ENERGY SPEED-GRADIENT CONTROL OF NONLINEAR SATELLITE OSCILLATIONS

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Abstract
Two problems of nonlinear oscillations control for satellite systems are considered. Firstly, a new solution to the problem of angular velocity stabilization for a spinning satellite is suggested. A satellite is assumed to be supplied with a passive inertial energy dissipater in the form of a spring-mass-dashpot and small resistojets. The motion of a satellite is subjected to a combination of a time varying excitation torque and a control torque. The energy-based speed-gradient (SG) control law is proposed. Numerical simulation results for *Intelsat–II* model are presented showing efficiency of the SG control strategy for suppression of possible chaotic motion. Secondly, the speed-gradient control method is applied to the excitation of oscillations with given amplitude for towed probe satellite. The modified speed-gradient control law for Hamiltonian systems is used to obtain the control algorithm. Robustness of the system with respect to the changes of satellite model and excitation torque amplitude is established by computer simulations.

Key words
Oscillations control, chaos, satellite, speed-gradient

1 Introduction

Most problems solved in nonlinear control so far are aimed at either regulation or tracking. In both cases the control objective can be described by specifying the desired plant trajectory $x(t)$ (e.g. by means of the reference model) with the aim of making the real behavior of the plant $x(t)$ close to the desired one:

$$\|x(t) - x_*(t)\| \to 0 \ \text{ as } \ t \to \infty. \qquad (1)$$

The above setting is typical for many problems of oscillations suppression. The recent interest in the field of the periodic and chaotic motions investigations demand for new settings, applicable to the problems not reducible immediately to standard regulation and tracking problems [Chen *et al.*, 2007; Ge *et al.*, 2007; Feng and Chen, 2006; Yau *et al.*, 2006; Bowong, 2005]. A row of papers are devoted to the satellite oscillation supression, see e.g. [Meehan and Asokanthan, 2002a; Meehan and Asokanthan, 2002b; Fradkov *et al.*, 2004; Meehan and Asokanthan, 2006; Bobtsov *et al.*, 2007a; Bobtsov *et al.*, 2007b; Fiorillo *et al.*, 2010]. The authors of [Pirozhenko and Khramov, 2010] consider the satellite gravitational stabilization system, including a tether connection to increase restoring moment and an additional constructive element increasing the efficiency of oscillations damping. Parametric optimization of a gravitational satellite-stabilizer system is considerred in [Mirer and Prilepskiy, 2010]. A nonlinear optimal and adaptive control design to control the attitude of a satellite using tether offset variations is presented in [Godard *et al.*, 2008; Godard *et al.*, 2010].

The well known problem of swinging up the pendulum can be recalled as example [Mori *et al.*, 1976; Wiklund *et al.*, 1993; Astrom and Furuta, 2000; Akulenko, 1991]. Solutions of such problems are often based on the energy considerations or specific tricks.

In this paper two control problems of nonlinear oscillators are considered based on a general approach: the so called speed-gradient (SG) method [Fradkov, 1990; Fradkov and Pogromsky, 1998]. In Section 2 the *speed-gradient* method based on energy goal function is described, following [Fradkov and Andrievsky, 2003; Andrievskii *et al.*, 1996]. Section 3 is devoted to a new solution to the problem of angular velocity stabilization for a spinning satellite. The energy-based (SG)-control law is proposed and numerical examination results for the closed-loop system are provided. In Section 4 the SG-control method is applied to the excitation of oscillations with given amplitude for towed probe satellite. Such a problem may arise when monitoring or dispersing some substance in a corridor on the earth or in the air. Robustness of the system with respect to the changes of satellite model and excitation torque amplitude is established by computer simulations.

2 Energy Speed-Gradient Method

Consider the controlled plant equation in the state space form:

$$\dot{x} = F(x, u, t), \quad t \geq 0, \tag{2}$$

where $x \in \mathbb{R}^n$ is a plant state vector, $u \in \mathbb{R}^m$ is an input vector, $F(\cdot) : \mathbb{R}^{n+m+1} \to \mathbb{R}^n$ is a vector-function, continuously differentiable in x, u.

