PHYSCON 2009, Catania, Italy, September, 1–September, 4 2009

CONTROL SYSTEM FOR SYNCHRONIZATION OF GENERALIZED CHUA'S CIRCUITS IN FPGA

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Abstract

In this study, feedback control system for synchronization of Generalized Chua's circuits (GCC) has been implemented on Field Programmable Gate Array (FPGA). The feedback control rule has been derived by feedback linearization method. In order to implement the designed synchronized system, *Matlab Simulink* design for the GCCs has been translated to *Xilinx System Generator* design to generate a *Very-High-Speed Integrated Circuits Hardware Description Language* (VHDL) code which is used to produce a bitstream file. By *Xilinx Integrated Software Environment* (ISE) program, a VHDL code is converted to a bitstream file which has been embedded into FPGA by *Field Upgradeable Systems Environment* (FUSE). Finally, the synchronized GCCs states and attractor have been observed on the HP 54540C oscilloscope.

Key words

Generalized Chua's circuit, Synchronization, FPGA.

1 Introduction

Synchronization of chaos is an important topic in the nonlinear science. There are various notions of chaos synchronizations such as generalized synchronization [Afraimovich, Verichev and Rabinovich 1987], complete synchronization [Pecora and Carrol 1991; Femat and Solis-Perales 2008], partial synchronization [Maistrenko and Popovych 2000] and phase synchronization [Rosenblum, Pikovsky and Kurths 1997]. The pioneering work [Pecora and Carrol 1991], has increased the interest in synchronization after having recently found many applications particularly in telecommunications [Abel and Schwarz 2002], in mechanical systems [Blekhman, Landa and Rosenblum] and in control theory [Nijmeijer 2001]. Some different forms of synchronization of chaotic systems such as practical synchronization and almost synchronization have been studied by [Femat and Solis-Perales 1999].

The paper is organized as follows: In Section 2 the complete synchronization problem and the feedback linearization method are explained based on the literature [Vidyasagar 1993; Fradkov 2007]. In Section 3, the control command for complete synchronization of GCCs have been derived. In Section 4, this control system is simulated by *Matlab Simulink* then the simulated design is converted to *Xilinx System Generator* design and the designed synchronized GCCs is implemented by FPGA by using ISE and FUSE programs. To the best of our knowledge, although FPGA implementation of chaotic circuits exist in the literature [Sobhy, Elkouny, Aseeri and Zakria 2003; Wang 2008], the implementation of synchronized chaotic system by FPGA is given as a first time by this manuscript. The implementation results of the GCCs have been observed on the HP54540 scope. Finally, in Section 5, conclusions are presented.

2 Complete Synchronization

In this study, the complete synchronization problem will be considered as the tracking of the master system trajectories by the slave system trajectories. The difference between master and slave system is called as the error system which can be constructed using the definition given below.

Definition 2.1: Let $\dot{x} = \mathbf{F}_M(x)$ and $\dot{y} = \mathbf{F}_S(y) + g(y)u(y)$ be two chaotic systems in a manifold $\mathbf{M} \subset \mathrm{R}^n$. $\mathbf{F}_M$, $\mathbf{F}_S$ smooth vector fields with scalar output functions $s_M = h(x)$, $s_S = h(y)$ and $x,\, y \in \mathrm{R}^n$ and $g(y) \in \mathrm{R}^n$ is a smooth input vector [Femat and Solis-Perales 2008] where subscripts M and S stands for master and slave, respectively.

$$\dot{x} = \mathbf{F}_M(x), \tag{1}$$

$$\dot{e} = \mathbf{F}_M(x) - \mathbf{F}_S(x,e) - g(x,e)u, \tag{2}$$

$$s_e = h(x,e), \tag{3}$$

where the error is defined as $e = x - y$ and s_e is the output of the synchronization error system then the extended synchronization error system can be defined in affine form as:

$$\dot{\mathbf{X}} = \mathbf{F}(X) + \mathbf{G}(X)u, \tag{4}$$

where $\mathbf{X} = [x, e]^T$, $\mathbf{F}(X) = [\mathbf{F}_M, \mathbf{F}_M - \mathbf{F}_S]^T$ and $\mathbf{G}(X) = [\mathbf{0}, -g(x, e)]^T$.

