COMPUTER AIDED SYMBOLIC MODELING AND PRECISE ADAPTIVE CONTROL OF COMPLEX LAGRANGIAN SYSTEMS

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Abstract
This paper deals with the problem of decomposition and precise control by complex objects. Decomposition is based on model reference adaptive control. Computer simulation demonstrates good results.

Key words
Complex Object, Lagrangian System, Decomposition, Adaptive Control, Lyapunov Function, Simulation

1. Introduction

As a complex we assume a mechanical object with some interconnected subsystems [1, 2]. In the wake of [3], by the Lagrangian systems are meant those where the controlled plant obeys a mathematical model (MM) of the form of Lagrange equations. A MM of such an object is usually multiconnected nonlinear with big number of degrees of freedom. Synthesis of control algorithms for such an object is not a simple problem. All the more it is difficult for precise control.

Qualitatively under precise control we mean the situation when the motion of any subsystem and the system in the whole are coincided with prescribed motions with prescribed accuracies.

Usual method for such an object control is decomposition [1, 2]. In this paper we assume that an object in the whole could be represented as a set of interconnected subsystems. For every subsystem a component of interconnections is selected and compensated on the base of adaptive control [4-6]. For this goal we use two steps of control. The first one is based on the programmed adaptive control. The second step is based on the model reference adaptive control. In this case special adaptive control algorithms are derived [8].

In this paper we assume that different subsystems have actuators with ideal MM.

2. Problem Statement

We consider controlled plants with the MM in the form of Lagrange equation

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q} - \frac{\partial T}{\partial q} = Q\,, \qquad (1)$$

where $q = (q_i)$ and $Q = (Q_i)$ $(i = \overline{1,n})$ are vectors of generalized coordinates and forces respectivly; $T = \frac{1}{2}\dot q^T A(q)\dot q$ is the kinetic energy; $A(q) = \left(a_{ij}(q)\right)$ $(i, j = \overline{1,n})$ is a symmetrical positive definite matrix (T denotes transposition).

By performing differentiation in (1), we proceed to the equation [7]

$$A(q)\ddot q + \sum_{s=1}^{n}\left[\dot q^T D_s(q)\dot q\right]e_s = S(q)M\,, \qquad (2)$$

where $M=(M_i)$ $(i=\overline{1,n})$ is a vector of control actions to the plant from a controller.

We assume ideal actuators. It means the equality

$$M_i=u_i \ \ (i=\overline{1,n}), \qquad (3)$$

where u_i are control algorithms to be discovered. So the equation (2) could be rewritten in the form

$$A(q)\ddot{q}+\sum_{s=1}^{n}\left[\dot{q}^T D_s(q)\dot{q}\right]e_s=S(q)U, \qquad (4)$$

where $U^{\mathrm{T}}=(u_1, u_2,...,u_n)$.

We assume that during the object operating:

- current matrices
  A(q), D_s(q), S(q) (s=$\overline{1, n}$)
  are known [8];
- vectors $q=q(t)$, $\dot{q}=\dot{q}(t)$ are measurable;
- for every q_i $(i=\overline{1, n})$ there exists a function $q_i^0(t)$ and an equation

$$\ddot{q}_i+d_i\dot{q}_i+k_i q=k_i q_i^0(t), \qquad (5)$$

where the function $q_i^0(t)$ and the numbers $k_i>0,\ d_i>0$ are prescribed in advance.

Problems:

- to discover algorithms for current matrices to be known;
- to discover control algorithms

$U=U(t,q,\dot{q})$ that guarantees the motion (5) for every generalized coordinates.

3. Computer Aided Symbolic Modeling of Complex Lagrangian Systems

Under computer aided modeling we mean to write formulas that connect matrices A(q), D_s(q), S(q) (s=$\overline{1, n}$) in (4) with constructive parameters of a mechanical system. These matrices are symbolic so such formulas need to be derived and written in a symbolic computer system. We use the system "Maple". Of course these formulas have to be convenient for a computer processing.

As a concrete mechanical system we consider the combination of (m+1) rigid bodies. As the example of such an object it could be a space robotic module (SRM) [3]. It consists of a supporting body and one or some manipulators. Let our SRM has only one manipulator with m links. In an inertial space the SRM position in common case is determined by N coordinates N =6(m +1), but connections which are imposed on relative positions of (m+1) bodies reduce this number to a value $n\le N$.

For the SRM object we assume that the supporting body position is determined by six coordinates q_j $(j=\overline{1,6})$ and the position of every link is determined by to coordinates q_{6+2i-1} and q_{6+2i} $(i=\overline{1,m})$ that shows the position of a link with respect to the preceding link. Let us consider these coordinates as the generalized ones $q^T=(q_1,q_2,...,q_n)$, where $n=6+2m$ is a maximum number of generalized coordinates. In turn the number n can be decreased by superposing of additional connections.

