

Pulse-Width Modulated Control via Singular Perturbation for a Spacecraft Attitude Maneuver ^{*}

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Abstract: The problem of controller design for a single-axis spacecraft reorientation maneuver in the presence of high-frequency pulse-width modulation (PWM) feedback is addressed. The Filippov's average PWM spacecraft attitude model controlled by a smooth function is used as the basis for a dynamical feedback controller design such that the controller dynamics is a singular perturbation with respect to the spacecraft dynamics. The explicit expressions for evaluation of the controller parameters are derived by fast and slow modes analysis that are induced in the closed-loop systems. The presented design methodology guarantees desired behavior for the Cayley-Rodrigues attitude parameter in the presence of nonlinearity and uncertainty of the spacecraft model. Simulation results are presented.

Keywords: spacecraft control; pulse-width modulation; singular perturbations.

1. INTRODUCTION

The problem of controller design for a spacecraft angle maneuver is of the great interest to practical applications and theoretical investigations due to a strong nonlinearity of spacecraft dynamics. In particular, the complexity of controller design increases in the presence of PWM control feedback which is usually implemented by a set of pairs of opposing pulse torque generators.

A number of the well known control system design methodologies are used for the solution of this problem, for example, the linearization techniques around the nominal point (Breakwell (1981)), the exact linearization approach based on nonlinear state feedback transformation and nonlinear feedback (Dwyer (1984)), the variable structure control (VSC) approach with linear sliding manifolds for the spacecraft equipped by PWM thrusters (Vadali (1986)), the general VSC approach based on nonlinear sliding manifolds (Sira-Ramirez (1986); Dwyer et al. (1987); Dwyer and Sira-Ramirez (1988)).

The equivalence between sliding modes of variable structure control and PWM control responses under the high frequency sampling was discussed by Sira-Ramirez (1989) and used for development of geometric approach to PWM control loop design. It was shown by Sira-Ramirez and Lischinsky-Arenas (1990), if PWM controller is not saturated and the sampling frequency tends to infinity, then the response of discontinuously controlled system coincides with an average model discussed by Filippov (1964) where control variable is represented by a smooth duty ratio function. Hence, the continuous-time duty ratio controller can be designed based on the Filippov's average model. For example, the duty ratio adaptive PWM controller

scheme for switch-regulated linearizable systems was discussed by Sira-Ramirez and Llanes-Santiago (1993) where its practical implementation may be too complicated due to the requirement for online estimation of the system parameters.

The objectives of this paper are the analysis and design of simple proportional-integral-derivative (PID) controller with PWM feedback loop for spacecraft reorientation maneuver based on the design methodology presented by Yurkevich (2004), that guarantees desired output transients by inducing of two-time-scale motions in the closed-loop system. Stability conditions imposed on the fast and slow modes and sufficiently large mode separation rate between fast and slow modes can ensure that the full-order closed-loop nonlinear system achieves the desired properties in such a way that the output transient performances are desired and insensitive to external disturbances and plant's parameter variations. The stability of fast-motion transients in the closed-loop system is provided by proper selection of controller parameters, as well as slow-motion transients correspond to the stable reference model of desired mapping from reference input into controlled output. The method of singular perturbations is used throughout the discussed controller design methodology (see: Tikhonov (1952); Klimushchev and Krasovskii (1962); Hoppensteadt (1966); Kokotović (1984); Kokotović et al. (1999); Naidu (2002)).

The main point of novelty in the paper is that the Filippov's average approach and singular perturbation method are combined together for a spacecraft attitude controller design in the presence of high-frequency pulse-width modulated feedback that allows to get simple procedure for robust controller design.

The paper is organized as follows. First, a simplified single-axis spacecraft reorientation maneuver model is discussed

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that describes the behavior of the Cayley-Rodrigues attitude parameter defined around skewed axis. Second, the Filippov's average model of the spacecraft maneuver is introduced given that the high-frequency pulse-width modulated discontinuous control is used. Third, the main steps of controller design procedure via singular perturbation is explained. Finally, simulation results are presented as well.

