LINEARIZATION AND OPTIMAL CONTROL FLOW IN ELECTROMAGNETIC BALL SUSPENSION SYSTEM

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Abstract
The electromagnetic ball suspension systems (EMBSS) have many applications especially in transport for high speed (Magnetic Levitation Trains) [Yadav et al., 2013]. It is impressive to observe a ball suspended in air without any support and while attracting many researchers. The aim of this research is to investigate the controllability and observability [Sontag, 2013] of the system and stability of the solution of the system. We perform a technique of optimization, known as linear quadratic regulator (LQR) [Basile and Marro, 1992] to find the optimal control input of the system. It is found that the EMBSS is both controllable and observable. Stability is guaranteed under certain conditions. The optimal control for EMBSS is performed adequately on both linearised models.

Key words
Controllability, Observability, Stability, Linearization and Optimal Control.

1 Introduction

This work is devoted to present the mathematical formulations in Control Theory. More emphasis is put on the modelling of EMBSS. Two models are considered, i.e. **model one** is when the input of the system is designed to be the voltage and **model two** is when the input of the system is designed to be the current. Ranks of two special matrices are considered (the controllability matrix and the observability matrix) to check if the above-mentioned systems are controllable or observable. Also the optimal control is performed so that the required input can be chosen for the desired output of the EMBSS; which gives the desired position of the ball from the end of the electromagnet.

2 Model Formulation

The EMBSS is a mechanism consisting of electromagnet and a steel ball m as it is shown in Figures 1 and 2. The system functions by regulating the current in electromagnet such that the steel ball of mass m is suspended at a fixed distance, y_0, from the end of electromagnet [Jayawant, 1982]. Two models are built, one model having the voltage as an input and the other with the current as an input. We shall use the following variables and parameters: I_2: the variable current through the electromagnet, R_2: the resistor of the circuit, L: the inductor of the electromagnet, y: the variable position of the ball, m: the mass of the ball, k: the spring constant, β: the damping coefficient due to air resistance and any other disturbance, C: the capacitor, I: the current input of the system, t : the variable time, R: the resistor of the circuit, L: the inductor of the electromagnet and V: the input voltage.

2.1 Model one: EMBSS with Voltage as Input

From Figure 1, Kirchhoffs voltage and current law give the following:

$$\frac{dI}{dt} = -\frac{R}{L}I + \frac{V}{L} \tag{1}$$

Also, from Figure 1, the force F has three components. The first component is the electromagnetic force $F(y, I) = \alpha(\frac{I}{y})^2$ [Suebsomran, 2014], which is the attractive force from the electromagnet and $\alpha = \frac{1}{4}\mu_0 N^2 A$. Here μ_0 is permeability, N is the number of turns of the coil and A is the cross area section of the electromagnet and I is the variable current through the electromagnet. The second component is the air resistance which is assumed to be proportional to the velocity of the steel ball and $F_{air} = -\beta\frac{dy}{dt}$, where β is the proportional coefficient, t is the time and y is the position of the steel ball. The third component is the effect due to the gravitation, $F_g = -mg$ where m is the mass and g is the gravitational acceleration; it is assumed to be $9.81m/sec^2$. Finally, another component is added which serves as a supportive force to the electromagnetic force. This supporting component is chosen to be

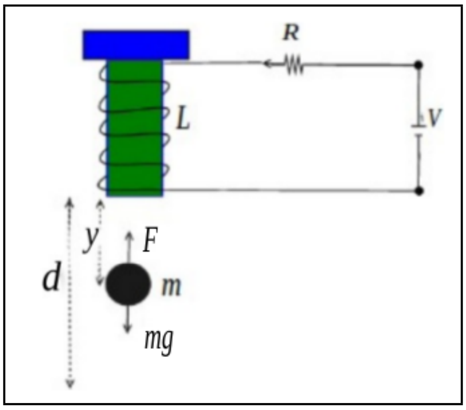

Figure 1. EMBSS with voltage as input.

