The effect of higher order hopping integrals on persistent current of a mesoscopic normal metal ring

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***Abstract*-The persistent current of a perfect metal nanoring in the presence of uniform magnetic field has been evaluated. In a perfect ring, the total persistent current versus the magnetic flux shows saw-tooth behavior and is diamagnetic for both even and odd number of electrons. By calculation the difference between persistent currents of various successive order hopping integrals for even or odd number of electrons, we have estimated that the enough number of hopping integrals is intensively depends on the number of the electrons in the ring.**

I. INTRODUCTION

Electronic transport through quantum-ring (QR) structures has been the subject of active research during the last years. QR's, with the capability of trapping magnetic flux in their interiors, are regarded as the ideal candidates for observing Aharonov-Bohm (AB) effects [1], such as energy oscillations, persistent currents [2-4], quantum Hall effects [5], the effects of electron phase coherence [6] and Fano antiresonances [7,8].

Persistent current is an indication of quantum coherence motion of charge carrier in the ring and varies periodically with magnetic flux. On the theoretical side, Cheung et al. [9] calculated the persistent currents and energy levels of the electron in a one-dimensional ring. Such currents are observed in many experiments, however their magnitudes are much larger than those predicted theoretically [2,10].

There are so many experimental results regarding the persistent current in these one-dimensional loops which cannot be explained clearly by theoretical papers even today [11,12,13].

The currents measured in metallic and semiconducting rings, either in a single ring or an array of many rings, generally exhibit an unexpectedly large amplitude, i.e., larger by at least one order of magnitude, than predicted by theoretical studies of electron models with either disorder or electron-electron interaction treated perturbatively [14].

As a matter of fact, QR structures could lead to the development of novel devices in the fields of quantum cryptography, quantum computation [15-18], optics, quantum information processing, optoelectronics, micro-electronics such as field-effect transistor [19], magnetic data-storage media [20], photonics and spintronic materials [21].

Here, we have achieved an analytic approach based on the tight-binding model to investigate the electronic transport properties of mesoscopic rings.

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II. THEORETICAL MODEL

The Hamiltonian describing a system of spinless electrons on a ring with N sites pierced by a magnetic flux Φ (see Fig.1), is given by,

$$H = \sum_i \varepsilon_i c_i^+ c_i + \sum_{i \neq j} v_{ij} [e^{i\theta_{ij}} c_i^+ c_j + h.c.] \tag{1}$$

where ε_i is the on site energy, c_i^+ (c_i) is the creation (annihilation) operator on site i, the hopping integral between any two sites i and j is $v_{ij} = v\, exp(\alpha\,(1-|i-j|))$ where v is the hopping strength between nearest sites. The physical domain of the parameter α can be determined from the boundedness of the energy spectrum.

With enhancement of α, the effects of higher order hopping integrals in the Hamiltonian decreases and we can neglect them in calculation of physical properties.

In our calculation, the parameter α has been considered a small value to modify the energy spectra and the persistent current of the ring due to the effects of higher order hopping integrals.

The Peierls phase factor is $e^{i\theta_{ij}}$ that describes the orbital response of the system to an external magnetic field:

$$\theta_{ij} = \frac{2\pi}{\Phi_0} \int_{R_j}^{R_i} A.dl = \frac{2\pi\Phi\,|i-j|}{\Phi_0 N} \tag{2}$$

where A is the vector potential and $\Phi_0 = hc/e$ is the flux quantum. we have used the units $e=h=c=1$ in this paper.

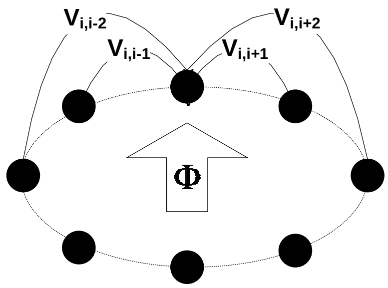

Fig.1. The metal ring threaded by a magnetic flux Φ.