2. SPACECRAFT REORIENTATION PROBLEM

The proposed approach to PWM feedback controller design is discussed based on the single-axis jet-controlled spacecraft reorientation maneuver model represented by Dwyer and Sira-Ramirez (1988), Sira-Ramirez and Lischinsky-Arenas (1990):

$$\dot{x} = 0.5(1 + x^2)\omega, \quad \dot{\omega} = J^{-1}u \quad (1)$$

where x is the Cayley-Rodrigues attitude parameter defined around skewed axis and x is considered as the measured output variable, ω is the angular velocity about the principal axis and ω is the unmeasured variable, J is the corresponding moment of inertia, u is the control torque provided by a pair of opposing torque generators.

Let us suppose that a pulse-width modulator is used in order to provide a discontinuous control strategy. The input signal of the pulse-width modulator is defined as the scalar variable χ which takes values in the interval $[-1, 1]$. The output signal of the pulse-width modulator is defined as the scalar variable u which takes one of three possible values in $\Omega_u := \{u^-, 0, u^+\}$.

Assumption 1. Assume that the symmetric switching function $u(t)$ is defined as the pulse-width modulated control strategy given by

$$u = \begin{cases} u^+ & \text{if } t_\kappa < t \leq t_\kappa + |\chi(t_\kappa)|T_s, \chi(t_\kappa) > 0 \\ 0 & \text{if } t_\kappa + |\chi(t_\kappa)|T_s < t \leq t_\kappa + T_s, \chi(t_\kappa) > 0 \\ 0 & \text{if } t_\kappa < t \leq t_\kappa + T_s, \chi(t_\kappa) = 0 \\ u^- & \text{if } t_\kappa < t \leq t_\kappa + |\chi(t_\kappa)|T_s, \chi(t_\kappa) < 0 \\ 0 & \text{if } t_\kappa + |\chi(t_\kappa)|T_s < t \leq t_\kappa + T_s, \chi(t_\kappa) < 0 \end{cases} \quad (2)$$

where T_s is the sampling period of the pulse-width modulation.

Remark 1. The duty ratio function is represented by D where $D \triangleq |\chi|$ and D takes values in the interval $[0, 1]$.

Remark 2. Note, the reorientation maneuver model given by (1) is used in this paper to show the applicability of the proposed design methodology for nonlinear systems while the single-axis jet-controlled spacecraft reorientation maneuver model can be represented without involving the Cayley-Rodrigues attitude parameter into consideration.

A control system is being designed so that the condition

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (3)$$

holds, where $e(t)$ is the error of the reference input realization; $e(t) \triangleq r(t) - x(t)$; $r(t)$ is the desired attitude (reference input). Moreover, the controlled transients $e(t) \rightarrow 0$ should have a desired behavior. These transients should not depend on the nonlinearity of the spacecraft model (1).

3. FILIPPOV'S AVERAGE MODEL

In accordance with (1), the single-axis jet-controlled spacecraft reorientation maneuver model corresponds to the class of exactly input-output linearizable systems without zero-dynamics and one may be rewritten in the normal canonical form.

From (1) the relationship

$$\omega = \frac{2\dot{x}}{1+x^2} \quad (4)$$

results. Then, the two-time differentiating $x(t)$ along the solutions of (1) with the help of (4) yield the following expression:

$$\ddot{x} = \frac{2x\dot{x}^2}{1+x^2} + \frac{1+x^2}{2J}u \quad (5)$$

and then, (5) can be rewritten, for short, as

$$\ddot{x} = f(X, u) \quad (6)$$

where $X = \{x, \dot{x}\}^T$.

