THE HOPF BIFURCATIONS IN THE WAVE MODELS OF TORSIONAL VIBRATIONS OF SUPERDEEP DRILL COLUMNS

Valery Gulyayev
Department of Mathematics
National Transport University
Ukraine
valery@gulyayev.com.ua

Sergey Hudoliy
Department of Mathematics
National Transport University
Ukraine
S_hoodoliy@gala.net

Olga Glushakova
Department of Mathematics
National Transport University
Ukraine
olvlgl@mail.ru

Abstract

With the aim to simulate the bifurcations of cycle birth in superdeep drill columns the nonlinear wave model of torsion pendulum is elaborated. The constitutive differential equation with delay argument is constructed. The analysis results testify that the self-excited torsional oscillations of the drill column proceed in the manner of quantized time, when the rotation speed $\dot{\varphi}$ continues to be constant during time quantum $\Delta\tau$ equal the duration of the torsion wave forth and back propagation through the DC length.

The states of the limit cycle birth and death in the superdeep DC are found, the modes of DC motion are constructed for the critical states. It is established that the auto-oscillations are predominant at low values of the DC rotation velocities.

Key words

Bifurcations, Discontinuous systems, Discrete-time systems.

Computer-aided methods, Hybrid systems, Modeling, Nonlinear systems.

1 Introduction

In the XX century the time of easy oil and gas finished and inasmuch as the readily accessible deposits of hydrocarbon fuels are practically depleted in the result of their intensive extraction during the two last centenaries, their drawing out from depths of 10km holds much promise. Taking into consideration that mechanical phenomena attending these processes are very complicated and there is no producing experience of such wells drivage, it may be concluded that the problems of their theoretical simulation are urgent.

At the present time, in accordance with requirements of economy, demands of oil-gas industry and its technological possibilities, the vertical, inclined and horizontal bore wells are drilled. Great attention is paid to the questions of drilling deep wells from ground surface and sea bottom. In the drilling technology the leading position belongs to the rotor method based on the use of a drill column with a bit.

When the fuel extraction is realized from great depths, the drill efficiency is associated with the problems of revealing the emergency regimes of the DC functioning.

One of the dynamic phenomena conducing the appearance of emergency situation during drilling is self-excitation of torsion vibrations of rotating drill columns [Brett, 1992; Tucker and Wang, 1999]. Inasmuch as a drill column (DC) represents a torsion pendulum (Fig.1) with energy outflow due to dissipative interaction between the bit and broken rock at its lower part, it can transit from a stationary state to the mode of torsional auto-oscillation at violation of the energy outflow conditions.

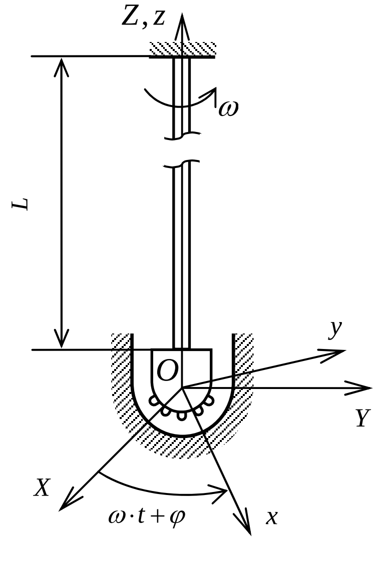

Figure 1. Drill column scheme.

In a general case the auto-oscillations constitute non-damping periodic motions of a non-linear dissipative system which are sustained by external non-vibrational source of energy [Jansen and Van den Steen, 1995; Tucker and Wang, 1999].

For their generating, the non-linear force interaction between separate parts of the system is of importance which regulates income and expenditure of the energy and by this means gives rise to finite amplitude vibration. In drill assemblies the reason of the torsion vibration self-excitation is bifurcational disturbance of the balance between elastic force moments in the DC and the non-linear moment of the bit interaction with the well wall.

In the theory of non-linear differential equations the periodical solutions are named cycles and the change of stationary solution by periodical one at transition of some distinctive parameter through a critical value is spoken of as a cycle generation or the Hopf (Poincare – Andronov – Hopf) bifurcation[Hassard, Kazarinoff and Wan, 1981]. In the problems of drill column dynamics the parameter exerting influence on their stationary and auto-oscillatory regimes is the angular velocity ω of their rotation.

In the cases when an additional impact is not necessary for a mechanical system transition from an initial (stationary) state into regime of auto-vibration the transition is designated as soft self-excitation. If the vibration begins to increase only after some initial threshold amplitude the self-excitation is termed rigid.