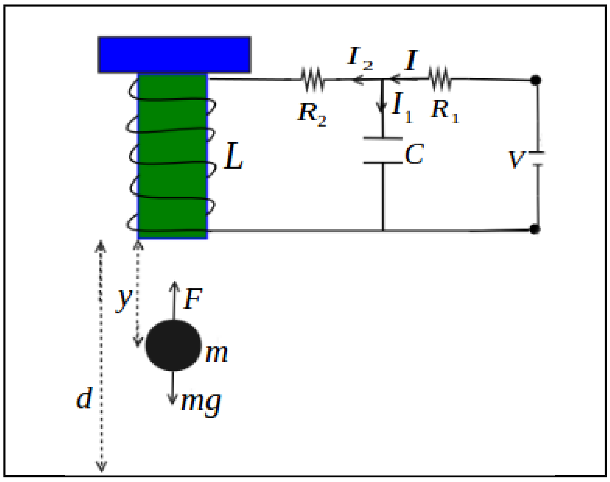

Figure 2. EMBSS with current as input.

the force due to a spring placed between the steel ball and the lower end of electromagnet. Hence, the Hook's law provides $F_s = ky$, where F_s is the force due to the spring elongation, k is spring constant and y is the position attained by the steel ball. Now putting all these forces together provide the following:

$$F = \alpha \left(\frac{I}{y}\right)^2 + ky - \beta \frac{dy}{dt} - mg \qquad (2)$$

Using Newton's laws of motion:

$$\frac{d^2y}{dt^2} = -g + \frac{\alpha}{m}\left(\frac{I}{y}\right)^2 + \frac{k}{m}y - \frac{\beta}{m}\frac{dy}{dt} \qquad (3)$$

The Equations (1) and (3) give the following system:

$$\begin{cases} \frac{dI}{dt} &= -\frac{R}{L}I + \frac{V}{L}, \\ \frac{d^2y}{dt^2} &= -g + \frac{\alpha}{m}\left(\frac{I}{y}\right)^2 + \frac{k}{m}y - \frac{\beta}{m}\frac{dy}{dt}. \end{cases} \qquad (4)$$

The variables domain are as follows:

$$\begin{cases} 0 \le I < \infty, \\ 0 < y < d < \infty, \\ -\infty < \frac{dI}{dt} < \infty, \\ -\infty < \frac{dy}{dt} < \infty. \end{cases} \qquad (5)$$

with $I = x_1$, $y = x_2$ and $\frac{dy}{dt} = x_3$ the system looks like:

$$\begin{cases} \frac{dx_1}{dt} = -\frac{R}{L}x_1 + \frac{V}{L}, \\ \frac{dx_2}{dt} = x_3, \\ \frac{dx_3}{dt} = -g + \frac{\alpha}{m}(\frac{x_1}{x_2})^2 + \frac{k}{m}x_2 - \frac{\beta}{m}x_3, \end{cases} \qquad (6)$$

$$Y(X) = x_2. \qquad (7)$$

The vector X is state of variable vector and it is given by the following equation $X = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$. The output of the system is $Y(X) = x_2$ which is the position of the ball from the electromagnet.

2.2 Model Two: EMBSS with Current as Input

The current acting as the input of the system from the source V is passing through the resistor R_1. Then the current is divided into two parts, one through the capacitor C and the other through the resistor R_2. This is shown in Figure 2. The electromagnet produces the attractive force which is capable to suspend the steel ball of mass m. Kirchhoffs voltage and current law yields

$$\frac{d^2I_2}{dt^2} = -\frac{R_2}{L}\frac{dI_2}{dt} - \frac{I_2}{LC} + \frac{I}{LC}. \qquad (8)$$

As stated before, the force F has three components, so $F = \alpha(\frac{I}{y})^2 + ky - \beta\frac{dy}{dt} - mg$. With Newton's laws of motion,

$$\frac{d^2y}{dt^2} = -g + \frac{\alpha}{m}\left(\frac{I_2}{y}\right)^2 + \frac{k}{m}y - \frac{\beta}{m}\frac{dy}{dt}. \qquad (9)$$

The Equations (9) and (8) give the following system:

$$\begin{cases} \frac{d^2I_2}{dt^2} &= -\frac{R_2}{L}\frac{dI_2}{dt} - \frac{I_2}{LC} + \frac{I}{LC}, \\ \frac{d^2y}{dt^2} &= -g + \frac{\alpha}{m}\left(\frac{I_2}{y}\right)^2 + \frac{k}{m}y - \frac{\beta}{m}\frac{dy}{dt}. \end{cases} \qquad (10)$$