Denote

$$\begin{aligned} f^+(X) &\triangleq f(X, u = u^+) = \frac{2x\dot{x}^2}{1+x^2} + \frac{1+x^2}{2J}u^+ \\ f^0(X) &\triangleq f(X, u = 0) = \frac{2x\dot{x}^2}{1+x^2} \\ f^-(X) &\triangleq f(X, u = u^-) = \frac{2x\dot{x}^2}{1+x^2} + \frac{1+x^2}{2J}u^- \end{aligned} \quad (7)$$

In accordance with (7), the system (1)-(2) can be rewritten in the form

$$\begin{aligned} \ddot{x} &= \frac{1}{2}[1 + \varphi(\nu)] [|\nu|f^+(X) + (1 - |\nu|)f^0(X)] \\ &\quad + \frac{1}{2}[1 - \varphi(\nu)] [|\nu|f^-(X) + (1 - |\nu|)f^0(X)] \end{aligned} \quad (8)$$

where ν is defined as the following switching function

$$\nu = \begin{cases} 1 & \text{if } t_\kappa < t \leq t_\kappa + |\chi(t_\kappa)|T_s, \chi(t_\kappa) > 0 \\ 0 & \text{if } t_\kappa + |\chi(t_\kappa)|T_s < t \leq t_\kappa + T_s, \chi(t_\kappa) > 0 \\ 0 & \text{if } t_\kappa < t \leq t_\kappa + T_s, \chi(t_\kappa) = 0 \\ -1 & \text{if } t_\kappa < t \leq t_\kappa + |\chi(t_\kappa)|T_s, \chi(t_\kappa) < 0 \\ 0 & \text{if } t_\kappa + |\chi(t_\kappa)|T_s < t \leq t_\kappa + T_s, \chi(t_\kappa) < 0 \end{cases} \quad (9)$$

and the function $\varphi(\nu)$ is defined as

$$\varphi(\nu) \triangleq \begin{cases} 1 & \text{for } \nu > 0 \\ 0 & \text{for } \nu = 0 \\ -1 & \text{for } \nu < 0 \end{cases} \quad (10)$$

The equation (8) can be rewritten as

$$\ddot{x} = f^0 + \left[\frac{1}{2}(f^+ + f^-) + \frac{1}{2}\varphi(\nu)(f^+ - f^-) - f^0 \right] |\nu|$$

or, for short, in the following form

$$\ddot{x} = f^0 + g(\nu) \quad (11)$$

where

$$g(\nu) = \begin{cases} (f^+ - f^0)|\nu| & \text{for } \nu > 0 \\ 0 & \text{for } \nu = 0 \\ (f^- - f^0)|\nu| & \text{for } \nu < 0 \end{cases} \quad (12)$$

Assumption 2. Let us take u^+ and u^- such that the condition

$$u^+ = -u^- = \bar{u} \quad (13)$$

holds where $\bar{u} > 0$.

From (7), (12), and (13), we get

$$g(\nu) = \begin{cases} \bar{g}|\nu| & \text{for } \nu > 0 \\ 0 & \text{for } \nu = 0 \\ -\bar{g}|\nu| & \text{for } \nu < 0 \end{cases} \quad (14)$$

that is the same as $g(\nu) = \bar{g}\nu$ for all ν where

$$\bar{g}(X, \bar{u}) \triangleq \frac{1+x^2}{2J} \bar{u}. \quad (15)$$

Finally, in accordance with (14) and (15), the equation (11) can be rewritten as

$$\ddot{x} = f^0(X) + \bar{g}(X, \bar{u})\nu \quad (16)$$

Assumption 3. Assume that the pulse-width modulator represented by (2) is not saturated, that is the following inequality $-1 < \chi < 1$ holds.

Assumption 4. The sampling period T_s is assumed to be sufficiently small in compare with time constants associated with the dynamics of the system (1) or, in other words, assume that the sampling frequency $f_s := 1/T_s$ tends to infinity.

In accordance with Assumptions 3 and 4, by following to the Filippov's approach (Filippov (1964)), the geometric approach to PWM control (Sira-Ramirez (1989)), and Theorem A.1 in the paper by Sira-Ramirez and Lischinsky-Arenas (1990), the response of discontinuously controlled system given by (16) and (9) coincides with Filippov's average model

$$\ddot{x} = f^0(X) + \bar{g}(X, \bar{u})\chi \quad (17)$$

where χ takes values in the open interval $(-1, 1)$.

The Filippov's average PWM spacecraft attitude model given by (17) is used below as the basis for a continuous-time dynamical feedback controller design.