The amplitude and frequency of the self-oscillation are determined only by the system parameters. This is its distinction from natural vibration, whose frequency is determined by the system properties but the phase and amplitude are dictated by the initial conditions, as well as from forced vibration, whose amplitude, frequency and phase are governed by the external force.

In the phase space the periodic auto-vibration corresponds to a closed trajectory attracting all the neighboring trajectories. So such a curve is generally referred as a stable limit cycle (or attractor).

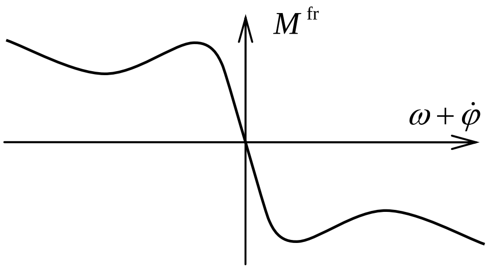

Figure 2. Friction moment function.

Auto-vibrational systems with several degrees of freedom and systems with distributed parameters are characterized by such phenomena as synchronization and competition of vibrations. In many cases this phenomena are responsible for initiation of well organized, complicated modes of periodic motions in dissipative unstable systems.

As applied to the phenomena accompanying drill column rotation, investigation of their auto-oscillation generating permits one to the provide the answers to three important questions: 1) what values of the system parameters and manners of functioning are responsible for the torsion auto-oscillation generation; 2) what type of the oscillation self-excitation (soft or rigid) does occur; 3) what precautions should be taken to prevent the possible mode of the torsion auto-oscillation.

For the drill columns in comparatively shallow wells, the answers to these questions can be received with the help of simplified mathematical model constructed issuing from the consideration of an appropriate torsion pendulum with non-linear friction forces applied to its fly-wheel. In doing so the fly-wheel and the DC elements can be considered to perform torsional oscillations with the same phase and in consequence the overall elastic system can be changed by one oscillator with one DOF.

However if the DC is long, application of the torsion pendulum model for analysis of its dynamics is not justified, as vibrations of its elements cease to be synchronized. So their simulation should be performed on the basis of the wave theory.

Under real conditions this simplification is not met as the time of the torsion wave propagation through the DC length is not multiple to the period of the lower fly-wheel vibration and for this reason its motion can attain a complicated mode. This effect can be essentially favored by the bit stick-slip dynamics inherent in the systems with dry friction. It consists in short-term stoppings of the bit rotation in the time intervals, when the sum of all the moments of active and inertia forces is less than some threshold moment of friction forces which should be overcome to begin the fly-wheel slewing. During these intervals the drive device at the upper end of the DC continues to rotate with constant angular velocity ω, the DC twists and accumulate potential energy of elastic strains. When elastic torque achieves a magnitude which is equal to the threshold value of the friction moment, the lower fly-wheel begins to rotate, the DC untwists and its potential energy begins to transform into kinetic energy of the DC and fly-wheel rotation. This rotation continues till the sum of elastic moment of the DC and inertia force moment of the fly-wheel again begins to be under the threshold value of the friction moment. As the result of this, the fly-wheel stops again and etc. Inasmuch as the functions of angular velocity and acceleration begin to be discontinuous in the described motion, the DC rotation acquires a shock character representing severe danger for the dynamic strength and stability of the whole system. It is not rational to describe these vibrations by trigonometric functions, so numerical methods should be used.

This theory contains an important factor complicating the considered phenomenon and the problem statement. It is the effect of torsion wave action on the fly-wheel the bit. The waves are formed as a result of elastic interaction between the fly-wheel and the DC. They achieve the DC top end, reflect and return with the delay to the lower end. Influence of this effect has not been studied yet and as shown below it reveals itself in the quantized character of the bit motion with the time quantum equal the wave passage time from one end of the DC to another and reverse.

In this paper, on the basis of taking into account non-linear frictional interaction of a bit and broken rock and influence of incident wave delay effect the problem about analysis of self-excitation of wave and vibrational twisting motions in a vertical deep DC is stated and solved.

2 Statement of the problem

For an extended analysis of mechanism of the DC torsion auto-oscillation generation, assume that the system can be simulated as a wave torsional pendulum (Fig. 1). Consider the case of stationary rotation of the DC top end with constant velocity ω. Introduce inertial coordinate system $OXYZ$ with its origin at the bit mass center and axis OZ in line with the DC axis, as well as the coordinate system $Oxyz$ rotating together with the DC top end.

