Chaotic regimes in coupled phase systems

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Abstract

It is shown that the use of the properties of collective dynamics may be effective for solution of the problem of generation of chaotically modulated oscillations.

1. The idea of using chaotic self-oscillations as carriers in communication systems that was put forward in was recently intensively discussed in the literature. Dynamical chaotic oscillations are the kinds of wideband and ultrawideband signals. Chaotic oscillations provide rich opportunities for controlling and modulation. These are the reasons which make them highly promising in the communication field of research.

Traditional communication systems with regular oscillations as carriers are usually based on phase-locked loops (PLL). Such systems allow effective solution of the whole complex of problems arising at transmission and reception of information, namely, generation of stabilized carrier oscillations, modulation of carrier oscillations by information signal, optimal noise filtration, and others. There arises a question: Is it possible to construct PLL-based non-traditional promising communication systems with chaotic oscillations as carriers? Specifically, can PLL be a useful tool for generation and synchronization of carrier chaotic oscillations that is the key task in communication systems? This problem is very scantily considered in the literature. The present work is concerned with generation of carrier chaotic oscillations using PLL.

2. A phase system is a typical system for automatic control of generator frequency that is intended for locking of periodic oscillations of a voltage controlled generator in the PLL by a reference oscillation. The mathematical model of such a phase system is generally written in the form:

$$\frac{p\varphi}{\Omega} + K(p)\sin\varphi = \frac{\Delta\omega_0}{\Omega}. \qquad (1)$$

Here $p \equiv d/dt$, Ω is the maximal deviation of voltage-controlled generator frequency that can balance the control loop, φ is the current phase mismatch of the voltage controlled generator relative to the reference signal, $\Delta\omega_0$ is the initial frequency mismatch of the voltage controlled generator relative to the reference signal, $K(p)$ is the transfer coefficient of the filter, and $\sin\varphi$ is the nonlinearity of the phase detector.

Dynamic properties of phase systems are determined by the structure of the control loop, in particular, by the type of the low-frequency filter. The simplest filter with which a phase system can generate dynamic chaos is a second-order filter with the transfer coefficient $K(p) = (1 + a_1 p + a_2 p^2)^{-1}$. In this case, (1) can be written as a differential equation

$$\mu \dddot{\varphi} + \varepsilon\ddot{\varphi} + \dot{\varphi} + \sin\varphi = \gamma \qquad (2)$$

in the cylindrical phase space $U = \{\varphi(\mathrm{mod}2\pi), \dot{\varphi}, \ddot{\varphi}\}$. Here, $\gamma = \Delta\omega_0/\Omega$, $\varepsilon = a_1\Omega, \mu = a_2\Omega^2$ are dimensionless parameters of the system.

The analysis of model (2) shows that for the phase system with a second-order filter the following dynamic modes are typical:

– *Synchronization of the generator by a reference signal*, i.e., the frequencies of the controlled generator and of the reference signal become equal, and the phase difference takes on a constant value. A stable equilibrium state with co-ordinates $\varphi_1 = \arcsin\gamma, \dot{\varphi}_1 = 0, \ddot{\varphi}_1 = 0$ (the projection onto the $\varphi, \dot{\varphi}$-plane is given in Fig. 1(a)) corresponds to this mode in the phase space U.

– *Quasi-synchronization*, when the frequency of the controlled generator is modulated regularly or chaotically around the average frequency stabilized by the

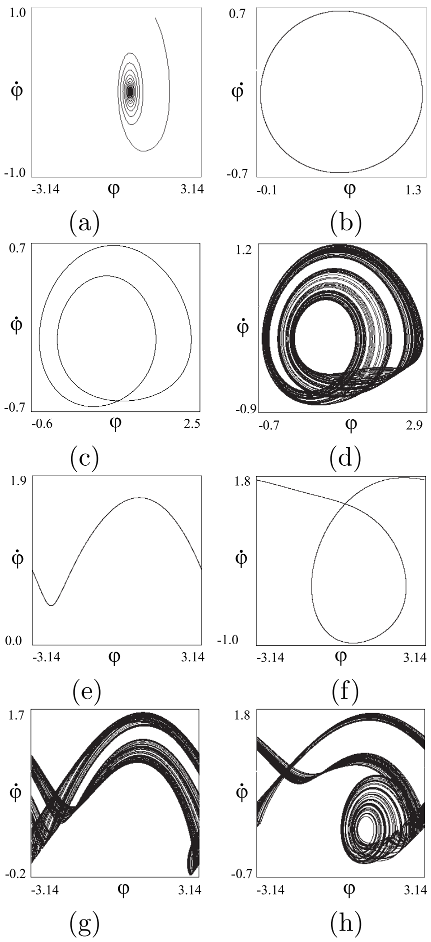

Figure 1: Examples of attractors of system (2). Units are arbitrary.