4. REFERENCE MODEL OF REORIENTATION

Consider the reference model of the single axis spacecraft reorientation maneuver in the following form

$$x(s) = G^d(s)r(s), \quad (18)$$

where

$$G^d(s) = \frac{1}{T^2s^2 + a^dT_s + 1} \quad (19)$$

and the parameters T , a^d are selected in accordance with the desired transient performances for $x(t)$. From $G^d(s)$, the reference model of the desired behavior for $x(t)$ in the form of the 2-nd order stable differential equation

$$\ddot{x} = -\frac{a^d}{T}\dot{x} - \frac{1}{T^2}x + \frac{1}{T^2}r \quad (20)$$

results. Let us rewrite the equation (20), for short, as

$$\ddot{x} = F(X, r) \quad (21)$$

where $x(t)$ exponentially converges to r if $r = \text{const}$.

Denote $e^F \triangleq F(X, R) - \ddot{x}$, where e^F is the realization error of the desired behavior assigned by (21) and \ddot{x} is defined by (6). Accordingly, if the condition

$$e^F = 0 \quad (22)$$

holds, then the behavior of \ddot{x} with prescribed dynamics of (21) is fulfilled, that is the same as (18). Accordingly,

$e(s) = [1 - G^d(s)]r(s)$, thus $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for $r = \text{const}$.

Hence, the output regulation problem given by (3) has been reformulated as the requirement (22), that is

$$F(X, r) - f(X, u) = 0. \quad (23)$$

Expression (22) (or (23)) is called as the insensitivity condition for the behavior of the output $x(t)$ with respect to the nonlinearity of the spacecraft model (6).

Remark 3. In accordance with (5), the unique isolated root of (23) exists, which can be denoted as

$$u^{id} = \frac{2J}{1+x^2} \left[F(X, r) - \frac{2x\dot{x}^2}{1+x^2} \right] \quad (24)$$

where the control function $u^{id}(t)$ is the nonlinear inverse dynamics solution in the average sense and one corresponds to the desired output behavior of (1) prescribed by (21).

Remark 4. If the condition (22) holds, then we get

- (i) robust zero steady-state error of the reference input realization;
- (ii) desired output performance specifications such as overshoot, settling time, and system type;
- (iii) insensitivity of the output transient behavior with respect to nonlinearity of the system (1).

Remark 5. In accordance with (17), the condition (22) can be rewritten as

$$F(X, r) - f^0(X) - \bar{g}(X, \bar{u})\chi = 0 \quad (25)$$

where the unique isolated root of (25) is given by the function $\chi(t) = \chi^{id}(t)$ and

$$\chi^{id} = \frac{F(X, r) - f^0(X)}{\bar{g}(X, \bar{u})}. \quad (26)$$

Hence, χ^{id} is the inverse dynamics solution with respect to the function χ .

Assumption 5. Assume that the values of u^+ and u^- are selected with the help of (24) such that the condition

$$u^- < u^{id} < u^+ \quad (27)$$

holds for all $X \in \Omega_X$ and for all $r \in \Omega_r$, where Ω_X and Ω_r are some bounded convex sets of workspace.

Remark 6. From (27) the non-saturation condition given by $-1 < \chi^{id} < 1$ results, that is

$$-\bar{g}(X, \bar{u}) < F(X, r) - f^0(X) < \bar{g}(X, \bar{u}), \\ \forall X \in \Omega_X, \quad \forall r \in \Omega_r.$$

5. CONTROL VIA SINGULAR PERTURBATION

5.1 Control Law

By following to the design methodology discussed by Yurkevich (2004), consider the continuous-time feedback control law given by the following differential equation:

$$\mu^2 \ddot{\chi} + d_1\mu \dot{\chi} + d_0\chi = k[F(X, r) - \ddot{x}], \quad (28)$$

that is

$$\mu^2 \ddot{\chi} + d_1\mu \dot{\chi} + d_0\chi = k \left[-\ddot{x} - \frac{a^d}{T}\dot{x} - \frac{1}{T^2}x + \frac{1}{T^2}r \right], \quad (29)$$

where μ is a small positive parameter of the controller.