The complete synchronization is achieved if and only if all the states of both the master and the slave systems are exactly synchronized [Femat and Solis-Perales 2008]. In order to obtain complete synchronization, the synchronization error system in Eq. (2) should be stabilized around the point $e^* = 0$. The definitions and the theorems given in the following sequel will be used to find the proper invertible transformation which will be used to derive the control command for synchronization.

Definition 2.2: System (4) is said to have relative degree ρ, $\rho \leq n$ at point $x_0 \in \mathrm{R}^n$ with respect to the output

$$s_e = h(x)$$

if for any $x \in \Omega$, where Ω is some neighborhood of x_0, the following conditions are valid

(i)$L_G L_F^k h(x) = 0$, $k = 0, 1, ...\rho - 2$, $\forall x$ in a neighborhood of x_0 and $k < \rho - 1$,
(ii)$L_G L_F^{\rho-1} h(x_0) \neq 0$.

where $L_\psi \phi(x) \triangleq \sum_{i=1}^{n} \frac{\partial \phi}{\partial x_i} \psi(x)$ stands for the Lie derivative of the vector function ϕ along the vector field ψ. Relative degree ρ is exactly equal to the number of times one has to differentiate the output in order to have the input explicitly appearing in the equation which describes the evolution of $s_e^{(r)}(t)$ in the neighborhood of x_0 [Fradkov 2007; Vidyasagar 1993].

Theorem 2.3: System (4) is feedback linearizable in the neighborhood Ω of a point $x_0 \in \mathrm{R}^n$ if and only if there exists a smooth scalar function $h(x)$ defined in Ω such that the relative degree ρ of (3) and (4) is equal to n [Fradkov 2007].

Theorem 2.4: Consider the system (4). Suppose that there exist $2n - \rho$ functions $\Phi_i(x, e)$ such that $L_G \Phi_i(x, e) = 0$, $i = \rho + 1, ..., 2n$. This system is feedback linearizable at $(x, 0)$ if and only if there exists a function $h(x, e)$ such that
(i) $< \partial h, ad_F^{k-1}\mathbf{G} > (x, e) = 0$ for $k = 1, ..., \rho - 1; \rho > 1$ and (x, e) in a neighborhood Ω of $(x, 0)$,
(ii) $< \partial h, ad_F^i \mathbf{G} > (x, 0) \neq 0$ for $i = \rho, ..., n$ at $(x, 0)$, where $\rho = d$ stands for the dimension of the tangent space and the accessibility distribution function $\mathbf{C}(x, e)$ can be expressed as $\mathbf{C}_d = span\{ad_F^{d-1}\mathbf{G}\}$ where $ad_F = [\mathbf{F}, \mathbf{G}]$ and $ad_F^{d-1} = [\mathbf{F}, [\mathbf{F}, [..., [\mathbf{F}, \mathbf{G}], ...]]]$ for $d = 1, ..., n$ where $[\mathbf{F}, \mathbf{G}]$ is called the Lie bracket of $\mathbf{F}$ and $\mathbf{G}$ [Femat and Solis-Perales 2008].

Corollary 2.5: Two chaotic systems with the same order are completely synchronizable if and only if the dynamical error system is feedback linearizable at $(x, 0)$ [Femat and Solis-Perales 2008].