reference system, and the phase difference of the tuned and reference signals fluctuates around some average value. Regular attractors – limit cycles [Fig. 1(b),(c)] or chaotic oscillatory (with phase difference advance φ less than 2π) attractors [Fig. 1(d)] correspond to this mode in phase space.

– *Regular or chaotic beats*, when the phase difference of the tuned and reference signals grows without restriction, and the frequency difference is non-zero. Rotatory or oscillatory-rotatory (with phase difference advance φ more than 2π) attractors that may be either regular [Fig. 1(e),(f)] or chaotic [Fig. 1(g),(h)] correspond to this mode in phase space.

In the following we address in more detail the mode of quasi-synchronization with a chaotically modulated frequency near the average frequency stabilized with respect to the reference signal. This regime may be regarded as a mode of generation of chaotically modulated oscillations (CMO) at the phase system output, and is the most interesting one in terms of its application for implementing the idea of information transmission by a chaotic carrier.

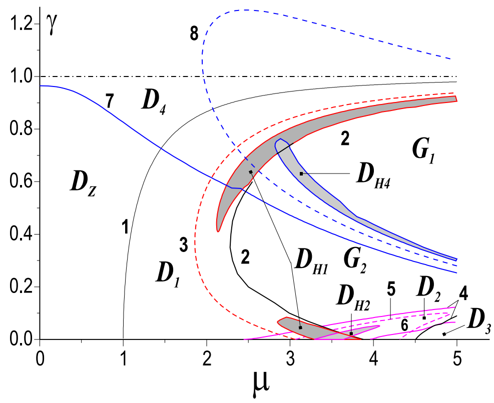

Figure 2: Domains of the existence of chaotic oscillatory attractors of model (2). Units are arbitrary.

Figure 2 illustrates distribution of the dynamic modes of model (2) in the space of the parameters. Figure 2 demonstrates the cross-section (μ, γ) for $\varepsilon = 1$, with domains where the following modes are realized: synchronization mode [Fig. 1(a)] – domain D_Z; regular quasi-synchronization mode [Fig. 1(b),(c)] – domains D_1 and D_2; regular mode of beats determined by rotatory limit cycle [Fig. 1(e)] – domain D_4; CMO mode [Fig. 1(d)] – domains D_{H1} and D_{H2}; chaotic mode of beats determined by rotatory chaotic attractor [Fig. 1(g)] – domain D_{H4}; regular or chaotic modes of beats determined by oscillatory-rotatory, regular [Fig. 1(f)] or chaotic [Fig. 1(h)] attractors – domains G_1 and G_2. Investigations of the CMO mode through motions in the phase space of (2) revealed that the domain of the existence of this mode in the space of parameters is relatively small [Fig. 2], which may impede practical application of such systems as CMO generators. Attempts to expand regions of generation of chaotically modulated oscillations by changing nonlinearity of the phase discriminator, by varying parameters of the filter, or by changing the structure of the local control loop proved to be ineffective. However, the approach based on the use of the properties of collective dynamics of small ensembles of coupled PLL systems seems to be more efficient. Below we present results of the investigation of collective dynamics of a system of two cascade-coupled phase systems with second-order filters in the control loops. We consider, in particular, CMO excitation and study domains of the existence of CMO in the space of the parameters (in the case of cascade coupling the output of the first phase system is the input of the second phase