Consider the problem of finding the control law $u(t) = U\{x(s), u(s) : 0 \leq s \leq t\}$, ensuring the control goal

$$Q_t \to 0 \ \text{ as } t \to \infty \tag{3}$$

where Q_t is some objective functional,

$$Q_t = Q\big(x(s), u(s) : 0 \leq s \leq t\big).$$

To design the speed-gradient algorithm for a typical case $Q_t = Q(x(t), t)$, where $Q(x, t) \geq 0$ is a scalar smooth objective function, determine a function $\omega(x, u, t)$ as the speed of change Q_t along the trajectories of the system (2): $\omega(x, u, t) = \big(\nabla_x Q\big)^{\mathrm{T}} F(x, u, t) + \partial Q/\partial t$.

SG algorithm changes the control action along the gradient of $\omega(x, u, t)$ in u. The combined form of the SG algorithm looks as follows [Fradkov, 1990]:

$$\frac{d}{dt}\big(u + \psi(x, u, t)\big) = -\Gamma \nabla_u \omega(x, u, t), \tag{4}$$

where $\psi(\cdot)$ satisfies the pseudogradient condition $\psi^{\mathrm{T}} \nabla_u \omega \geq 0$, $\Gamma = \Gamma^{\mathrm{T}} > 0$ is $m \times m$ gain matrix. The main special cases of (4) are speed gradient algorithm in differential form and speed gradient algorithm in finite form

$$u = \psi(x, u, t) \tag{5}$$

having, in turn, the linear and relay versions:

$$u = -\Gamma \nabla_u \omega(x, u, t), \ \ \Gamma_0 > 0, \tag{6}$$

$$u = -\Gamma_1 \operatorname{sign}\big(\nabla_u \omega(x, u, t)\big), \tag{7}$$
$$\Gamma_1 = \operatorname{diag}\{\gamma_i\}, \ \ \gamma_i > 0,$$

where components of vector $\operatorname{sign}(z)$ are the signs of the corresponding components of vector z. The main idea of algorithms (4)–(7) is decreasing $\dot{Q}$ along the trajectories of the closed loop system. Then for sufficiently large t under some additional conditions the relation $\dot{Q} < 0$ holds and $Q(t)$ begins to decrease.

A broad class of technical systems with negligible dissipation, weak environment resistance etc. may be described by conservative models. For example, conservative models with one degree of freedom are: an ideal rotor, a physical pendulum, a conical pendulum, and an oscillator with nonlinear recovering force. These models may be described in canonical Hamiltonian form as follows:

$$\dot{p} = \left(\frac{\partial H}{\partial q}\right)^{\mathrm{T}} + Bu, \ \ \dot{q} = \left(\frac{\partial H}{\partial p}\right)^{\mathrm{T}}, \tag{8}$$

where $p, q \in \mathbb{R}^n$ are generalized coordinates and momenta; $H = H(p, q)$ is Hamiltonian function (total energy of the system); $u = u(t)$ is input (generalized force); $B(p, q)$ is $m \times n$ matrix-function, $B \in \mathbb{R}^{m \times n}$, $m \leq n$.

Formalize the control goal as approaching the given total system energy level:

$$H\big(p(t), q(t)\big) \to H_* \ \text{ as } t \to \infty. \tag{9}$$

This goal can be rewritten in the form (3) with $x = (p^T, q^T)^T$ and the goal function

$$Q(p, q) = \frac{1}{2}\big(H(p, q) - H_*\big)^2. \tag{10}$$

To design the SG-algorithm calculate $\dot{Q}$ – the derivative of (10) along the trajectories of (8):

$$\dot{Q} = (H - H_*)\left(\frac{\partial H}{\partial p}\right)^{\mathrm{T}} Bu, \tag{11}$$

and then calculate the partial derivative in u. The finite forms (6), (7) look as follows:

$$u = -\gamma (H - H_*) B^{\mathrm{T}} \left(\frac{\partial H}{\partial p}\right), \tag{12}$$

$$u = -\gamma \operatorname{sign}\left((H - H_*) B^{\mathrm{T}} \left(\frac{\partial H}{\partial p}\right)\right). \tag{13}$$

3 Stabilization of the Spinning Spacecraft

3.1 Model of the System Dynamics

For the sake of simplicity, the 1-DOF model of the satellite angular motion is used below. The degrees of freedom of the system describe the damper mass displacement and rotation of the satellite. The damper is centered on the body fixed X-axis and has a point mass m. That mass moves along an axis perpendicular to X-axis at the some distance of the principal axis Z.