The domain of variables is as follows:

$$\begin{cases} 0 \le I_2 < \infty, \\ 0 < y < d < \infty, \\ -\infty < \frac{dI_2}{dt} < \infty, \\ -\infty < \frac{dy}{dt} < \infty. \end{cases} \qquad (11)$$

with $I_2 = x_1, \frac{dI_2}{dt} = x_2$, $y = x_3$ and $\frac{dy}{dt} = x_4$, the system is:

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -\frac{R_2}{L}x_2 - \frac{x_1}{Lc} + \frac{I}{LC}, \\ \frac{dx_3}{dt} = x_4, \\ \frac{dx_4}{dt} = -g + \frac{\alpha}{m}\left(\frac{x_1}{x_3}\right)^2 + \frac{k}{m}x_3 - \frac{\beta}{m}x_4. \end{cases} \tag{12}$$

$$Y(X) = x_3. \tag{13}$$

The vector X is state of variables vector and it is given by the following equation $X = \begin{pmatrix} x_1 \ x_2 \ x_3 \ x_4 \end{pmatrix}^T$. The output of the system is $Y(X) = x_3$ which is the position of the ball from the electromagnet.

2.3 Linearized Model of EMBSS with Voltage Input

The equilibrium point and Jacobian matrix evaluated at that point linearizes the model one given by the system (6) as follows: Let $\delta x_1 = x_1 - x_{1e}$, $\delta x_2 = x_2 - x_{2e}$, $\delta x_3 = x_3 - x_{3e}$ and $\delta v = v - v_e$, then we have the following:

$$\begin{pmatrix} \frac{d\delta x_1}{dt} \\ \frac{d\delta x_2}{dt} \\ \frac{d\delta x_3}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ \frac{2\,\alpha\sqrt{\frac{gm-kx_{2e}}{\alpha}}}{mx_{2e}} & -\frac{2\,gm-3\,kx_{2e}}{mx_{2e}} & -\frac{\beta}{m} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \end{pmatrix} \delta v. \tag{14}$$

$$\delta x_2 = \begin{pmatrix} 0 \ 1 \ 1 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix} + 0\delta v. \tag{15}$$

The equation (14) is representing the dynamics of the linearised model of EMBSS with the voltage input δv. The equation (15) represents the output of the system δx_2. From the system (14) we identify the following:

$$A = \begin{pmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ \frac{2\,\alpha\sqrt{\frac{gm-kx_{2e}}{\alpha}}}{mx_{2e}} & -\frac{2\,gm-3\,kx_{2e}}{mx_{2e}} & -\frac{\beta}{m} \end{pmatrix}, \tag{16}$$

$$B = \begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 \ 1 \ 0 \end{pmatrix} \quad \text{and} \quad D = 0. \tag{17}$$

2.3.1 Stability Analysis of the Solution for Linearized Model of EMBSS with Voltage as Input

This section is investigating the stability of the equilibrium point. Different cases that are going to be discussed here are the following: Case one: when $\beta = 0$ and $k = 0$, Case two: when $\beta = 0$ and $k \neq 0$, Case three: when $\beta \neq 0$ and $k = 0$, Case four: when $\beta \neq 0$ and $k \neq 0$. The equilibrium point of the System (6) is stable if all eigenvalues of the Jacobian matrix A evaluated at this point have negative real parts. These eigenvalues of the matrix A from equation (14), are given by the following equations:

$$\lambda_1 = -\frac{\beta x_{2e} + \sqrt{-8\,gm^2x_{2e} + (\beta^2 + 12\,km)x_{2e}^2}}{2\,mx_{2e}}, \tag{18}$$

$$\lambda_2 = -\frac{\beta x_{2e} - \sqrt{-8\,gm^2x_{2e} + (\beta^2 + 12\,km)x_{2e}^2}}{2\,mx_{2e}}, \tag{19}$$

$$\lambda_3 = -\frac{R}{L}. \tag{20}$$

The Equation (20) shows that the real part of the eigenvalue λ_3 is negative. But it is not clear whether to draw a conclusion for eigenvalues λ_1 and λ_2 given by equations (18) and (19) respectively. It needs to be used case by case basis, Table 1 forms the conclusions on the stability of this equilibrium point.