Feedback linearization problem: The system in (4) is called feedback linearizable if there exist a smooth reversible change of coordinates $z = \Phi(x, e)$ and smooth transformation of the feedback [Vidyasagar 1993; Andrievskii and Fradkov 2003].

$$u = \lambda(x, e) + \mu(x, e)v, \tag{5}$$

where $v \in \mathrm{R}^m$ is the new control if the closed-loop is linear and then the resulting variables z and v satisfy linear dynamical system in the form of

$$\dot{z} = Az + bv \tag{6}$$

$$z = \Phi(x, e) = [h(x, e), L_F h(x, e), ..., L_F^{\rho-1} h(x, e)]^T$$
$$u = \frac{1}{L_G L_F^{\rho-1} h(x, e)}(-L_F^\rho h(x, e) + v) \tag{7}$$

$$\lambda(x, e) = \frac{-L_F^\rho h(x, e)}{L_G L_F^{\rho-1} h(x, e)}$$
$$\mu(x, e) = \frac{1}{L_G L_F^{\rho-1} h(x, e)}$$
$$\nu = \mathrm{K}_i(z_i - z_i^*)$$

where K_i with $i = 1, ..., \rho$ are the control gains and chosen in such a way that the closed-loop subsystem $\dot{z}$ converges to the origin and z_i^*'s are the coordinates of the stabilization point. In order to achieve complete synchronization z_i^*'s are set to zero.

3 Synchronization of Generalized Chua's circuits

In this manuscript master system has been chosen as the GCC [Suykens and Vandewalle 1997] which is described by

$$\begin{aligned} \dot{x_1} &= \alpha[x_2 - f(x_1)] \\ \dot{x_2} &= x_1 - x_2 + x_3 \\ \dot{x_3} &= -\beta x_2 \end{aligned} \tag{8}$$

where

$$f(x_1) = m_{2q-1}x_1 + \frac{1}{2}\sum_{i=1}^{2q-1}(m_{i-1} - m_i)(|x_1 + c_i| - |x_1 - c_i|)$$

In order to obtain 7-scroll attractor in the GCC, the parameters have been chosen [Yalcin, Suykens and Vandewalle 2005] as below:

$$\begin{aligned} \alpha &= 9 \\ \beta &= 100/7 \\ q &= 3 \\ c &= [1 \quad 2.15 \quad 3.6 \quad 6.2 \quad 9 \quad 14 \quad 25] \end{aligned}$$

$$\begin{aligned} m = [0.9/7 \quad -3/7 \quad 3.5/7 \quad -2.4/7 \\ 2.52/7 \quad -1.68/7 \quad 2.52/7 \quad -1.68/7] \end{aligned}$$

Driving the master system by control input u and changing the parameter β to γ in Eq. (8) then describing equations for the slave system can be written as:

$$\begin{aligned} \dot{y_1} &= \alpha[y_2 - f(y_1)] + g_1(y)u \\ \dot{y_2} &= y_1 - y_2 + y_3 + g_2(y)u \\ \dot{y_3} &= -\gamma y_2 + g_3(y)u \end{aligned} \tag{9}$$

and the error dynamics in Eq. (2) is written as:

$$\begin{aligned} \dot{e_1} &= \alpha[e_2 - f(e_1)] - g_1(x,e)u \\ \dot{e_2} &= e_1 - e_2 + e_3 - g_2(x,e)u \\ \dot{e_3} &= -\beta x_2 + \gamma(x_2 - e_2) - g_3(x,e)u \end{aligned} \tag{10}$$

then, the extended synchronization error system dynamics in Eqs. (1), (2) for GCCs can be obtained as:

$$\begin{aligned} \dot{x_1} &= \alpha[x_2 - f(x_1)] \\ \dot{x_2} &= x_1 - x_2 + x_3 \\ \dot{x_3} &= -\beta x_2 \\ \dot{e_1} &= \alpha[e_2 - f(e_1)] - g_1(x,e)u \\ \dot{e_2} &= e_1 - e_2 + e_3 - g_2(x,e)u \\ \dot{e_3} &= -\beta x_2 + \gamma(x_2 - e_2) - g_3(x,e)u \end{aligned} \tag{11}$$