Then the angle of the bit rotation relative to system $OXYZ$ is $\omega t + \varphi(0)$, where ωt is the angle of the DC top end rotation; t is the time; $\varphi = \varphi(z)$ is the angle of the DC element elastic twist relative to the $Oxyz$ system.

If to separate by convention the bit from the DC and to consider its dynamic equilibrium, the equation of elastic oscillation of the torsional pendulum can be represented in the form of d'Alembert's principle

$$M^{in} + M^{fr} + M^{el} = 0 \qquad (1)$$

Here $M^{in} = M^{in}(\ddot{\varphi})$ is the moment of inertia forces acting on the bit; $M^{fr} = M^{fr}(\omega + \dot{\varphi})$ the moment of the friction forces formed between the bit and the broken rock; $M^{el} = M^{el}(\varphi)$ the moment of elastic forces acting on the bit at the DC twist; the dots above φ denote differentiation with respect to time t.

Value M^{in} is calculated through the formula

$$M^{in} = -J \cdot \ddot{\varphi}, \qquad (2)$$

where J is the bit inertia moment.

Moment M^{el} is determined by the equality

$$M^{el} = GI_z \frac{\partial \varphi}{\partial z}, \qquad (3)$$

where G is the DC material elasticity module in shear; I_z the DC cross-section area inertia moment.

The question about the M^{fr} determining is more complicated. The models of the M^{fr} dependence on the rotary velocity $\omega + \dot{\varphi}$ of the bit relative to the rock medium are constructed in accordance with the tribological properties of rubbing materials and their friction interaction conditions. The most commonly encountered relationships are represented by the Coulomb friction law. In its diagram the vertical segment $\omega + \dot{\varphi}$ determines the static friction moment M_{st}^{fr}, it is realized in the absence of sliding between bodies. Its limit value is equal to dynamic moment M_{dyn}^{fr}, which occurs in the bit rotation and remains constant for any value of the relative angular velocity $\omega + \dot{\varphi}$.

The friction force moment model with nonlinear dynamic moment is also widespread. Its feature is that the dynamic moment M_{dyn}^{fr} is less than the limit static moment M_{st}^{fr}. It should be recorded that the stick-slip effect connected with stoppings of the bit rotation relative to inertial coordinate system $OXYZ$ is inherent in both these models. Its mathematical explanation is associated with the presence of non -linearities in the M^{fr} expression which cannot be linearized.

If lubricating liquid is between the rubbing bodies the function $M^{fr}(\omega + \dot{\varphi})$ can gain the form shown in Fig. 2. It can be represented with the help of approximating function

$$M^{fr} = \frac{a_1(\omega+\dot{\varphi}) + a_3(\omega+\dot{\varphi})^3}{1 + a_2(\omega+\dot{\varphi})^2} + \frac{a_5(\omega+\dot{\varphi})^5 + a_7(\omega+\dot{\varphi})^7}{1 + a_2(\omega+\dot{\varphi})^2} + \frac{a_9(\omega+\dot{\varphi})^9}{1 + a_2(\omega+\dot{\varphi})^2}, \qquad (4)$$

where coefficients a_i $(i = 1,2,...,9)$ are found experimentally.

Rotary dynamics of a bit hanged at the end of a long drill column possesses specificities typical of waveguide systems. As in these systems a disturbance applied to its one end attains other end in a finite time interval, one is forced to take into consideration the disturbance delay. Indeed, if for example the DC is manufactured from steel then the velocities of longitudinal and transversal waves expressed through the elasticity moduli E, G and density ρ are equal to $\alpha = \sqrt{E/\rho} \approx 5100$ m/s, $\beta = \sqrt{G/\rho} \approx 3200$ m/s, correspondingly. So if the DC length $L = 6500$ m the torsional disturbance applied to one of its ends reaches another one and returns back in 4s only.

For this reason the DC torsion oscillation should be studied on the basis of the wave equation

$$\rho I_z \frac{\partial^2 \varphi}{\partial t^2} - GI_z \frac{\partial^2 \varphi}{\partial z^2} = 0. \qquad (5)$$

where G is the DC material elasticity module in shear; ρ the material density; I_z the DC cross-section area inertia moment.