2.4 Linearized Model of EMBSS with Current Input

The equilibrium point and Jacobian matrix evaluated at that point linearizes the **model one** provided by the system (12) as follows: Let $\delta x_1 = x_1 - x_{1e}$, $\delta x_2 = x_2 - x_{2e}$, $\delta x_3 = x_3 - x_{3e}$, $\delta x_4 = x_4 - x_{4e}$ and $\delta v = v - v_e$, then one has the following:

$$\begin{pmatrix} \frac{d\delta x_1}{dt} \\ \frac{d\delta x_2}{dt} \\ \frac{d\delta x_3}{dt} \\ \frac{d\delta x_4}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{1}{CL} & -\frac{R_2}{L} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2\,\alpha\sqrt{\frac{gm-kx_{3e}}{\alpha}}}{mx_{3e}} & 0 & -\frac{2\,gm-3\,kx_{3e}}{mx_{3e}} & -\frac{\beta}{m} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{LC} \\ 0 \\ 0 \end{pmatrix} \delta v. \tag{21}$$

$$\delta x_3 = \begin{pmatrix} 0 \ 0 \ 1 \ 0 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{pmatrix} + 0\delta I. \tag{22}$$

The equation (21) represents the dynamics of the linearised model of EMBSS with the current input δI. The equation (22) represents the output of the system δx_3. From the system (21) we identify the following:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{1}{CL} & -\frac{R_2}{L} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{2\,\alpha\sqrt{\frac{gm-kx_{3e}}{\alpha}}}{mx_{3e}} & 0 & -\frac{2\,gm-3\,kx_{3e}}{mx_{3e}} & -\frac{\beta}{m} \end{pmatrix} \tag{23}$$

$$B = \begin{pmatrix} 0 \\ \frac{1}{LC} \\ 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 \ 0 \ 1 \ 0 \end{pmatrix} \quad \text{and} \quad D = O. \tag{24}$$

Models	Case one: $k=0, \beta=0$	Case two: $k \neq 0, \beta = 0$
Input voltage	Not stable	Not stable
Input current	Not stable	Not stable

Models	Case three: $k=0, \beta \neq 0$	Case four: $k \neq 0, \beta \neq 0$
Input voltage	Stable	Not stable if $\frac{2gm}{3x_{2e}} \leq k < \frac{gm}{x_{2e}}$.
		Stable if $k < \frac{2gm}{3x_{2e}}$.
Input current	Stable	Not stable if $\frac{2gm}{x_{3e}} \leq k < \frac{gm}{x_{3e}}$.
		Stable if $k < \frac{2gm}{x_{3e}}$.

Table 1. Stability analysis

3 Stability Analysis of the Solution for Linearized Model of EMBSS with Current Input

This subsection is investigating the stability of the equilibrium point. Different cases that are going to be discussed here are the following: Case one: when $\beta = 0$ and $k = 0$, Case two: when $\beta = 0$ and $k \neq 0$, Case three: when $\beta \neq 0$ and $k = 0$ and Case four: when $\beta \neq 0$ and $k \neq 0$. The equilibrium point of the System (12) is stable if all eigenvalues of the Jacobian matrix A evaluated at this point has negative real parts. These eigenvalues of the matrix A from equation (21), are given by the following equations:

$$\lambda_1 = -\frac{CR_2 + \sqrt{C^2R_2^2 - 4\,CL}}{2\,CL}, \tag{25}$$

$$\lambda_2 = -\frac{CR_2 - \sqrt{C^2R_2^2 - 4\,CL}}{2\,CL}, \tag{26}$$

$$\lambda_3 = -\frac{\beta x_{3e} + \sqrt{-8\,gm^2x_{3e} + (\beta^2 + 12\,km)x_{3e}^2}}{2\,mx_{3e}}, \tag{27}$$

$$\lambda_4 = -\frac{\beta x_{3e} - \sqrt{-8\,gm^2x_{3e} + (\beta^2 + 12\,km)x_{3e}^2}}{2\,mx_{3e}}. \tag{28}$$