In principle the derivation of a mechanical system equations of motion in Lagrange's form is not a difficult problem but if the number of degrees of freedom is big ($n=6\div 25$ and more) the task becomes too much unwieldy. For operative modeling it is necessary to choose the most constructive method among the different approaches [4], [5]. On the authors opinion such a constructability there is in Kulakov's method [5]. The main idea of this method is to receive a sequence of recursive relations that could be easy realized on computer.

Not to examine Kulakov's method in detail we will fix some nodal points that are necessary for our tasks solution. For every body of the mechanical system the kinetic energy is defined by equality

$$T_i=\frac{1}{2}\dot{x}_i^T B_i\dot{x}_i, \qquad (3)$$

where $\dot{x}_i^T = (\omega_i^T, v_i^T)$, ω_i is a vector of angular quasirates, v_i is a vector of linear quasirates for pole, B_i is a matrix for concrete mass-inertia parameters of the body [5].

Vectors x_i in (3) are connected with the vector q by equalities

$$\omega_i = W_i \dot{q},\ v_i = V_i \dot{q} \quad \left(i = \overline{0, m} \right), \tag{4}$$

where matrices W_i, V_i have the forms

$$W_i = \alpha_{i,i-1} W_{i-1} + I_i^W,$$
$$V_i = \alpha_{i,i-1} V_{i-1} - \alpha_{i,i-1} \left(Ox_i^a + Ox_i^g \right) W_{i-1} + I_i^V$$
$$(i = \overline{0, m}). \tag{5}$$

In (5) $\alpha_{i,\ i-1}$ are matrices of direction cosines; matrices Ox_i^a Ox_i^g, I_i^W, I_i^V are determined by construction parameters of i-th body and its generalized coordinates q_{6+2i-1}, q_{6+2i} $(i = \overline{0, m})$. An index $(i-1)$ for i = 0 is relating to an inertial coordinate system.

From (3) – (5) we receive the equality $\dot{x} = C\dot{q}$, where

$$\dot{x}^T = \left(\omega_0^T, v_0^T, \omega_1^T, v_1^T, ..., \omega_m^T, v_m^T \right),$$
$$C^T = \left(W_0^T, V_0^T, W_1^T, V_1^T, ..., W_m^T, V_m^T \right). \tag{6}$$

With taking into account that the mechanical system possesses by scleronomous connections we can write the equality $T = \frac{1}{2} \dot{q}^T A(q) \dot{q}$ where $A(q) = \left(a_{ij}(q) \right)$ $(i, j = \overline{1, n})$ has the form

$$A(q) = C^T B C, \tag{7}$$

and $B = diag\left(B_0, B_1, ..., B_m \right)$.

After differentiation in (1) we receive the equality

$$A(q)\ddot{q} + \sum_{s=1}^{n} \left[\dot{q}^T D_s(q) \dot{q} \right] e_s = Q \tag{8}$$

where e_s is the vector of $(n \times 1)$-dimension with all zero components except s-th to be equal to 1; matrices $D_s(q) = \left(d_{it}^s(q) \right)$ $(i, t, s = \overline{1, n})$ are determined by their elements in the form

$$d_{it}^s(q) = \frac{1}{2} \left[\frac{\partial a_{si}(q)}{\partial q_t} + \frac{\partial a_{st}(q)}{\partial q_i} - \frac{\partial a_{it}(q)}{\partial q_s} \right]. \tag{9}$$

Let us assume that control actions in every body are acted along the axis's of the coordinate system connected with the body and with respect to these axis's. Then such a virtual vector could be written in the form $M_V{}^T = \left(M_{V1}, M_{V2}, ..., M_{VN} \right)$ with N components.

Let G be a vector

$$G^T = \left(G_0{}^T, F_0{}^T, G_1{}^T, F_1^T, ..., G_m{}^T, F_m{}^T \right),$$

where G_i, F_i $\left(i = \overline{0, m} \right)$ are the main momentum and the main force vectors respectively for every body presented in projections to the connected coordinate system of the body.

In this case it is possible to write the equality

$$Q = C^T G. \tag{10}$$

To derive a formula for matrix $S(q)$ in the MM (4) it is necessary to use equality

$$S(q)U = C^{\mathrm{T}} G \tag{11}$$

and to determine the equality

$$G = ZLU. \tag{12}$$

Matrices Z and L, that in (??) was named as the informational-force and informational matrices respectively, are determined by concrete active axes of an object.

From (10) – (12) we obtain the formula

$$S(q) = C^T Z L. \tag{13}$$

So the problem 1 is solved: matrices A(q), D_s(q), S(q) $(s=\overline{1,\ n})$ in the MM (4) are derived in the formulas (7), (9) and (10) that are convenient for computer calculation.