Under these assumptions the system satellite-damper model can be written as follows [Meehan and Asokanthan, 2002a; Meehan and Asokanthan, 2002b; Meehan and Asokanthan, 2006]:

$$\begin{cases} (I + m(1-\mu)y^2)\dot{\omega} + 2m(1-\mu)y\dot{y}\omega \\ \qquad -mb\ddot{y} = M(t), \\ m(1-\mu)\ddot{y} + c\dot{y} + \big(k - (1-\mu)\omega^2\big)y \\ \qquad -b\dot{\omega} = 0, \end{cases} \tag{14}$$

where ω, y denote satellite angular velocity and damper mass displacement; I, m, k, c stand for the satellite moment of inertia about Z-axis, damper mass, spring constant and viscous resistance gain; $\mu = m/m_T$, where m_T denotes a total mass of the considered system. The external torque $M(t)$ is a sum of the excitation torque and the control torque, i.e. $M(t) = M_E(t) + M_C(t)$. It is assumed that $|M_C(t)| \le \bar{M}$, where $\bar{M}$ represents restriction on the control torque.

The system (14) examinations show that if $M(t) \equiv 0$ and initial conditions belong to some region, the system is dissipative and is attracted to the equilibrium state of constant angular velocity ω^* and no damper mass deflection [Meehan and Asokanthan, 2002a; Meehan and Asokanthan, 2002b; Meehan and Asokanthan, 2006]. If these conditions are violated, the amplitude of $y(t)$ becomes inadmissible large and the system can perform chaotic jumps between two stable equilibrium points. To improve the system performance let us use, in addition, the active damping by means of the resistojets torque M_C .

3.2 Control Law Design

The control aim is to stabilize the desired state $[y, \dot{y}, \omega]^{\mathrm{T}} = [0, 0, \omega_{\text{ref}}]^{\mathrm{T}}$. This aim corresponds to the desired constant rotation rate $\omega(t) \equiv \omega_{\text{ref}}$ and zero displacement of the damper mass $y(t) \equiv 0$. Following [Andrievskii *et al.*, 1996; Fradkov and Andrievsky, 2003] let us use an energy-based approach and apply the speed-gradient (SG) method to control law design.

The total energy H of the system (14) may be derived as

$$H(y, \dot{y}, \omega) = 0.5\big((m(1-\mu) + k)y^2 + I\big)\omega^2 \\ - mb\dot{y}\omega - 0.5m(1-\mu)\dot{y}^2. \tag{15}$$

Substitution of $y = \dot{y} = 0$, $\omega = \omega_{\text{ref}}$ to (15) gives the desired energy H_{ref} as $H_{\text{ref}} = 0.5I\omega_{\text{ref}}^2$. Let us introduce the *goal function* $Q = (H - H_{\text{ref}})^2$ and derive the SG control laws in the finite form. It gives the "proportional" and relay algorithms as follows:

$$M_C = \gamma\big(H_{\text{ref}} - H(y, \dot{y}, \omega)\big) \\ \times \big(\omega + \dot{\tilde{y}}(\tilde{I} + \tilde{y}^2 - 1)^{-1}\big), \tag{16}$$

$$M_C = \gamma \operatorname{sign}\big(H_{\text{ref}} - H(y, \dot{y}, \omega)\big) \\ \times \operatorname{sign}\big(\omega + \dot{\tilde{y}}(\tilde{I} + \tilde{y}^2 - 1)^{-1}\big), \tag{17}$$