The Equations (25) and (26), show that the real part of the eigenvalue λ_1 and λ_2 is negative. But it is not clear to draw a conclusion for eigenvalues λ_3 and λ_4 given by equations (27) and (28) respectively. We need to investigate on case by case basis to draw the conclusion on the stability of this equilibrium point. (See Table:1)

4 Non-Linear Simulation of EMBSS

In this section the solutions of the models are presented with appropriate simulations. Four different cases are considered in order to visualize the behaviours of the solution of the non-linear system. These are the following: Case one: $k = 0$ and $\beta = 0$, Case two: $k \neq 0$ and $\beta = 0$, Case three: $k \neq 0$ and $\beta \neq 0$, Case four: $k = 0$ and $\beta \neq 0$.

Without other stated information on parameters the following Tables 2 and 3 is are useful:

Parameters	R_2	L	m	α	β
Values	1	1	0.5	0.0001	0.8
Units	Ω	Henry	kg	$\frac{Nm^2}{A^2}$	$\frac{Ns}{m}$

Table 2. Parameter values

Parameters	C	x_{3e}	g	k	R	x_{2e}
Values	1	0.5	9.81	3	1	0.5
Units	Farad	m	$\frac{m}{s^2}$	$\frac{N}{m}$	Ω	m

Table 3. Parameter values

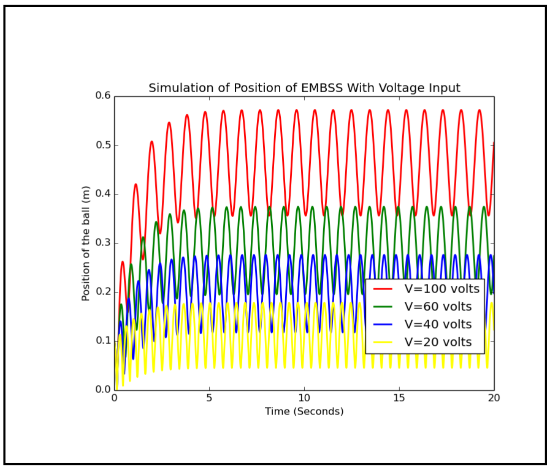

Figure 3. Case two: Position of the ball when $k = 0$ and $\beta = 0$ (input voltage).

The Figures 3, 4, 5 and 6 represent the simulations of the model one, with the position of the ball and and the current distribution of the EMBSS given by equation (6), ie when the input of the system is the voltage. Whereas Figures 7, 8, 9 and 10 represent the simulations of the model two which is the EMBSS given by the Equation (12), i.e when the input of the system is the current.

5 Controllability and Observability of the Linearized Models

This subsection is investigating the controllability and observability of the systems provided by equations (14) and (21). We shall discuss for which conditions or relationship the involved parameters in the system must satisfy such that the system is controllable and observable. We also identify the parameters on which controllability and observability could depend. We calculate

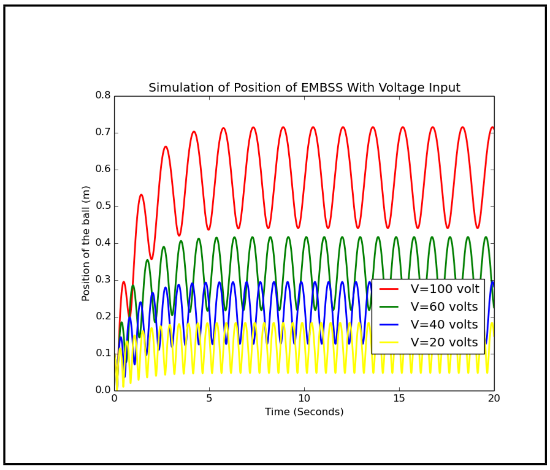

Figure 4. Case two: Position of the ball when $k \neq 0$ and $\beta = 0$ (input voltage).