Remark 7. Note, the highest derivative \ddot{x} is presented at feedback loop given by (29). However, the discussed control law (29) may be expressed in terms of transfer functions

$$\chi(s) = -\frac{k(s^2 + \frac{a^d}{T}s + \frac{1}{T^2})}{\mu^2 s^2 + d_1 \mu s + d_0} x(s) + \frac{\frac{k}{T^2}}{\mu^2 s^2 + d_1 \mu s + d_0} r(s). \quad (30)$$

Hence, the control law (29) is proper and implemented without an ideal differentiation of $x(t)$ or $r(t)$.

Remark 8. In case of $d_0 = 0$, from (30) the conventional proper PID controller given by

$$\chi(s) = \frac{k}{\mu(\mu s + d_1)} \left\{ \frac{1}{sT^2} [r(s) - x(s)] - \left(s + \frac{a^d}{T} \right) x(s) \right\}$$

results.

5.2 Implementation of Control Law

In order to practical implementation, the discussed control law (29) can be rewritten in the form given by

$$\chi^{(2)} + a_1 \chi^{(1)} + a_0 \chi = b_2 x^{(2)} + b_1 x^{(1)} + b_0 x + c_0 r \quad (31)$$

where

$$a_1 = \frac{d_1}{\mu}, \quad a_0 = \frac{d_0}{\mu^2}, \quad c_0 = \frac{k}{\mu^2 T^2},$$

$$b_2 = -\frac{k}{\mu^2}, \quad b_1 = -\frac{k a_1^d}{\mu^2 T}, \quad b_0 = -c_0.$$

Then, from (31), we may get the equations of the controller in the state-space form, for example, that are

$$\begin{aligned} \dot{\chi}_1 &= -a_1 \chi_1 + \chi_2 + (b_1 - a_1 b_2) x \\ \dot{\chi}_2 &= -a_0 \chi_1 + (b_0 - a_0 b_2) x + c_0 r \\ \chi &= \chi_1 + b_2 x + \chi_0 \end{aligned} \quad (32)$$

where $\chi_0 = \text{const}$ and $\chi(0) = \chi_0$ if $\chi_1(0) = -b_2 x(0)$.

Note, the measured output variable x is used in feedback merely without informational needs for measure of \dot{x} or \ddot{x} .

5.3 Two-Time-Scale Motions Analysis

In this section the closed-loop system properties are treated based on the average model (17) given that the both Assumptions 1 and 2 are satisfied. Substitution of (17) into (28) yields the average closed-loop system equations in the form

$$\ddot{x} = f^0(X) + \bar{g}(X, \bar{u}) \chi \quad (33)$$

$$\mu^2 \ddot{\chi} + d_1 \mu \dot{\chi} + [d_0 + k \bar{g}(X, \bar{u})] \chi = k[F(X, r) - f^0(X)]. \quad (34)$$

Denote $x_1 = x$, $x_2 = \dot{x}$, $\chi_1 = \chi$, $\chi_2 = \mu \dot{\chi}$. Hence, the average closed-loop system may be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f^0(X) + \bar{g}(X, \bar{u}) \chi_1 \end{aligned} \quad (35)$$

$$\mu \dot{\chi}_1 = \chi_2$$

$$\mu \dot{\chi}_2 = -[d_0 + k \bar{g}(X, \bar{u})] \chi_1 - d_1 \chi_2 + k[F(X, r) - f^0(X)].$$

Since μ is the small positive parameter, the above equations (35) are the singularly perturbed differential equations. If $\mu \rightarrow 0$, then fast and slow modes are forced in the

closed-loop system and the time-scale separation between these modes depends on the parameter μ .

The main qualitative property of the singularly perturbed systems is that: if an isolated equilibrium point of the fast-motion subsystem (FMS) exists and one is exponentially stable, then there exists $\mu^* > 0$ such that for all $\mu \in (0, \mu^*)$ the trajectories of the singularly perturbed system approximate to the trajectories of the slow-motion subsystem (SMS) (see: Tikhonov (1952); Klimushchev and Krasovskii (1962); Hoppensteadt (1966); Kokotović (1984); Kokotović et al. (1999); Naidu (2002)).

