of members of additive model is equivalent to the description of nonlinear dynamic system in the expanded linear phase space with nonlinear coordinates $\Phi(p)$ - members of additive models (2). The expanded linear phase space $\Phi(p)$ is obtained from the space of conditions of dynamic system by its addition with the nonlinear coordinates $y(t)$, $y^2(t)$, y^3, $,dy(t)/dt$,, etc., corresponding to the nonlinear members of the model (2).

In such statement the problem of identification of dynamic systems is similar to a problem of the discriminant analysis. Thus, instead of the expanded space of attributes $X(p)$ the expanded phase space $\Phi(p)$, and instead of training data from the discriminated classes $x_j(1)$ and $x_j(2)$ - the pieces of time process $\vec{y}(T_1)$ and $\vec{y}(T_2)$, observable in adjacent windows T_1 and, T_2 and as discriminant function – the additive model (2) are used. The parameterization of the model can be spent on the basis of various criteria, for example, Fisher's criterion:

$$H_i = \max\left\{\frac{[M_1(d_i) - M_2(d_i)]^2}{\sigma_1^2(d_i) + \sigma_2^2(d_i)}\right\}, \quad i = 1,\ldots,n. \tag{3}$$

Where $M_1(d_i)$ and $M_2(d_i)$ - average values; $\sigma_1^2(d_i)$ and $\sigma^2(d_i)$ - the dispersions of modeling function (2), being calculated at adjacent time windows.

The denominator of the expression (3) is a root-mean-square measure of a deviation of the observable process $\vec{y}(t)$ from the modeling process $\vec{z}(t)$, being calculated on a time interval $T_1 + T_2$. In it's turn, the square of a difference of average values $[M_1(d_i) - M_2(d_i)]^2$, standing in the numerator of the Fisher criterion and liable to maximization serves as a measure of difference of the description of the observable process $\vec{y}(t)$ by the model $\vec{d}_{\text{mod}}[\vec{y}(t),\vec{a}]=0$ on an interval T_1 from its description at an interval T_2, i.e. is being a measure of nonstationarity description of the observable process within the limits of the chosen model.

In ideal and stationary conditions without noise when the chosen dynamic model (2) coincides completely with the initial system $\vec{d}[\vec{y}(t),\vec{A}]=0$, the criterion H aspires to zero, and the values of parameters of the model $\vec{a}$ aspire to values of the parameters of the initial system $\vec{A}$. If the chosen model contains the superfluous items then the parameters also aspire to zero, that allows excluding them from the model.

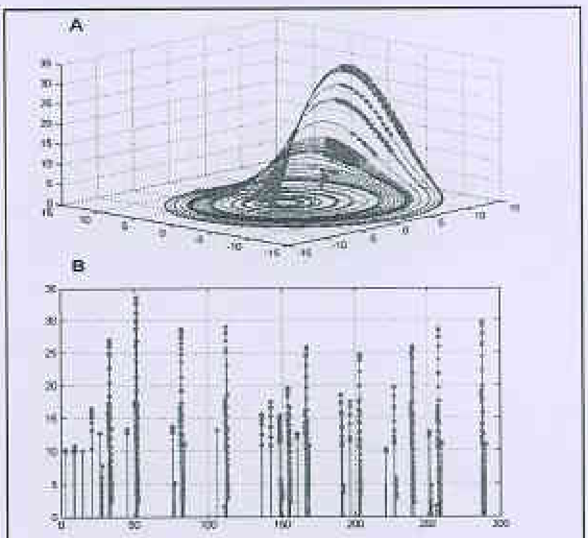

Fig.2. A - a phase portrait of Rossler system (1); B – The Fisher's criterion constructed at reconstruction of exact model of the first system equation (1). The points at the figuries correspond to the values $H(t)>1$.

In that case when the windows slide on an axis of time, the criterion H and parameters $\vec{a}$ become dependent on the time: $H=H(t)$ and $\vec{a}=\vec{a}(t)$. Analyzing these dependences, it is possible to judge bout the quality of the description of the observable system model with constant parameters and by that to identify the areas of the change of the reconstruction quality.

During the testing this method at various nonlinear models, the inexplicable jumps of the Fisher criterion (fig.2) were found out. It was possible to explain these jumps after the analysis of the results [5].

Really, from fig.2 it is visible, that on a level $H(t)>1$ in system (1) two areas of jokers were found out, one of which corresponds to the area of unstable movement (see fig.1), and in another dimension of the phase space from three up to two (the further evolution of system occurs at $z\approx 0$) locally varies. Similar results subsequently have been received and for the other well-known chaotic systems - Lorentz, Henon, etc. From the definition channel and jokers communication with local Lyapunov parameter and movement in phase space is obvious enough. This communication is shown on fig.3, where z-component of the Rossler system (a thick line) and the local maximal lyapunov parameter being calculated by the means of the decision of the equations in the variations for indignations vectors and subsequent of the Gramma-Shmidta orthogonalization.

As follows from the figure, in the field of $z >> 0$ the local lyapunov parameter increases, specifying unstable character of the system movement of in this area. Remembering that with the increase in dimension of system is connected with increase of the lyapunov parameter, it would be right to assume the presence of a joker in this area.

Besides numerical experiments have shown, that the sizes of areas of jokers increase with the increase of noise level.