$\mathbf{F}(X)$ and $\mathbf{G}(X)$ can be found as in Eq. (4)

$$\mathbf{F} = \begin{bmatrix} \alpha[x_2 - f(x_1)] \\ x_1 - x_2 + x_3 \\ -\beta x_2 \\ \alpha[e_2 - f(e_1)] \\ e_1 - e_2 + e_3 \\ (\gamma - \beta)x_2 + -\gamma e_2 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g_1(x,e) \\ -g_2(x,e) \\ -g_3(x,e) \end{bmatrix} \tag{12}$$

The corresponding accessibility distribution function $\mathbf{C}_3(x,e)$ can be calculated where $\mathbf{C}_3(x,e) = span\{\mathbf{G}, ad_F\mathbf{G}, ad_F^2\mathbf{G}\}$. Let $\mathbf{G}$ be defined as $\mathbf{G} = [0,\ 0,\ 0,\ -g_1,\ -g_2,\ -g_3]^T$ with g_1, g_2 and g_3 constants, then $ad_F\mathbf{G}$ and $ad_F^2\mathbf{G}$ will be calculated to obtain $\mathbf{C}_3(x,e)$.

$$\mathbf{C}_3(x,e) = span\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -g_1 & -g_1\alpha f(\dot{e_1}) + g_2\alpha & a^* \\ -g_2 & g_1 - g_2 + g_3 & b^* \\ -g_3 & -\gamma g_2 & c^* \end{bmatrix}\right\} \tag{13}$$

$$\begin{aligned} a^* &= \alpha f(\dot{e_1})[g_2\alpha - g_1\alpha f(\dot{e_1})] + \alpha(g_2 - g_1 - g_3) \\ b^* &= g_1[\alpha f(\dot{e_1}) + 1] + g_2(\gamma - \alpha - 1) + g_3 \\ c^* &= \gamma(g_1 - g_2 + g_3) \end{aligned}$$

For simplicity by setting $g_1 = 0, g_2 = 0$ and $g_3 = 1$ then $C_3(x,0)$ can be written as in Eq. 14.

$$\mathbf{C}_3(x,0) = span\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\alpha \\ 0 & 1 & 1 \\ -1 & 0 & \gamma \end{bmatrix}\right\} \tag{14}$$

In order to derive the control command, we need to determine the dimension of the tangent space d which is generated by the corresponding distribution so the conditions of *Theorem 2.4* must be satisfied. The dimension of the tangent space d is determined to be equal to 3 since the conditions below have been satisfied.

$$\begin{aligned} -\frac{\partial h}{\partial e_3} &= 0 \\ \frac{\partial h}{\partial e_2} &= 0 \\ -\alpha\frac{\partial h}{\partial e_1} + \frac{\partial h}{\partial e_2} + \gamma\frac{\partial h}{\partial e_2} &\neq 0 \end{aligned}$$

For this case $h(x,e) = e_1$ can be chosen as an output function which satisfies conditions of *Theorem 2.4* then $d = \rho = Dim(\mathbf{C}_3(x,0)) = 3, \forall x \in \mathrm{R}^3$ and considering *Theorem 2.3*, to have feedback linearizable system for $\rho = 3$, the conditions below must be fulfilled.

$$
\begin{aligned}
L_G h(x,e) &= 0 \\
L_G L_F h(x,e) &= 0 \\
L_G L_F^2 h(x,e) &\neq 0
\end{aligned}
$$

then the transformation can be found as

$$
\begin{aligned}
z_1 &= h(x,e) = e_1 \\
z_2 &= L_F h(x,e) = \dot{e_1} = \alpha[e_2 - f(e_1)] \qquad (15) \\
z_3 &= L_F^2 h(x,e) = \ddot{e_1} = \alpha[e_1 - e_2 + e_3 - f(\dot{e_1})]
\end{aligned}
$$