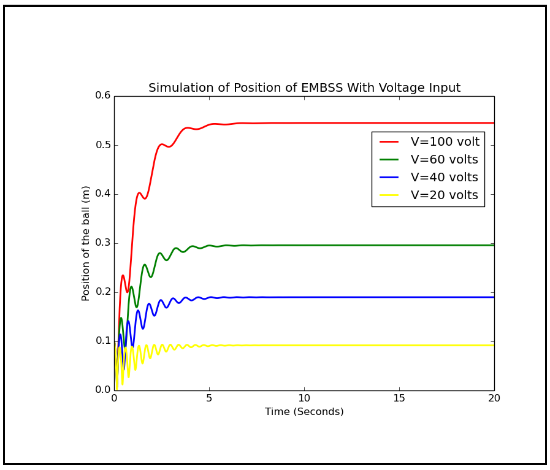

Figure 5. Cases three: Position of the ball when $k \neq 0$ and $\beta \neq 0$ (input voltage).

first of all the controllability and observability matrices CO and OBS respectively and their corresponding ranks. The following two theorems help to conclude about controllability and observability of a system.

Theorem 5.1 (R. Kalman's Criteria). *The linear continuous time-invariant system is controllable if and only if the controllability matrix* $\mathbf{C}$ *has a full rank, that is* $rank\ (\mathbf{C}) = n$.

Theorem 5.2 (R. Kalman's Criteria). *The linear continuous time-invariant system is observable if and only if the observability matrix* $\mathbf{O}$ *has a full rank, that is* $rank\ (\mathbf{O}) = n$.

Proof. The proof of the above theorems can be found in several literatures for example [Davis et al., 2009].

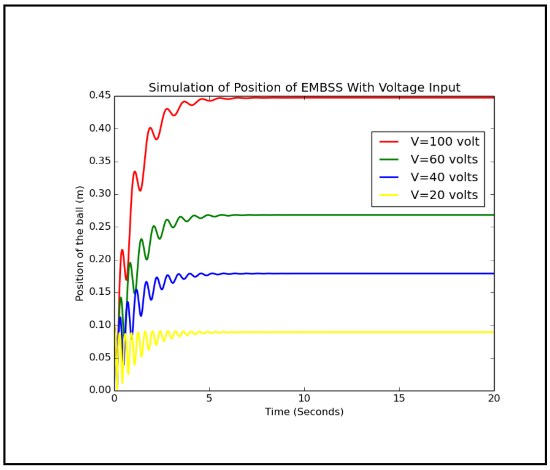

Figure 6. Cases four: Position of the ball when $k = 0$ and $\beta \neq 0$ (input voltage).

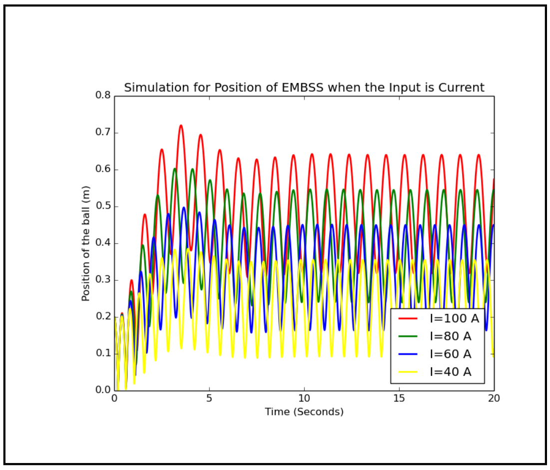

Figure 7. Case one: Position of the ball when $k = 0$ and $\beta = 0$ (input current).

5.1 Controllability and Observability of the Linearized Model of EMBSS with Voltage as Input

Noting that $n = 3$, the controllability matrix CO is calculated as follows: $\begin{pmatrix} B & AB & A^2B \end{pmatrix}$, where matrices A and B are given by Equations (16) and (17). Its determinant is given by the following equation:

$$Det(CO) = -\frac{4\,(\alpha g m - \alpha k x_{2e})}{L^3 m^2 x_{2e}^2}. \qquad (29)$$

Also noting that $n = 3$, the observability matrix OBS is calculated as follows: $\begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$, where matrices A and C are given by Equations (16) and (17). Its determinant is given by the following equation:

$$Det(OBS) = \frac{2\,\alpha\sqrt{\frac{gm - k x_{2e}}{\alpha}}}{m x_{2e}}. \qquad (30)$$