In order to enable usage of the standard technique for two-time-scale motions analysis, take $t = \mu t_0$. Hence, from (35), the system

$$\begin{aligned} \frac{dx_1}{dt_0} &= \mu x_2 \\ \frac{dx_2}{dt_0} &= \mu \{f^0(X) + \bar{g}(X, \bar{u}) \chi_1\} \\ \frac{d\chi_1}{dt_0} &= \chi_2 \\ \frac{d\chi_2}{dt_0} &= -\{d_0 + k \bar{g}(X, \bar{u})\} \chi_1 - d_1 \chi_2 \\ &\quad + k[F(X, r) - f^0(X)] \end{aligned} \quad (36)$$

results. By setting $\mu = 0$ we get the system given by

$$\begin{aligned} \frac{dx_1}{dt_0} &= 0 \\ \frac{dx_2}{dt_0} &= 0 \\ \frac{d\chi_1}{dt_0} &= \chi_2 \\ \frac{d\chi_2}{dt_0} &= -\{d_0 + k \bar{g}(X, \bar{u})\} \chi_1 - d_1 \chi_2 \\ &\quad + k[F(X, r) - f^0(X)] \end{aligned} \quad (37)$$

Then the inverse replacement $t_0 = \mu^{-1} t$ yields the average fast-motion subsystem (FMS) given by

$$\begin{aligned} \mu \dot{\chi}_1 &= \chi_2 \\ \mu \dot{\chi}_2 &= -\{d_0 + k \bar{g}(X, \bar{u})\} \chi_1 - d_1 \chi_2 \\ &\quad + k[F(X, r) - f^0(X)] \end{aligned} \quad (38)$$

where x_1 and x_2 are treated as the frozen variables during the transients in (38). Finally, from (38), the average FMS

$$\begin{aligned} \mu^2 \ddot{\chi} + d_1 \mu \dot{\chi} + \{d_0 + k \bar{g}(X, \bar{u})\} \chi \\ = k[F(X, r) - f^0(X)] \end{aligned} \quad (39)$$

results, where X is the frozen vector during the transients in (39).

Remark 9. The FMS given by (39) describes the average fast-motion transients behavior given that the sampling frequency of PWM controller is large enough such that the FMS (39) displays a low-pass filtering property.

Remark 10. From (39), by taking $\ddot{\chi} = 0$ and $\dot{\chi} = 0$, the unique isolated equilibrium point (steady state) of the FMS can be found, that is

$$\chi^s = \frac{k[F(X, r) - f^0(X)]}{d_0 + k\bar{g}(X, \bar{u})}, \quad (40)$$

where $\chi^s = \chi^{id}$ when $d_0 = 0$.

Assumption 6. Assume that the parameters μ , k , d_1 d_0 are selected such that the equilibrium point χ^s of FMS transients is exponentially stable and after the rapid decay of transients in (39), we get the steady state (more precisely, quasi-steady state) for the average FMS, where $\dot{\chi} = 0$, $\ddot{\chi} = 0$, $\chi(t) = \chi^s(t)$, and $\chi^s(t)$ is given by (40).

Substitution of $\chi = \chi^s$ into (33) yields the average slow-motion subsystem (SMS) given by

$$\ddot{x} = F(X, r) + \frac{d_0[f^0(X) - F(X, r)]}{d_0 + k\bar{g}(X, \bar{u})}, \quad (41)$$

which is the same as the reference model (21) when $d_0 = 0$.

So, if a sufficient time-scale separation between the fast and slow modes in the closed-loop system and exponential convergence of FMS transients to equilibrium are provided, then after the damping of fast transients the desired output behavior prescribed by (21) is fulfilled despite that $f(X, u)$ is unknown complex function. Thus, the output transient performance indices are insensitive to the non-linearity of the system (1), by that the solution of the discussed control problem (3) is maintained.