The complementary functions which should satisfy such that $L_G \Phi_i(x,e) = 0, i = 2n - \rho, ...2n$ can be found as

$$
\begin{aligned}
z_4 &= x_1 \\
z_5 &= x_2 \qquad (16) \\
z_6 &= x_3
\end{aligned}
$$

Eqs. (16) and (17) constitute an invertible transformation around $(x,0)$ which means there is a proper control command $u(x,e)$ such that the slave system trajectory tracks master system trajectory exactly. The transformed system can then be written as follows:

$$
\begin{aligned}
\dot{z_1} &= z_2 \\
\dot{z_2} &= z_3 \\
\dot{z_3} &= L_F^3 h(x,e) + L_G L_F^2 u(x,e) \\
\dot{z_4} &= \dot{x_1} \\
\dot{z_5} &= \dot{x_2} \\
\dot{z_6} &= \dot{x_3}
\end{aligned}
$$

where the control command in Eq. (7) can be found as:

$$u(x,e) = \frac{1}{L_G L_F^2 h(x,e)}(-L_F^3 h(x,e) + \kappa_1 z_1 + \kappa_2 z_2 + \kappa_3 z_3)$$

and hence based on Eq. (15) and *Definition 2.2* the control command can be obtained as:

$$u(x,e) = \alpha[e_2 - f(e_1)] - (e_1 - e_2 + e_3) + (\gamma - \beta x_2 - \gamma e_2 - f(\ddot{e_1}) - \frac{1}{\alpha}\{\kappa_1 e_1 + \kappa_2 \alpha[e_2 - f(e_1)] + \kappa_3 \alpha[e_1 - e_2 + e_3 - f(\dot{e_1})]\}$$

$\kappa_1 = -50$, $\kappa_2 = -25$ and $\kappa_3 = -20$ have been chosen to make the subsystem $(z_1, z_2, z_3)^T$ is stable at $(x,0)$.

4 Simulation and implementation results of the designed synchronized system

After deriving the control command for the GCCs with different initial conditions and different parameters, *Matlab Simulink* has been used to simulate proposed system. The GCC master and the slave system block and the designed control system is built with *Simulink* blocks.

The simulation results of the waveforms $x_1(t)$ and $y_1(t)$; $x_2(t)$ and $y_2(t)$; $x_3(t)$ and $y_3(t)$ can be seen in Fig. (1). In the same figure, the error signals $e_1(t)$, $e_2(t)$ and $e_3(t)$ approaching to zero after some short transient can be seen.

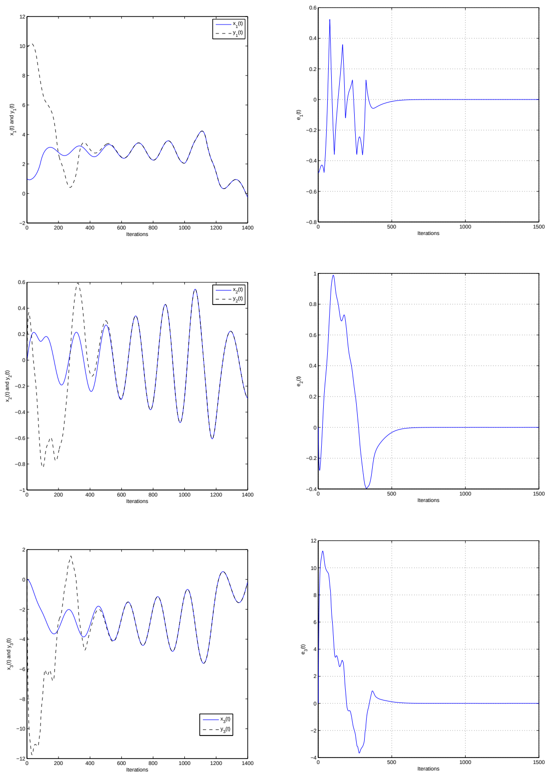

Figure 1. *Simulink* - 7-scroll Chua's Master and Slave System state variables $(x_1(t), y_1(t))$, $(x_2(t), y_2(t))$, $(x_3(t), y_3(t))$ and $e_1(t)$, $e_2(t)$, $e_3(t)$, respectively.