After substitution $\beta = \sqrt{G/\rho}$ this equation is converted to the standard form

$$\frac{\partial^2 \varphi}{\partial t^2} - \beta^2 \frac{\partial^2 \varphi}{\partial z^2} = 0. \qquad (6)$$

It has the d'Alembert solution

$$\varphi(z,t) = f(z-\beta t) + g(z+\beta t), \qquad (7)$$

where $f(z-\beta t)$, $g(z+\beta t)$ are the arbitrary continuous functions. The first of them determines the wave propagating in the direction of the Oz axis and the reverse is true for the second one. As the waves are not dispersive they propagate without varying their profile, resulting in essential simplification of the problem solving.

Indeed, in this case functions $f(z-\beta t)$, $g(z+\beta t)$ are determined only by the initial conditions

$$f(z-0) = f_0(z), \quad g(z+0) = g_0(z) \qquad (8)$$

and boundary conditions

$$F[f(0-\beta t), g(0+\beta t)] = 0, \\ f(L-\beta t) + g(L+\beta t) = 0, \qquad (9)$$

where F is the non-linear differential operator determining the bit motion.

Using equation (1) of the drill column bit equilibrium, one gains the constitutive differential equation of the wave pendulum vibration with delay argument

$$J\left\{\ddot{f}(-\beta t) - \ddot{f}\left[-\beta\left(t-\frac{2L}{\beta}\right)\right]\right\} - M^{fr} + \frac{G\cdot I_z}{\beta}\left\{\dot{f}(-\beta t) + \dot{f}\left[-\beta\left(t-\frac{2L}{\beta}\right)\right]\right\} = 0. \qquad (10)$$

In this equation J is the bit inertia moment. Moment M^{fr} was chosen as shown in Fig.2.

Equation (10) is integrated numerically at a constant angular velocity ω and prescribed initial conditions $q_1(0) = q_1^{(0)}$, $q_2(0) = q_2^{(0)}$. The found solutions allow determining the drill regimes accompanied by the DC torsion oscillation self-excitation, to construct their modes and to select the drill conditions excluding the system auto-oscillation.

The stated problem belongs to the case of stationary rotation, when $\omega = const$. But its formulation can be easily extended for non-stationary cases of the DC rotation connected with the starting and braking regimes.

3 Analysis of the results

In the process of functioning the DC can be either in the state of stationary rotation or of torsional self-excited oscillation depending on the chosen regime of drilling. Types of these states are dictated by the equation (10) solutions which primary are determined by the DC length L and angular velocity ω. Since the DC stiffness, its inertial properties and time delay $2L/\beta$ of wave disturbances depend on the DC length L, in the simulation of the auto-oscillation effect the value L was chosen to be equal to $600\ m$ and $1200\ m$.

Use of the wave torsion pendulum model for investigation of drill column vibration self-excitation permitted not only to reflect general regularities of limit cycle birth bifurcations, established on the basis of simplified 1 DOF oscillator model, but also to find radically new feature unique only to wave systems. It is associated with formation of the so-called quantized time with the resulting effect of constant angular velocity staying during time segment $\Delta\tau$, which is equal to the time duration of the torsion wave passing the path from the bit to the upper end and backward

$$\Delta\tau = 2L/\beta. \qquad (11)$$

Fig.3 presents $\dot{\varphi}$ as a function of t in the segment $0 \le t \le 130\ s$, constructed by the way of equation (10) integration with the use of the Runge-Kutta method.

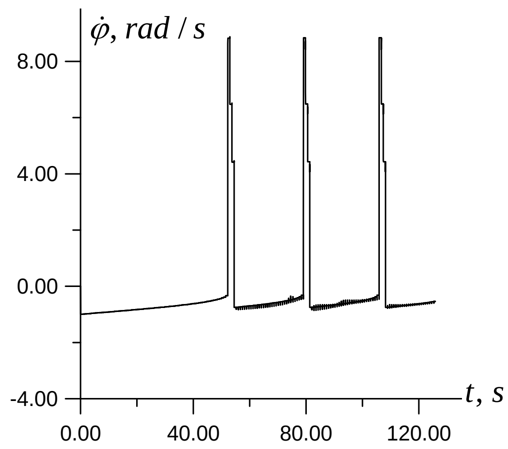

Figure 3.Angular velocity function.