5.4 Selection of Controller Parameters

Denote

$$\gamma = d_0 + k\bar{g}(X, \bar{u}).$$

From (39), the FMS characteristic polynomial

$$\mu^2 s^2 + d_1 \mu s + \gamma$$

results, where the FMS time constant is given by $\tau_{fms} = \mu/\sqrt{\gamma}$. From (19), the SMS characteristic polynomial

$$T^2 s^2 + a^d T s + 1$$

follows, where the SMS time constant is given by $\tau_{sms} = T$. The controller parameters μ , k , d_1 d_0 are selected in accordance with the requirements on FMS stability and the desired degree of time-scale separation between the fast and slow modes in the closed-loop system (35).

The requirement for degree of time-scale separation between the fast and slow modes in the system (35) can be represented by $\tau_{fms} \leq \tau_{sms}/\eta$, where, for example, $\eta = 10$. The last inequality yields the upper bound for μ given by $\mu \leq \mu_{max} = T\sqrt{\gamma_{min}}/\eta$ where

$$\gamma_{min} = \min_{X \in \Omega_X} \{d_0 + k\bar{g}(X, \bar{u})\}.$$

Note that by selection of d_1 the desired damping of fast transients is provide. The parameter d_0 is taken one of two possible values, $d_0 = 1$ or $d_0 = 0$, where the integral action is incorporated into the control loop when $d_0 = 0$ and, accordingly, the robust zero steady-state error of the reference input realization is maintained for finite value of k . In accordance with (41), the parameter k is selected such that the condition $k\bar{g}(X, \bar{u}) \gg 1$ holds when $d_0 = 1$.

6. SIMULATION RESULTS

Simulation results provided below were run for a spacecraft model (1) with $J = 90 \text{ kg m}^2$, where the pulse-width

modulated control is defined as the switching function $u(t)$ given by (2) with $T_s = 2 \text{ s}$, $u^+ = 0.55$, and $u^- = -0.55$. The controller is given by (32), where the controller parameters are selected as $T = 8 \text{ s}$, $a^d = 2$, $k = 10J$, $d_1 = 5$, $d_0 = 0$, and $\mu = 1 \text{ s}$. The simulation results of the closed-loop system given by (1) and (32) with the pulse-width modulated control (2) are shown in Figs. 1–4, where the initial conditions are zero.

The control system for numerical simulation is shown in Fig. 5, where the plant (P) is represented by the spacecraft model (1), the controller (C) is given by (32). In order to provide the fuel saving mode, the control system is supplemented by the additional limiter with dead zone Δ , where $\Delta = 0.02$.

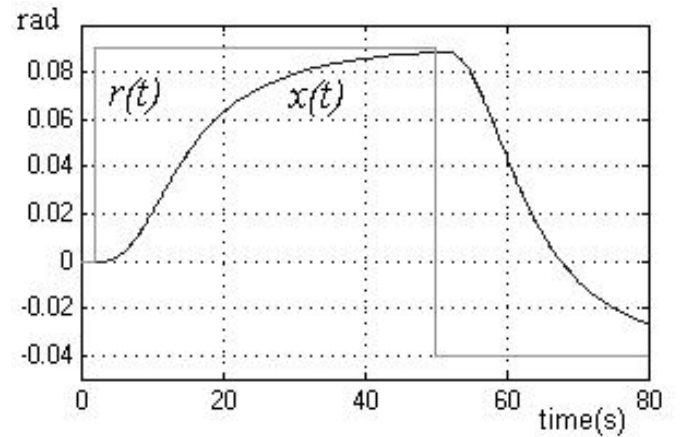


Fig. 1. Plots of $r(t)$ and $x(t)$ in the system (1),(32) with the pulse-width modulated control (2).