After simulating the designed synchronized GCCs system, in order to generate a VHDL code, *Xilinx System Generator* blocks can be used. *System Generator* is a very useful tool since *Simulink* design is easily converted to *System Generator* blocks and it works under *Matlab* as a toolbox which provides re-simulation of the designed synchronized for generating a VHDL code, and the new results are bit and cycle accurate. *System Generator* blocks representation of the GCC master system can be seen in Fig. (2).

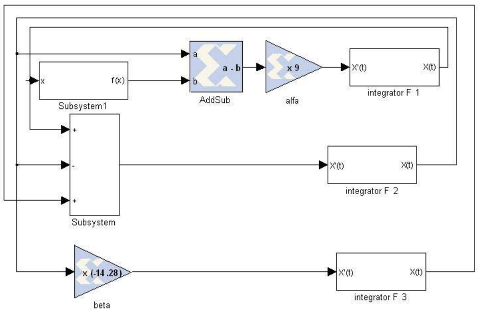

Figure 2. 7-scroll Chua's Master System with *System Generator* Blocks.

System Generator simulation results can be seen in Figs. (3) and (4). Since these results are the same with Simulink results hence the conversion operation is achieved successfully. After assigning proper pinout specifications for *XtremeDSP Development Kit for Virtex-4* and defining clock period then the VHDL code has been generated.

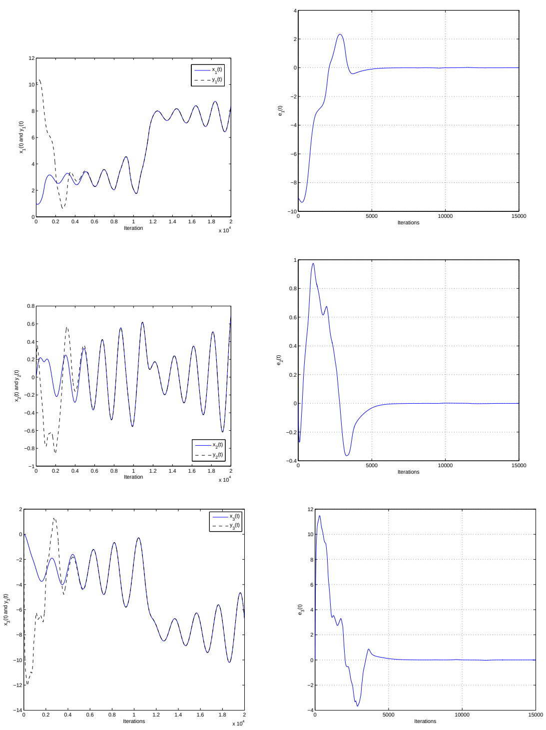

Figure 3. *System Generator* - 7-scroll Chua's Master and Slave System state variables $(x_1(t), y_1(t))$, $(x_2(t), y_2(t))$, $(x_3(t), y_3(t))$ and $e_1(t), e_2(t), e_3(t)$, respectively.

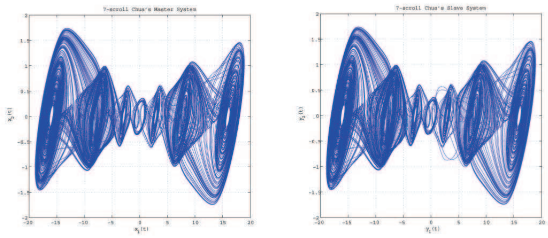

Figure 4. 7-scroll Chua's Master and Slave System Generators with *Simulink* and *System Generator* Blocks, respectively.