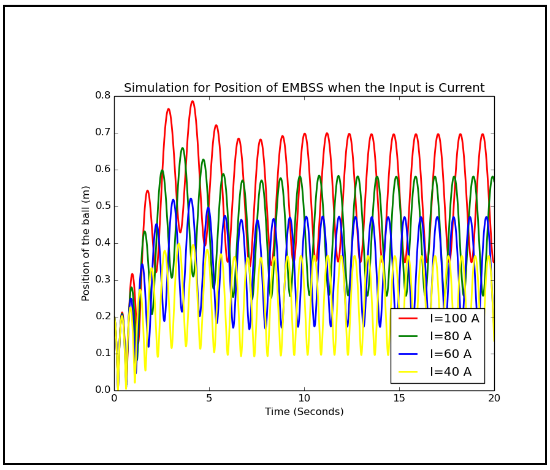

Figure 8. Case two: Position of the ball when $k \neq 0$ and $\beta = 0$ (input current).

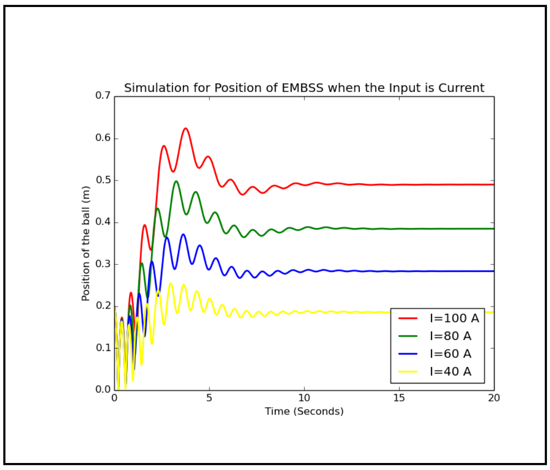

Figure 9. Cases three: Position of the ball when $k \neq 0$ and $\beta \neq 0$ (input current).

These calculations introduce the following theorem.

Theorem 5.3. *The system given by the Equation* (14) *is controllable and observable if and only if the relation* $\frac{gm}{x_{2e}} > k$ *holds.*

Proof. We prove this theorem, using the two theorems (5.1) and (5.2). That is , one needs to show that the matrices CO and OBS have full rank if and only if this inequality $\frac{gm}{x_{2e}} > k$ holds. Since all the two matrices CO and OBS are square matrices, one only has to show that their determinants are different from zero if and only if $\frac{gm}{x_{2e}} > k$ holds. This can be seen obviously from (29) and (30).

5.2 Controllability and Observability of the Linearized Model of EMBSS with Current Input

In this subsection also R. Kalman's Criteria for controllability and observability are going to be used here to check the controllability and observability of EMBSS with current input. Theorems (5.1) and (5.2) are

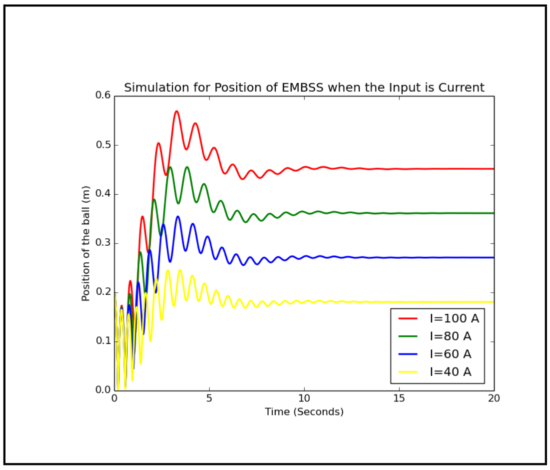

Figure 10. Cases four: Position of the ball when $k = 0$ and $\beta \neq 0$ (input current).

the powerful tool to be used here. The following theorem guarantee us that EMBSS with current input is both controllable and observable under certain conditions.