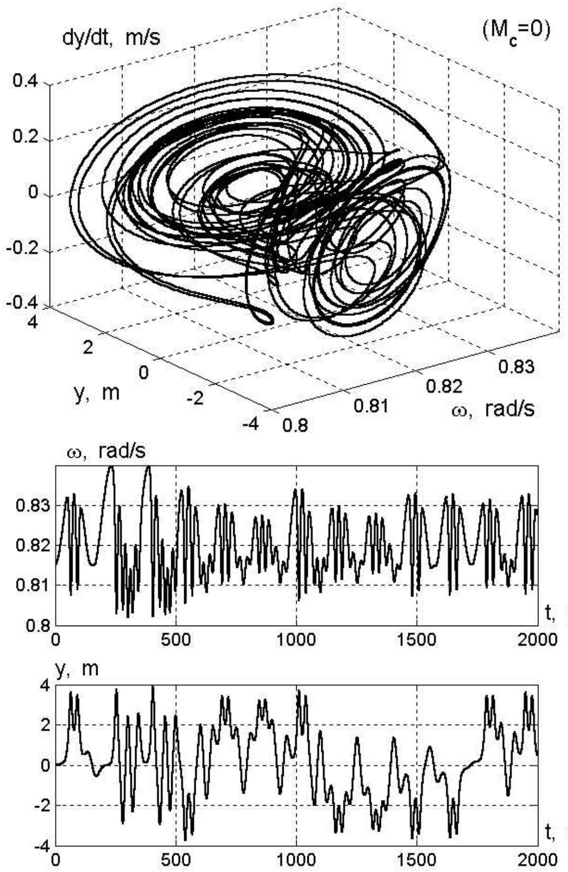

Figure 1. Chaotic oscillations for the case of uncontrollable motion.

where $\tilde{y} = (1-\mu)b^{-1}y$, $\tilde{I} = (1-\mu)m^{-1}b^{-2}I$ are introduced. The control law (17) can be directly implemented by means of the on-off operating resistojets. In such a case the gain γ gives the control torque amplitude: $\bar{M} = \gamma$. The pulse-width modulation can be used for implementation of the "proportional" control law (16) by means of the on-off device.

3.3 Simulation Results

For numerical examination the parameters of the spinning spacecraft with circumferential nutational damper were chosen to be similar to that of *Intelsat-II* being $m = 0.3$ kg, $b = 1$ m, $k = 0.2$ N/m, $\mu = 0.01$,$I = 100$ kgm^2, $c = 0.002$ Ns/m [Meehan and Asokanthan, 2002a; Meehan and Asokanthan, 2002b; Meehan and Asokanthan, 2006]. The harmonic disturbance torque M_E is taken: $M_E(t) = \bar{M}_E \sin\Omega t$. The excitation frequency $\Omega = 0.04$ s^{-1} and the amplitude $\bar{M}_E = 0.05$ Nm. Following initial conditions are picked up for the simulations: $\omega(0) = 0.815$ s^{-1}, $y(0) = 0$, $\dot{y}(0) = 0$. Two cases of the control torque amplitude $\bar{M}$ are studied: *a*) $\bar{M} = 0.0225$ Nm, $\bar{M} < \bar{M}_E$, and *b*) $\bar{M} = 0.055$ Nm, $\bar{M} = 1.1\bar{M}_E > \bar{M}_E$.

Some simulation results are shown in Figs. 1–3. The simulation results for the case of active damping absence $M_C \equiv 0$ are plotted in Fig. 1. One sees that the chaotic motion with a large magnitude of $y(t)$ appears. (Note that in practice $y(t)$ is restricted due to travel limits, but it is seen that the damper can not be effective in that case.) The effect of the feedback control via relay SG-law (17) is demonstrated in Fig. 2. It is taken $H_{\text{ref}} = 33$ kgm^{2}s^{-2}, which corresponds to $\omega_{\text{ref}} = 8.124$ rad/s. It is seen that even in the case when the amplitude of control torque is less that one of the disturbance, $\bar{M} = 0.5\bar{M}_E$ (see Fig. 2*a*), the system behavior is improved in a great extent in comparison with the uncontrollable case. Perfect suppression of oscillations is obtained for the case $\bar{M} = 1.1\bar{M}_E$ (Fig. 2*b*). Note that in [Meehan and Asokanthan, 2002a] the ratio $\bar{M}/\bar{M}_E$ is about 15. Therefore the proposed method is characterized as a low-level control.