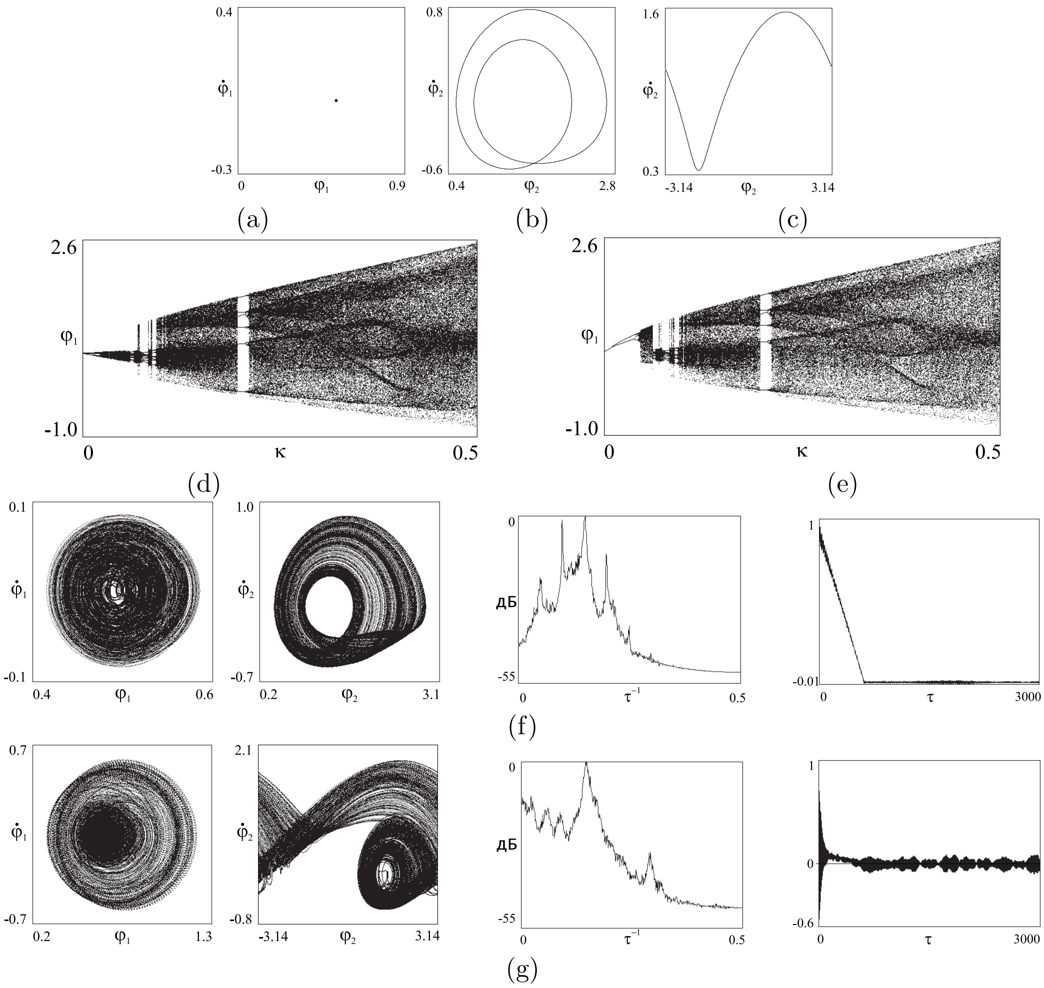

Figure 3: Projections of attractors of model (3) for $\kappa = 0$ characterizing modes of synchronization of the first generator (a), and quasi-synchronization (b) and beats (c) of the second generator; the one-parametric bifurcation diagram of Poincare map illustrating the dependence of the coordinate φ_1 of the trajectory intersection points with the plane $\varphi_2 = \pi/2$ on parameter κ (d,e); projections of the phase portraits of the attractors, power spectra and autocorrelation functions calculated from the time series $\varphi_1(t)$ for $\kappa = 0.04, 0.23$ (f,g), respectively. Units are arbitrary.

system).

3. The dynamics of two cascade-coupled phase systems with additional coupling through error signals considered is described by the following system of equations

$$\mu_1 \dddot{\varphi}_1 + \varepsilon_1 \ddot{\varphi}_1 + \dot{\varphi}_1 + \sin\varphi_1 + \kappa \sin(\varphi_2 - \varphi_1) = \gamma_1, \quad (3)$$
$$\mu_2 \dddot{\varphi}_2 + \varepsilon_2 \ddot{\varphi}_2 + \dot{\varphi}_2 + \sin(\varphi_2 - \varphi_1) = \gamma_2.$$

Here, the phase variables φ_i and parameters $\gamma_i, \varepsilon_i, \mu_i$ are introduced as in equation (2), (i=1, 2). The parameter κ stands for additional coupling.