Theorem 5.4. *The system provided by the Equation* (21) *is controllable and observable if and only if the relation* $\frac{gm}{x_{3e}} > k$ *holds.*

Proof. One needs to show only that the determinant of the controllability and observability matrices CO and OBS respectively is different from zero when $\frac{gm}{x_{3e}} > k$. We calculate the CO as follows: $\left(B\ AB\ A^2B\ A^3B\right)$, where matrices A and B are given by Equations (23) and (24) respectively. It's determinant is given by the following:

$$Det(CO) = \frac{4\alpha}{C^4L^4m^2x_{3e}^2}\left(gm - kx_{3e}\right). \quad (31)$$

The observability matrix OBS is given by the following matrix: $\begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{pmatrix}$, where matrices A and C are given by Equations (23) and (24) respectively. It's determinant is given by the following:

$$Det(OBS) = \frac{4\alpha}{mx_{3e}^2}\left(gm - kx_{3e}\right). \quad (32)$$

Using the information given by equations (31) and (32), one concludes that the linearized model of EMMBSS with current input is both controllable and observable when $\frac{gm}{x_{3e}} > k$.

Theorem 5.5. *The system given by the Equation* (21) *is controllable and observable if and only if the relation* $\frac{gm}{x_{2e}} > k$ *holds.*

6 Optimal Control for the Linearized EMBSS Model with Input Voltage

The linearized model given by the equation (14) is being optimized in this section. We need to chose positive matrices R and Q. Let them be $R = 100$ and

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (33)$$

The remaining is to solve for P in the Riccati's equation $A^T P + PA - PBR^{-1}B^T P + Q = 0$. The matrices A and B are given by the equations (16) and (17). The octave function $lqr()$ is used to produce the following matrix P:

$$\begin{pmatrix} 4.9902 \times 10^{-1} & -1.2836 \times 10^{-4} & 7.1108 \times 10^{-3} \\ -1.2836 \times 10^{-4} & 1.3126 & 3.0864 \times 10^{-1} \\ 7.1108 \times 10^{-3} & 3.0864 \times 10^{-1} & 5.0540 \times 10^{-1} \end{pmatrix}. \quad (34)$$

Then K is calculated using the formula, $K = -R^{-1}B^T P$ to obtain the following:

$$K = \begin{pmatrix} 4.9902 & -1.2836 \times 10^{-3} & 0.071108 \end{pmatrix} \times 10^{-3}. \quad (35)$$

The input of the system is then calculated by the input voltage with KX, where $X = \begin{pmatrix} \delta x_1 & \delta x_2 & \delta x_3 \end{pmatrix}^T$.

7 Optimal Control for Linearized EMBSS Model with input current

The linearized model given by the equation (21) is being optimized in this section. One needs to chose positive matrices R and Q. Let them be $R = 1000$ and

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (36)$$

The remaining is to solve for P in the Riccati's equation $A^T P + PA - PBR^{-1}B^T P + Q = 0$. The matrices A and B are given by the equations (23) and (24). The octave function $lqr()$ is used to produce the following P:

$$\begin{pmatrix} 1.4981701 & 0.4991131 & -0.0010295 & 0.0097175 \\ 0.4991131 & 0.9941712 & -0.0043303 & 0.0020641 \\ -0.0010295 & -0.0043303 & 1.3125769 & 0.3086419 \\ 0.0097175 & 0.0020641 & 0.3086419 & 0.5054012 \end{pmatrix}. \quad (37)$$

Then K is calculated using the formula, $K = -R^{-1}B^T P$ to obtain the following:

$$K = \begin{pmatrix} 4.9911 & 9.9417 & -4.3303 & 2.0641 \end{pmatrix} \times 10^{-3}. \quad (38)$$

The input of the system is then calculated by the input voltage with KX, where $X = \begin{pmatrix} \delta x_1 & \delta x_2 & \delta x_3 & \delta x_4 \end{pmatrix}^T$.

8 Conclusion

Two models were built depending on whether the input is voltage or current. All these models are nonlinear that why for better analysis linearization techniques were used. They were also represented in state space representation form and then the linearization was made around their corresponding equilibrium points. The controllability and observability of the two models together with their corresponding stability were investigated. They have been guaranteed under certain conditions and combination of parameters k, β and the position of the ball. It was found out that the two models of EMBSS are all both controllable and observable. The controllability and observability of the EMBSS with the input as voltage were guaranteed when $\frac{gm}{x_{2e}} > k$. This is the same as the case of EMBSS with the input as current where the controllability and observability were guaranteed when $\frac{gm}{x_{3e}} > k$. In both cases x_{2e} and x_{3e} stand for the position of the ball from the end of the electromagnet.

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