4. Decomposition on the Base of Adaptive Programmed Control

Let us rewrite the MM (4) in the form

$$\ddot{q} = A^{-1}(q)S(q)U + f(t), \qquad (14)$$

where

$$f(t,q,\dot{q}) = -A^{-1}(q)\sum_{s=1}^{n}\left[\dot{q}^T D_s(q)\dot{q}\right]e_s\ .(15)$$

During the real mechanical system processing vectors q and $\dot{q}$ are functions of time, so the vector-function $f(t,q,\dot{q})$ could be denote as $f(t)$. Now from the equation (14) it is evident a programmed computer aided adaptive control. Really, an adaptive control algorithm could be taken in the form

$$U = [S(q)]^{-1}A(q)[K(q^0 - q) - D\dot{q} + L]. \qquad (16)$$

Really, in the control algorithm (16) every term is known besides matrices $K,\ D$ and a vector L which we may take to realize the motion (5).

The equation (14) together with the algorithm (16) could be represented in the form

$$\ddot{q} + D\dot{q} + Kq = Kq^0(t) + [f(t) + L]. \qquad (17)$$

Let us take matrices $K,\ D$ as diagonal ones that is

$$\begin{aligned} K &= diag(k_1,\ k_2,...,k_n), \\ D &= diag(d_1,\ d_2,...,d_n) \end{aligned} \qquad (18)$$

where $k_i,\ d_i\ (i=\overline{1,\ n})$ are coincided with desired numbers in (5).

Then the system (17) is decomposed to n interconnected equations

$$\ddot{q}_i + d_i\dot{q}_i + k_i q_i = k_i q_i^{\,0}(t) + [f_i(t) + L_i]$$
$$(i=\overline{1,\ n}). \qquad (19)$$

From the equation (19) we see that if it is valid the equality

$$[f_i(t) + S_i] \equiv 0 \quad (i=\overline{1,\ n}) \qquad (20)$$

then the problem for generalized coordinates $q_i \quad (i=\overline{1,\ n})$ is solved.

5. Model Reference Adaptive Control

Let us set up the problem to compensate the action of the term $f_i(t)\ \ (i=\overline{1,\ n})$ in the equation (19) on the desired movement $q_i = q_i(t)$ with the help of a purposeful variation of the vector L in the control algorithm (16). For this goal we use the well known principle of model reference adaptive control [5].

Let us take a reference model in the form

$$\ddot{q}_{mi} + d_i\dot{q}_{mi} + k_i q_{mi} = k_i q_i^{\,0}(t)\ . \qquad (21)$$

From (19) and (21) we receive an equation with respect to the error $\varepsilon_i = q_i - q_{mi}$ in the form

$$\ddot{\varepsilon}_i + d_i\dot{\varepsilon}_i + k_i\varepsilon_i = [f_i(t) + L_i]. \qquad (22)$$

The equation (22) can be rewritten in a matrix form

$$\begin{aligned} \dot{x}_i &= A_i x_i + \rho_i(y_i), \\ \dot{y}_i &= \psi_i + \mu_i(t) \end{aligned} \qquad (23)$$

where $\varepsilon_i = x_{i1},\ \dot{\varepsilon}_i = x_{i2},\ f_i(t) + L_i = y_i,$ $\dot{f}_i(t) = \mu_i(t),\ \ \dot{L}_i = \psi_i,\ \ x_i^T = (x_{i1}\ \ x_{i2}),$ $\rho_i^T(y_i) = (0\ \ y_i),\ A_i = \begin{pmatrix} 0 & 1 \\ -k_i & -d_i \end{pmatrix}.$

Now we can choose an algorithm for L_i purposeful variation from the condition of an asymptotical convergence of the system (23) with respect to the movement

$$x_i \equiv 0,\ \ y_i \equiv 0. \qquad (24)$$

For this goal we take Lyapunov's function in the form

$$V_i(x_i, y_i) = \kappa_i x_i^T P_i x_i + y_i^2 \qquad (25)$$

where P_i is a positive definite matrix, $\kappa_i = const > 0$. The derivative of $V_i(x_i, y_i)$ with respect to the time on the strength of the system (23) is determined by the equality

$$\begin{aligned}\dot{V}_i(x_i, y_i) &= \kappa_i x_i^T Q_i x_i + \\ &+ 2y_i[\sigma_i + \mu_i(t) + \psi_i]\end{aligned} \qquad (26)$$

where Q_i is the prescribed negative definite matrix, $\sigma_i = (p_{21}^i x_{i1} + p_{22}^i x_{i2})$, p_{jk}^i are elements of the matrix $P_i = (p_{jk}^i)$ $(j,k = 1,2)$.