At zero temperature (T=0), the persistent current carried by nth energy level with energy E_n is calculated as

$$I_n(\Phi) = -c\frac{\partial E_n(\Phi)}{\partial \Phi} \quad (3)$$

An important condition for I_n to be nonzero is that the wave functions of the charge carriers should stay coherent along the circumference L of the ring. The persistent current defined above is not a transport current, but rather an equilibrium property of the ring.

For a perfect ring, we have considered $\varepsilon_i = 0$ for all sites in the equation (1). By utilizing the boundary condition in the form $\psi_n(x+L) = \exp(2\pi i\frac{\Phi}{\Phi_0})\psi_n(x)$, the energy of the nth eigenstate can be achieved as

$$E_n(\Phi) = \sum_{p=1}^{p_0} 2v\exp[\alpha(1-p)]\cos[\frac{2\pi p}{N}(n+\Phi)] \quad (4)$$

And the persistent current related to this energy level is

$$I_n(\Phi) = (\frac{4\pi v}{N})\sum_{p=1}^{p_0} p\exp[\alpha(1-p)]\times\sin[\frac{2\pi p}{N}(n+\Phi)] \quad (5)$$

where p is an integer. In equations (4) and (5), $p_0 = 1,2,3,\ldots$ for the ring with NNH (nearest-neighbor hopping), SNH (second-neighbor hopping), TNH (third-neighbor hopping) and other successive higher order hopping integrals, respectively.

The total persistent current is given by

$$I(\Phi) = \sum_n I_n(\Phi) \quad (6)$$

The physical domain of n is $-[N_e/2] \le n < [N_e/2]$ and N_e is the number of electrons.

We have presented the diagrams of the total persistent current versus the magnetic flux for even (odd) number of electrons that show saw-tooth behavior and are diamagnetic for both even and odd number of electrons, in consequence of it's negative slope (see Fig. 2). The current is an odd periodic function of Φ with period Φ_0.

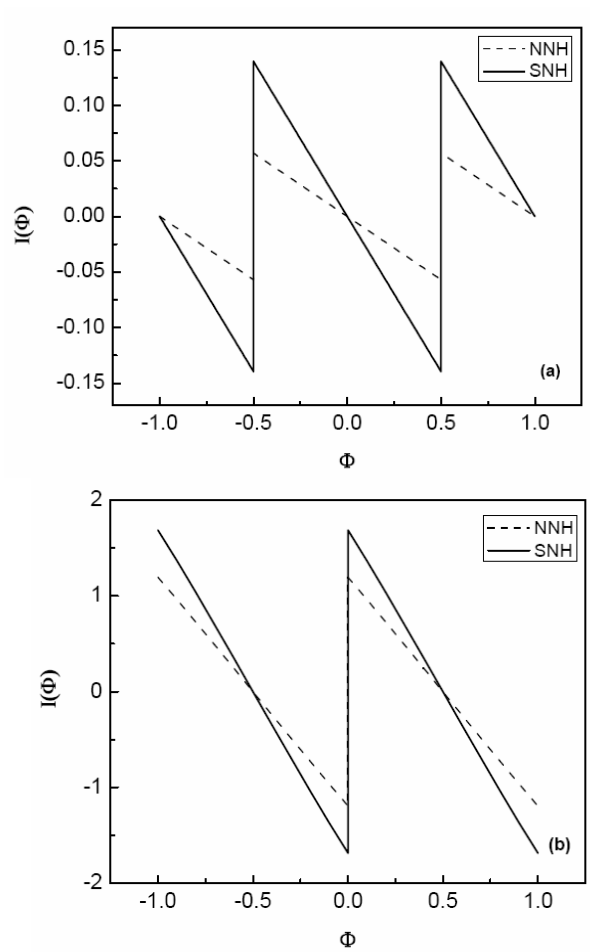

Fig. 2. Persistent current vs. magnetic flux for a perfect ring with N=100, α =0.9, v=-1. (a) N_e=15 and (b) N_e= 20. The dashed and solid lines are for a ring with NNH and SNH, respectively.