Equations (3) are defined in the cylindrical phase space $V=\{\varphi_1(\mathrm{mod}2\pi), \dot{\varphi}_1, \ddot{\varphi}_1, \ \varphi_2(\mathrm{mod}2\pi), \dot{\varphi}_2, \ddot{\varphi}_2\}$. Attractors of model (2) characterize collective dynamics of the ensemble. But the behaviour of the generators of the first and second phase systems are determined by the projections of these attractors onto subspaces $V_1 = \{\varphi_1(\mathrm{mod}2\pi), \dot{\varphi}_1, \ddot{\varphi}_1\}$ for the first generator and $V_2 = \{\varphi_2(\mathrm{mod}2\pi), \dot{\varphi}_2, \ddot{\varphi}_2\}$ for the second one. If the attractor projection of model (2) onto subspace V_i contracts into one point, then the i-th generator is in a synchronized regime. If the projection is bounded along the φ_i-coordinate, then the i-th generator is quasi-synchronized, otherwise it is in the mode of beats.

Modelling of system (3) and analysis of the mechanisms of excitation of chaotic oscillations reveal a rich diversity of bifurcation transitions. We find that the chaotic dynamics is not so much influenced by the complex dynamics of each coupled subsystem as by coupling between both the elements. To support this statement, we consider the process of chaotization of oscillations in the two coupled phase systems with second-order filters due to additional coupling in the control loops for the case when both coupled subsystems possess a simple individual dynamics.

Let γ_1=0.5, ε_1=1, μ_1=1, γ_2=0.69, ε_2=1, μ_2=2.37. Then, for $\kappa = 0$, the synchronization mode is realized in the first subsystem [Fig. 3(a)]; while in the second subsystem it may be either a regular mode of quasi-synchronization [Fig. 3(b)] or a regular mode of beats [Fig. 3(c)], depending on the initial conditions. The evolution of the dynamics for increasing κ is illustrated in the one-parametric bifurcation diagram of Poincare map [Fig. 3(d)], when the second generator is in the quasi-synchronization mode. Clearly, the introduction of a weak additional coupling ($\kappa \approx 0.014$) stimulates chaotization of oscillations at the output of the first generator; CMO is also observed at the input of the second generator [Fig. 3(f)]. When the coupling force is increased up to $\kappa = 0.07$, the second generator starts to operate in the mode of chaotic beats, whereas the first generator persists to function in the CMO mode [Fig. 3(g)]. When the coupling force reaches $\kappa = 0.5$, both generators operate in the mode of chaotic beats.

If the mode of beats is taken as the initial state (for κ=0) of the second generator, then arising of CMO at the output of the first generator [Fig. 3(e)] with an increase in κ coincides qualitatively with the scenario of the evolution of chaotic oscillations considered above. Then chaotic oscillations also appear as a result of a cascade of period doubling bifurcations at small $\kappa \approx 0.045$, and the bifurcation diagram $\{\kappa, \varphi_1\}$ changes only slightly in the interval $\kappa \in (0, 0.1)$. It is worth noting that by varying the force of coupling κ it is possible to make the second generator function in the quasi-synchronous mode, even a chaotic one.

Thus, by introducing additional coupling it is possible to excite CMO at the output of the first generator, independent of the mode of operation (quasi-synchronization or beats) of the second generator. This is a very important conclusion because it completely lifts restrictions on the dynamics of the second generator. Moreover, the oscillatory-rotatory attractors corresponding to the mode of beats of the second generator generate at the output of the first generator chaotic oscillations with a broader spectrum than the oscillatory attractors.

4. In this paper we have studied collective dynamics of coupled phase systems. It has been shown that collective dynamics provides rich opportunities for generation of various chaotic oscillations in such systems, chaotically modulated oscillations included. Our most important result is that domains of CMO generation are much larger in coupled phase systems than in an individual phase system. This is promising for creation of new communication systems in which dynamic chaos is used.

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