In this paper we suppose that the sign of the coordinate y_i is known. Then we choose the desired algorithm in the form

$$\psi_i = -\sigma_i - \bar{k}_i sign(y_i) \qquad (27)$$

where $\bar{k}_i > 0$ and

$$\bar{k}_i > |\mu_i(t)| \,. \qquad (28)$$

Then we have inequalities

$$V_i(x_i, y_i) > 0, \qquad \dot{V}_{mi}(x_{mi}, y_{mi}) < 0 \qquad (29)$$

which ensure the solution of the problem.

6. Simulation Results

In simulation results we demonstrate the model reference adaptive control. We assume that the decomposition problem is solved and now it is necessary to show that algorithms (27) really provide precise control of $q_i(t)$ with respect to $q_i^0(t)$ in (23).

Let $k_i = 1$ and $d_i = 1.4$ in (14), $Q_i = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ in (20). Then $P_{mi} = \begin{pmatrix} 2.83 & 1 \\ 1 & 1.43 \end{pmatrix}$ in (19) and $\sigma_{mi} = (1.43\dot{\varepsilon}_{mi} + \varepsilon_{mi})$ in (21).

In fig.1 we see $q_{mi}(t)$, $q_i(t)$ with the disturbance $f_i(t)$ but without adaptation that is $L_i \equiv 0$. The difference between $q_{mi}(t)$ and $q_i(t)$ is essential.

In fig.2 we see the same coordinates but with adaptation that is $L_i \neq 0$. The difference between $q_{mi}(t)$ and $q_i(t)$ is practically null.

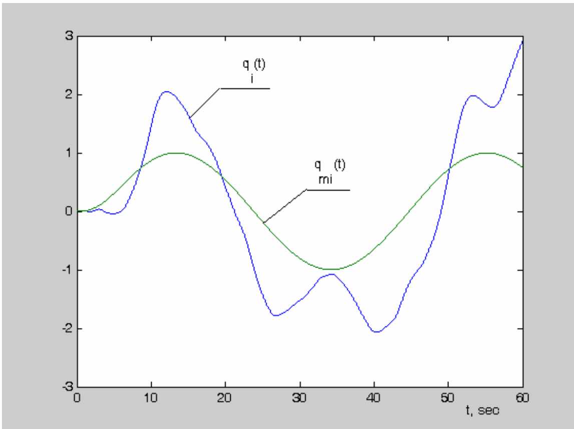

Fig. 1.

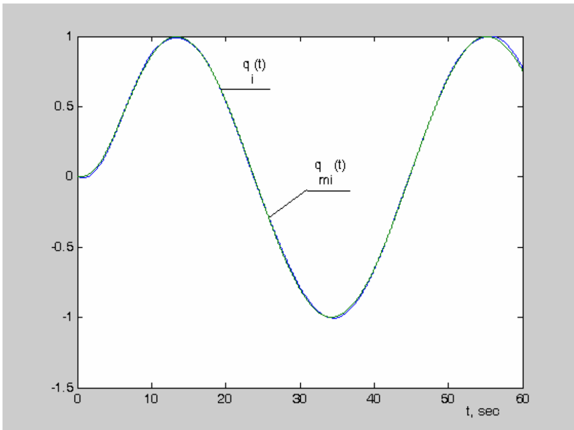

Fig. 2.

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References

Voronov A.A., *Fundamentals of the Theory of Automatic Control*, Energiya, Leningrad, 1965.

Šiljak D., *Decentralized Control of Complex Systems,* Academic Press, Inc., New York, 1991.

Pyatnitskii E.S., *Controllability of Classes of Lagrangian Systems with Bounded Controls* //Avtomatica i Telemechanica. 1996, No 12, pp.29 – 37.

Zemlyakov S.D. and Rutkovsky V.Yu. *Synthesis of Model Reference Adaptive Control System* // Avtomatica i Telemechanica. 1966, No 3, pp.70 – 77.

Petrov B.N., Rutkovsky V.Yu. and Zemlyakov S.D. *Adaptive Coordiate-Parametric Control of Nonstationary Objects.* Nauka, Moscow, 1980.

Boltyanskii V.G. *Mathematical Methods of Optimal Control.* Nauka, Moscow, 1969.

Rutkovsky V.Yu. and Zemlyakov S.D. *Decomposition of Lagrangian Systems on the Base of Technical Controllability Concept* // Proc. of Conf. PHYSCON-2003, St.Petersburg, 2003.

Zemlyakov S.D. and Glumov V.M. *Free-Flying Space Robotic Module – Adaptive Computer Aided Precize Control* // Proc. of 17-th Symp. on Aut. Contr. in Space, Toulouse, France, 2007.

Krutiko P.D. *Control of Robot Actuators.* Nauka, Moscow, 1991.

Raushenbakh B.V.and Tokar E.N. *Controlof Spacecraft Orientation*. Nauka, Moscow, 1974.

Utkin V.I. *Sliding Modes in Control and Optimization*, Springer, Heidelberg, 1992.