Speeding-up the satellite rotation from $\omega(0) = 0.6$ rad/s to given velocity ω_{ref} is demonstrated in Fig. 3 for the case $\bar{M} = 0.5\bar{M}_E$. In the case $\bar{M} = 1.1\bar{M}_E$ the finite-time convergence of $\omega(t)$ to ω_{ref} takes place. The transient time is about 360 s. The sliding motion with exact holding the desired state arises after the transient is finished. (The similar processes are pictured in Fig. 2 *a*.) The control algorithm with a dead-zone or a pulse-width modulation control can be used to reduce propellant consumption and working fluid discharge.

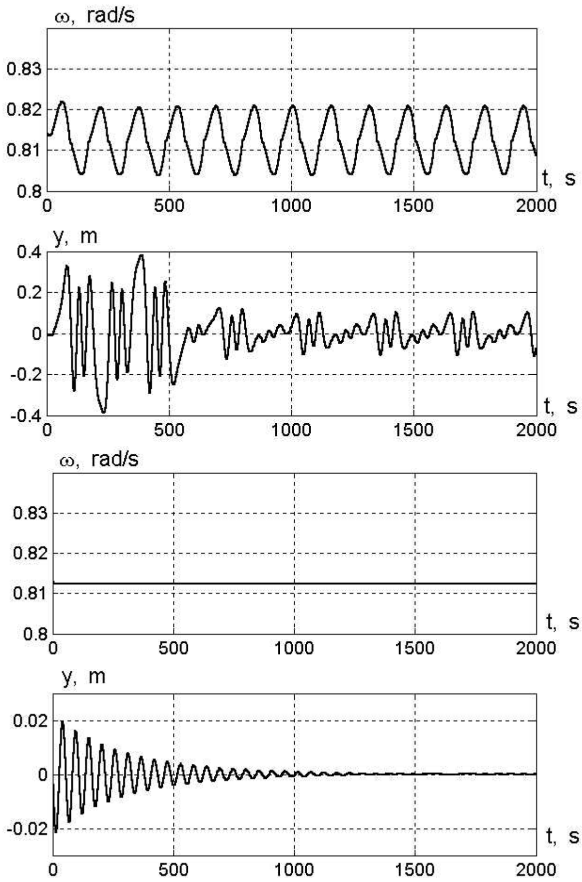

Figure 2. Active damping via control algorithm (17).

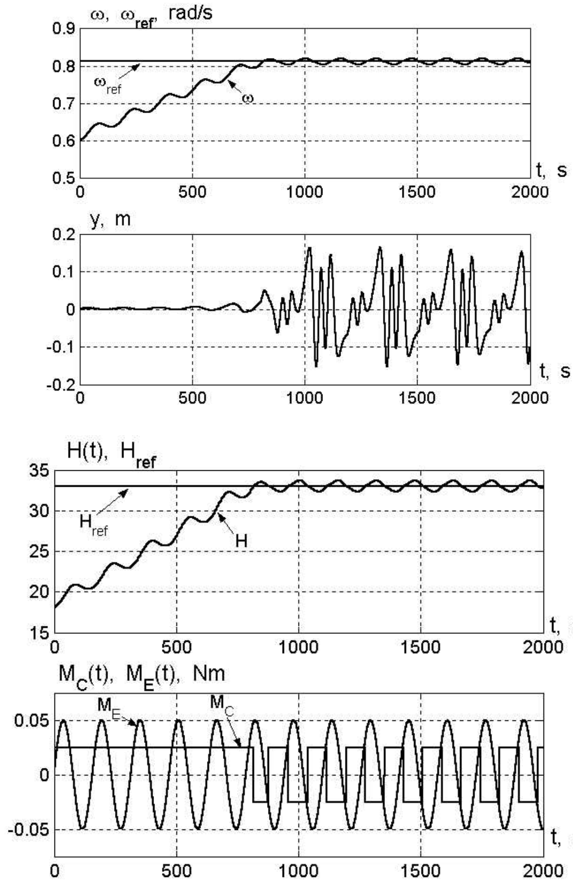

Figure 3. Satellite speeding-up via the control law (17), $\bar{M} = 0.5\bar{M}_E$.

The simulations demonstrate efficiency of the SG control strategy in eliminating chaotic instabilities in a spinning spacecraft and robustness properties with respect to excitation torque amplitude.