Logic Utilization	**Used**	**Available**	**Utilization**
Number of Slice Flip Flops	1,944	30,720	% 6
Number of 4 input LUTs	7,033	30,720	% 22
Number of occupied Slices	5,412	15,360	% 35
Number of DSP48s	189	192	% 98

Table 1. Device Utilization Summary

After generating a VHDL code, ISE program has been used to produce bitstream file. ISE program provides the report of device utilization to detect whether designed system is realizable, as can be seen in Table I. Synchronized 7-scroll Chua's circuits and the controller can be implemented on a *XtremeDSP Development Kit for Virtex-4* device as our device utilization constraints are fulfilled.

Bitstream file has been obtained with ISE then to configure *XtremeDSP Development Kit for Virtex-4* board FUSE program is used. After configuring FPGA, the implementation results have been observed on the scope as can be seen in Figure 5 and 6. These results exactly match with the results obtained with *Simulink* and *System Generator*, which show the successful implementation of synchronization of the GCCs.

5 Conclusion

The control system, the GCC master system and the GCC slave system have been embedded in FPGA. The control command has been designed to synchronize the GCC master and the slave system by feedback linearization method. Since FPGAs are digital circuits the robust nonlinear control command can thus be obtained. As a future work a chaotic communication system using GCC will be embedded in FPGA

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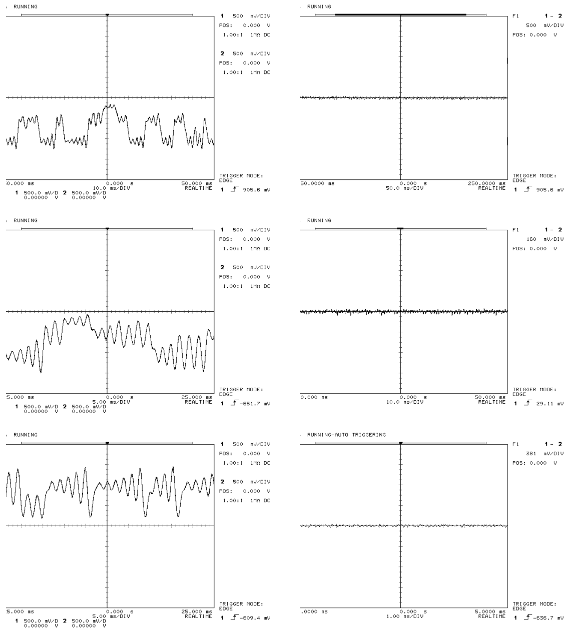

Figure 5. *The trajectories observed on the HP54540 scope* - The left column of the figures represents x_1 on y_1, x_2 on y_2 and x_3 on y_3; the right column of the figures represents corresponding error signals which are e_1, e_2 and e_3, respectively.

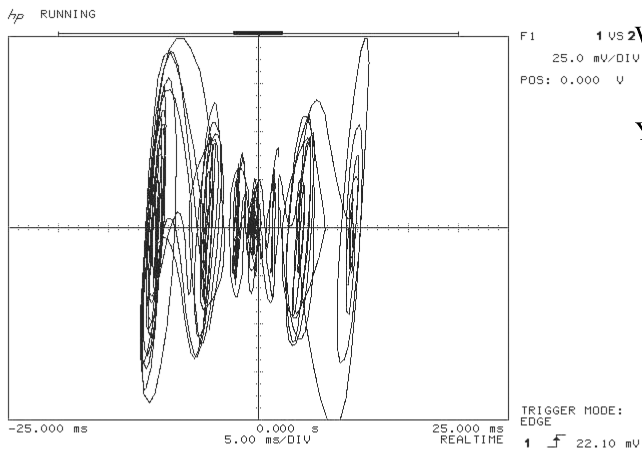

Figure 6. x_1 versus x_2 of 7-scroll Chua's system on the HP54540C scope.

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