The sharp transitions at $\Phi=\pm n\Phi_0/2$ with odd N_e (Fig. 2(a)) and at $\Phi = 0$ or $\Phi=\pm n\Phi_0$ with even N_e (Fig. 2(b)), for the perfect ring appear due to the degeneracy of the energy levels at these respective flux.

In the absence of the impurities and electron-electron interactions, the amplitude of persistent current has its maximum magnitude, since the bloch wave functions are not scattered.

We have also obtained the difference between persistent currents of various successive order hopping integrals versus the magnetic flux for even or odd number of electrons that is shown in Fig. 3.

Whenever this difference approximately approaches to zero, we can neglect the higher order hopping integrals in calculation of the physical transport properties of the system.

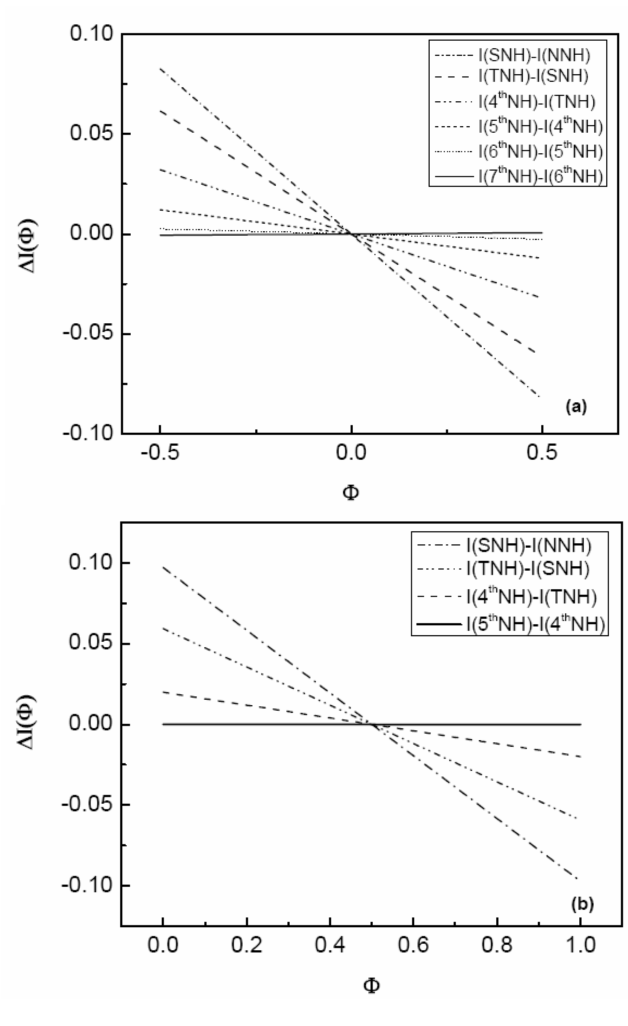

Fig. 3. Difference between persistent currents of various successive order hopping integrals vs. magnetic flux for a perfect ring with N=100, α=0.9, v=-1. (a) N_e=15 and (b) N_e=20.

As a result, the enough number of hopping integrals is intensively depends on the number of the electrons in the ring.

III. Conclusion

We have investigated the persistent current in a perfect mesoscopic metal ring. At an integer (N_e odd) or half-integer (N_e even) flux quantum, the energy is maximum or minimum; hence, at these values of Φ, the current is zero.

Transitions of current versus the magnetic flux are due to the degeneracy of the energy levels at their respective flux for even and odd number of electrons.

With considering higher order hopping integrals, the energy spectra and the persistent current are modified and the amplitude of the current increases.

The results of the diagrams show that for a constant number of sites, the enough number of hopping integrals in calculation of the persistent current varies with the number of electrons.

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