3. Predictability in channels and jokers areas

The central problem of the given work was the research of predictability and as the consequence - the range of the forecast. For this purpose the concept of a degree of determinacy $D(\tau)$ [6] was used

$$D(\tau) = \frac{< XZ >}{\sqrt{< X^2 >< Z^2 >}} \quad (4).$$

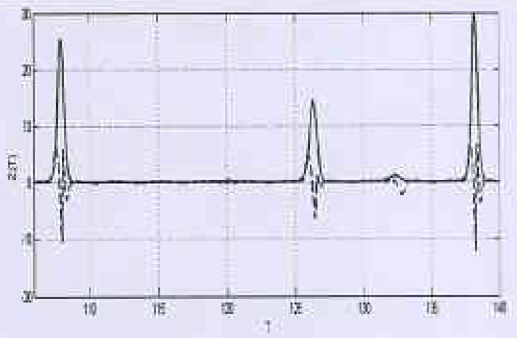

Fig.3. Dependences *Z (t)* - components of the Rossler system (a continuous line) and maximal lyapunov parameter (a shaped line) are shown.

In the given section the results of comparison of the quality of Rossler system reconstruction being lead at the following areas of space phase are presented: to channels, jokers, and full phase space.

On to the observable noisy process $\vec{X}(t) = (x(t), y(t), z(t))$, being born by the system, there was reconstructed the model of the system in the form of (1), all the factors of which were found by the means of reconstruction of the equations of dynamics on the whole phase space. Then the dependence of the Fisher criterion on time $H(t)$ was being construction and after it there the areas channels and jokers were defined, as areas, in which $H(t) < 1$ and $H(t) \geq 1$ accordingly. As a result we received three sets of points of phase the space: $\vec{X}_r$ - the points laying in channels; $\vec{X}_j$ - the points laying in jokers; and $\vec{X}$ - all set of the points of the phase space.

On each set of points the model system (1) was reconstructed by the means of least squares method (see for the example [3]) and in addition to the observable process $\vec{X}$ we received model processes $\vec{Z}_r$, $\vec{Z}_j$ and $\vec{Z}$, generated at the same initial conditions, as $\vec{X}(t)$.

Further there were estimated the quality of the forecast of modeling processes by means of the correlator (6). The values $D(\tau)$ close to unit, answer the satisfactory forecast whereas the small values $D(\tau)$ correspond to the not coordinated course of supervision and the forecast. As well as during [1] the time of predicted behavior is being defined from a condition: $D(\tau_{pred}) = 1/2$.

On fig.4. there are presented: 1) the dependence of a degree of the predictability $D(\tau)$ being calculated on the whole realization and in the field of channel; 2) the correlation function of the process; 3) the dependence of the degree of predictability $D(\tau)$, being calculated in the field of jokers. From fig.4. it is visible, that the time of predictability of the model system being reconstructed on the data from the area of jokers is essentially lower, than at the model systems, being reconstructed on the data of channel of the whole phase space. Thus, the thesis about the complexity and instability of phase trajectories in jokers proves to be true that is shown in the deterioration of the reconstruction on the data from these areas. At the same time the reconstruction on the channels has not increased essentially the duration of the forecast in comparison with reconstruction on all phase space. This fact can be explained so, that area of jokers borrows only 10 % of phase space. It is also necessary to note that the time of predictability of the model constructed on the data from the area of a joker is less, than the time of correlation which can be identified with a linear prediction.

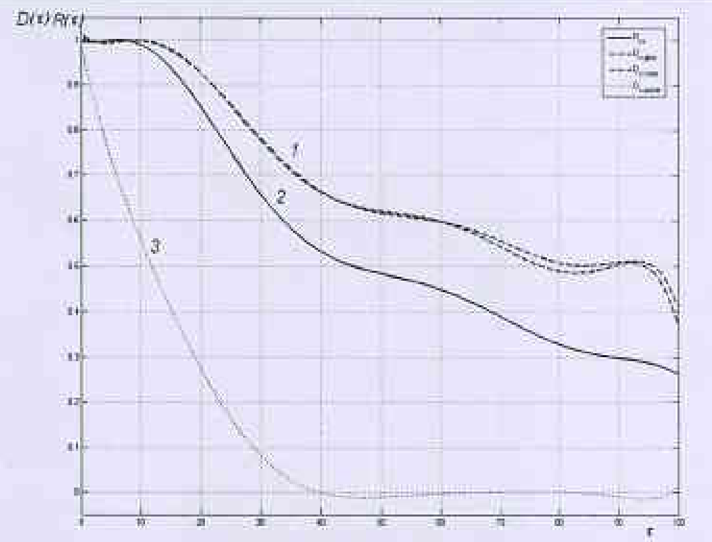

Рис.4. A degree of the predictability $D(\tau)$, the reconstructed model in the field of "русел" (1) and on all phase space; factor of correlation $R(\tau)$ of process (2); a degree of the predictability $D(\tau)$, the reconstructed model in the field of "jokers" (3).

4. The Conclusion

This clause is development and continuation of work [5] about the description of dynamic systems in terms of channels and jokers. The applicability of such approach for continuous systems also is shown in the work. It seems to us, that it can appear useful at modeling complex dynamic systems.

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