4 Speed-Gradient Control of Towed Probe Satellite Oscillations

Let us consider the problem of the probe satellite excitation. In the last two decades, space missions such as SEDS-1, SEDS-2, TSS-1, and TSS-1R have highlighted use of tethers for transport and deployment of satellites. Tether System Experiment (TSE) has been identified by the European Space Agency (ESA) to be an important initiative for the next decade. Recently, ESA sponsored a Phase 1 study of a mission, called ROGER. This mission was aimed at nonfunctional satellites currently in orbit – for inspection, capture, and transport to graveyard orbits [Mankala and Agrawal, 2004; Mankala and Agrawal, 2005].

The small mass satellite is midair towed by orbital spacecraft or station with long non-stretched cable in varying atmosphere density. Oscillations arise under the influence of aerodynamic dray and cable tension. Their period is defined by atmosphere density at the altitude of the spacecraft. The difference of amplitudes and the difference of half-cycles at the up and down deviations from the relative equilibrium are connected with atmosphere density gradient [Shahov, 1988; Schutte and Dooley, 2005; Kim and Hall, 2007].

According to the project [Bevilacqua and Chiarelli, 1986] the typical cable length is $L = 1 \div 100$ km, the mass of spacecraft is about 500kg, the mass of probe satellite is about 1 kg. To change the probing layer the methods of oscillations control may be used. The control of towed probe satellite can be realized by retracting and releasing of the cable. Measurable values are telemetric data of the outer vision about relative location of the probe satellite and the orbital station. This data contain the information of the probe satellite deviation angle. Similar telemetric system appears in other problems connected with tethered systems, particularly in oscillations dumping problem [Beletsky and Levin, 1993; Tang *et al.*, 2011].

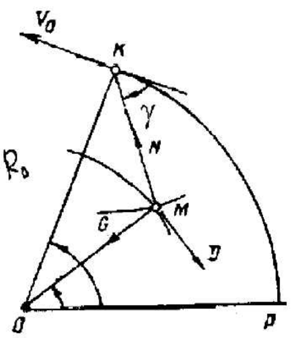

Figure 4. Model of the tethered system.

Following [Shahov, 1988] consider the relative motion of the probe satellite, connected to the spacecraft by flexible non-stretched inertialess cable. Let the fixed length of the cable be L, the mass of the satellite be m, and the mass of the spacecraft be much more than the mass of the satellite. Let the spacecraft and the satellite be described as mass points. Such a model of the tethered system is discussed and substantiated in [Beletsky and Levin, 1993]. The spacecraft moves along the circular orbit of the radius R_0 and the center in the point O (see Fig. 4). The peripheral velocity V_0 is constant. Assume that the wind is absent and the resistance force is directed to the tangent to the circle of radius R with center in O. The gravity force turns to the center and its value is $G = mg_0(R_0/R)^2$, where g_0 is the gravitational acceleration at the orbit R_0. Let us denote the force of aerodynamic resistance by D; the cable tension by N. The variable γ is shown in Fig. 4.

If oscillation frequency ω conditioned by aerodynamic resistance is much greater than the frequency of rotation ω_0, then the equation of satellite oscillations coincides with the equation of pendulum oscillations in the inertial frame [Shahov, 1988]

$$\ddot{\gamma} + Dm^{-1}L^{-1}\sin\gamma = 0 \tag{18}$$

under condition $\omega^2 = D_0 m^{-1}L^{-1} \gg g_0 R_0^{-1} = \omega_0^2$, which bounds the length of cable L: $L \ll a g_0^{-1} R_0$, $a = D_0 m^{-1}$.

More accurate model of nonlinear oscillations is as follows [Shahov, 1988]:

$$\ddot{\gamma} + (\omega^2 e^{\delta \sin\gamma} - 3\omega_0\cos\gamma)\sin\gamma = 0 \tag{19}$$

It admits the energy integral. Under initial conditions $t = 0$, $\dot{\gamma} = 0$ and $\gamma = \gamma_t$:

$$\begin{aligned} &\dot{\gamma}^2 + \int_{\gamma+}^{\gamma} F\, d\gamma = 0, \\ &F = (\omega^2 e^{\delta\sin\gamma} - 3\omega_0\cos\gamma)\sin\gamma. \end{aligned} \tag{20}$$