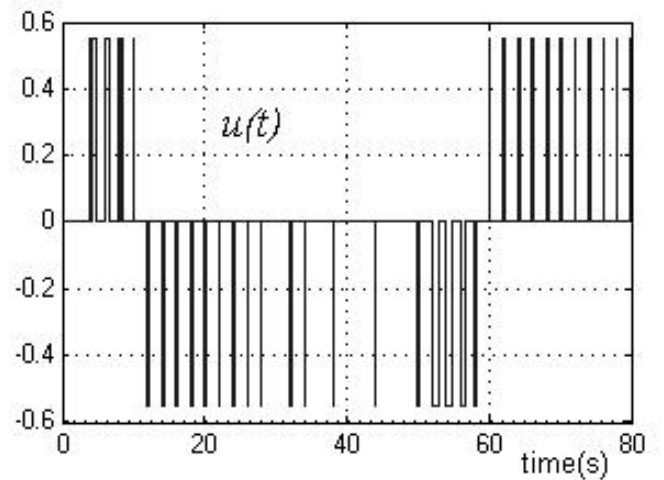


Fig. 2. Plot of $u(t)$ in the system (1),(32) with the pulse-width modulated control (2).

7. CONCLUSION

The proposed scheme for robust PID controller design via singular perturbation technique is based on the Filippov's average model with control variable represented by the smooth function given that the high-frequency pulse-width modulated discontinuous control is used in feedback loop.

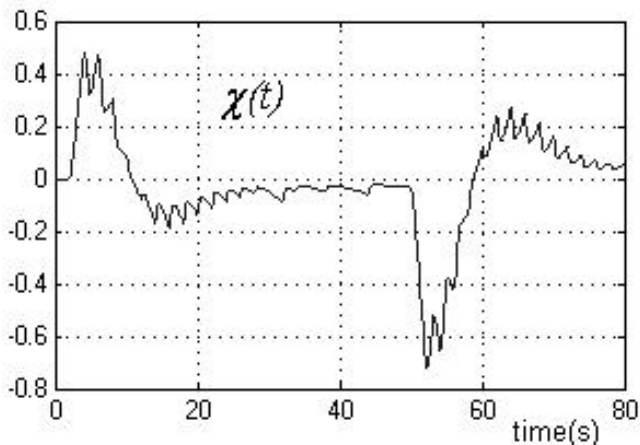


Fig. 3. Plot of $\chi(t)$ in the system (1),(32) with the pulse-width modulated control (2).

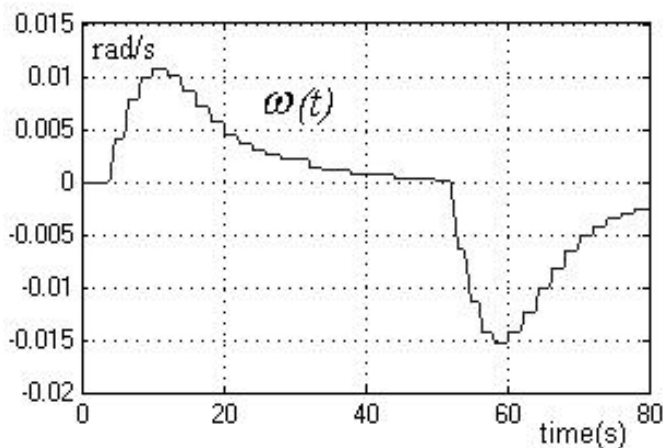


Fig. 4. Plot of $\omega(t)$ in the system (1),(32) with the pulse-width modulated control (2).

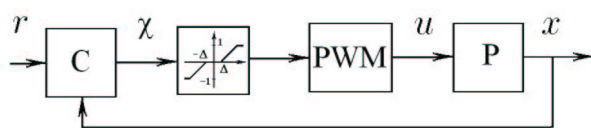


Fig. 5. Block diagram of control system given by (1) and (32) with the pulse-width modulated control (2), which is supplemented by the limiter with dead zone.

The presented design methodology guarantees desired behavior for the Cayley-Rodrigues attitude parameter in the presence of nonlinearity and uncertainty of the single-axis spacecraft reorientation maneuver model. The advantage, caused by two-time-scale technique for closed-loop system analysis, is that analytical expressions for selection of the controller parameters can be found, where controller parameters depend explicitly on the specifications of the desired output behavior. The presented design methodology may be useful for real-time control system design under uncertainties. The simulation results confirm the analytical calculation presented in the paper.

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