Initial condition $f(0) = 0$, $\dot{f}(0) = 0$ were assumed and integration step of time measured $\Delta t = 7.76898\cdot 10^{-6}\ s$. In doing so the system parameters were chosen to be $G = 8.077\cdot 10^{10}\ Pa$, $I_z = 3.12\cdot 10^{-5}\ m^4$, $J = 3.1\ kg\cdot m^2$; the rotation velocity $\omega = 1\ rad/s$. It should be noted, that the periodical oscillations with the period $T = 26.25\ s$ are set very rapidly and function $\dot{\varphi}(t)$ has the step-wise shape in the chosen scale, in spite of the function $M^{fr}(\dot{\varphi})$ smoothness. The step length coincides with the time quantum $\Delta\tau = 0.75\,s$ calculated by formula (11). It turned out to be $\Delta\tau = 9.6\cdot 10^4\,\Delta t$. The attempts to integrate equation (10) with other initial conditions led to the same results indicating to soft character of the oscillation self-excitation.

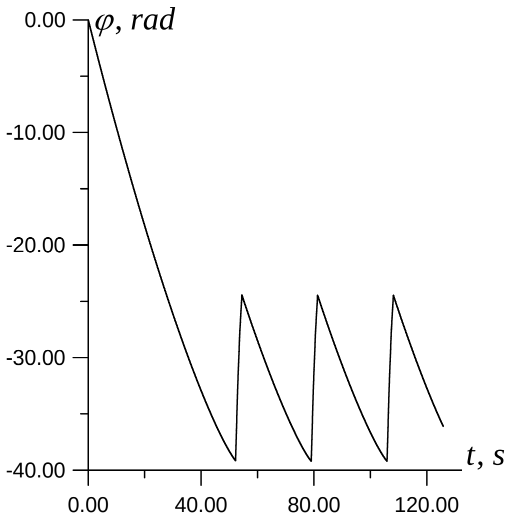

Figure 4. Torsion angle function.

The outline of function $\varphi(t)$ in Fig.4 testifies that the bit oscillations proceed with jerks accompanied by large acceleration at transfer from one angular velocity level to another.

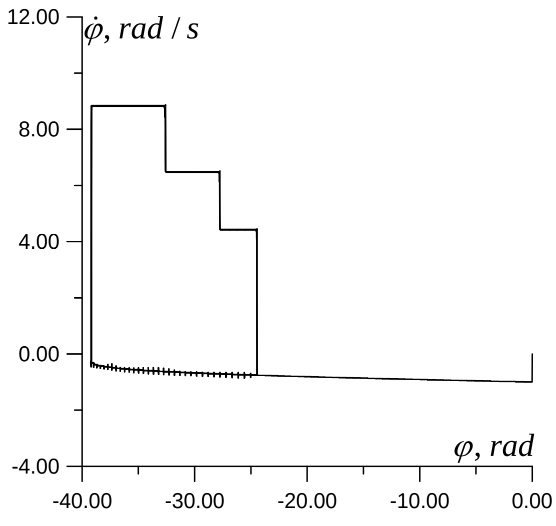

Figure 5. Phase portrait of the limit cycle.

The limit cycle properties are represented by its phase portrait (Fig. 5). It has complicated shape, also stipulated by the quantized character of the time.

4 Conclusions

The problem of analysis of limit cycle birth bifurcations in the torsion wave models of superdeep drill columns is set up. The constitutive differential equation with delay argument is constructed. Analysis of its solutions permitted to establish the following features of the drill column torsion oscillation self-excitation:

1. The limit cycles of the torsion wave pendulum do not depend on initial conditions, so the self-excitation has the soft character.
2. The self-excited oscillations proceed in the manner of quatized time. The time quantum duration equals the time of the torsion wave propagation through the column doubled length.
3. The auto-oscillations prevail at low values of the DC angular velocity.

References

Brett, J.F. (1992). The genesis of torsional drillstring vibrations. *SPE Drill. Engin.*, **7**, pp.168-174.

Challamel, N. (2000). Rock destruction effect on the stability of a drilling structure. *Journ. Sound Vibr.*, **233** (2), pp. 235-254.

Jansen, J.D., Van den Steen, L. (1995). Active damping of self-excited torsional vibrations in oil well drillstrings. *Journ. Sound Vibr.*, **179** (4), pp. 647-668.

Tucker, R.W., Wang, C. (1999). On the effective control of torsional vibrations in drilling systems. *Journ. Sound Vibr.*, **224** (1), pp. 101-122.

Hassard, B.D., Kazarinoff, N.D., Wan, Y. - H. (1981). Theory and Applications of Hopf Bifurcation. Cambridge University Press. Cambridge, London, New York, New Rochelle, Melbourne, Sydney.