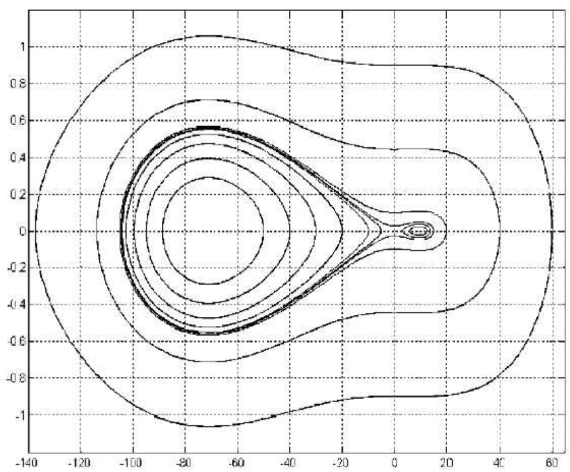

Figure 5. Phase plots of tethered system model (19).

Phase plot family given for different initial conditions (Fig. 5) confirms the conservativity of the system (19).

Consider the problem of swinging the probe satellite until the amplitude of its angle achieves the given value γ. To solve the problem we apply the energy SG method, see Sec. 2.

4.1 Control Algorithm for Control of Probe Satellite Oscillations

Let the variation of cable length after unfolding of tethered system be control variable. Applying the method described in Sec. 2 to the reduced model (18), we obtain the following algorithm:

$$\begin{aligned} &u'_k = -\alpha(\gamma_{\max} - \gamma_*)\dot{\gamma}\sin\gamma, \\ &u_k = \begin{cases} u'_k, & \text{if } |u'_k| \le \bar{u}, \\ \bar{u}\,\mathrm{sign}\, u', & \text{otherwise,} \end{cases} \end{aligned} \tag{21}$$

where $\bar{u}$ is a maximum variation of cable length, $\gamma > 0$ is the gain coefficient. Good swinging abilities of the proposed algorithm are confirmed by its efficiency for a model probe satellite (19) established by computer simulations. The results are shown in Figs. 6, 7 for $m = 2$kg, $l = 1000$m, $\alpha = 10^4$, $D_0 = 1$, $\bar{u} = 0.15l$, $\omega_0 = 0.1$, $\delta = 1$, $\gamma(t = 0) = 10^o$, $\gamma_* = 40^o$. It is seen that the control goal is achieved. It is worth noting that after the end of the control process the cable length remains unchanged.

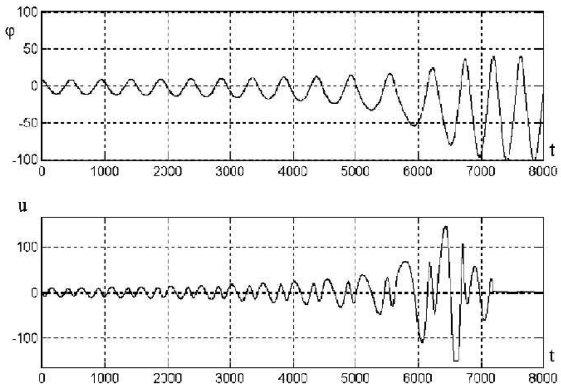

Figure 6. γ and u vs t for closed-loop system (19),(21).

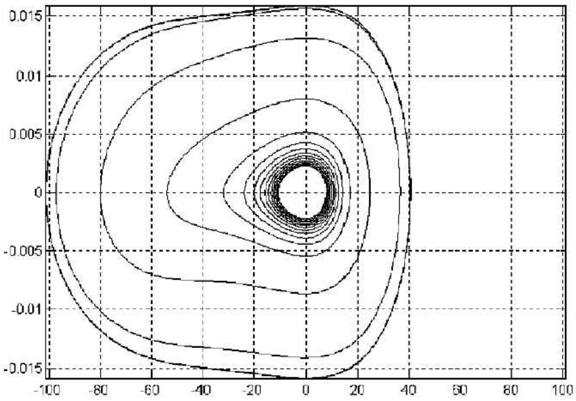

Figure 7. Phase plot family for closed-loop system (19), (21).

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