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NONLINEAR LAWS FOR GUIDANCE AND ATTITUDE CONTROL OF AN AGILE LAND-SURVEY SATELLITE

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Abstract

We consider problems on attitude guidance and control of the agile spacecraft for surveying the Earth surface. We present developed methods for synthesis of nonlinear guidance and attitude control laws, results on dynamic research of the spacecraft attitude control system under disturbances and digital control of the gyro moment clusters by two excessive gyrodine schemes.

Key words

spacecraft, land-survey, guidance, attitude control

1 Introduction

Dynamic requirements to an attitude control system (ACS) for a land-survey spacecraft (SC) are as follows: (i) guidance the telescope's line-of-sight to a predetermined part of the Earth surface with the scan in designated direction; (ii) stabilization of an image motion at the onboard optical telescope focal plane. These requirements are expressed by the SC rapid angular manoeuvering and spatial compensative motion with a variable angular rate vector. Attitude guidance laws of a land-survey SC are presented into a sequence of time intervals for the observing scanning routes (SRs) and rotational maneuvers (RMs), Figs. 1 and 2.

Lifetime up to 10 years, exactness of spatial rotation maneuvers with effective damping the SC flexible structure oscillations, robustness, fault tolerance as well as the reasonable mass, size and energy characteristics have motivated intensive development of ACS with the gyro moment clusters (GMCs) based on excessive number of gyrodines (GDs) – single-gimbal control moment gyros.

In the paper we present new results on synthesis of nonlinear laws for guidance and gyromoment attitude control of an agile land-survey satellite.

Figure 1. The space observing scanning routes on a map

2 Models and the Problem Statement

We have applied standard reference frames – the inertial reference frame (IRF) $\mathbf{I}_\oplus$ $(\mathrm{O}_\oplus \mathrm{X}_\mathrm{e}^\mathrm{I}\mathrm{Y}_\mathrm{e}^\mathrm{I}\mathrm{Z}_\mathrm{e}^\mathrm{I})$, the geodesic Greenwich reference frame (GRF) $\mathbf{E}_\mathrm{e}$ $(\mathrm{O}_\oplus \mathrm{X}^\mathrm{e}\mathrm{Y}^\mathrm{e}\mathrm{Z}^\mathrm{e})$ rotated with respect to IRF by angular rate vector $\boldsymbol{\omega}_\oplus \equiv \boldsymbol{\omega}_\mathrm{e}$ and the geodesic horizon reference frame (HRF) $\mathbf{E}_\mathrm{e}^\mathrm{h}$ $(\mathrm{C}\,\mathrm{X}_\mathrm{c}^\mathrm{h}\mathrm{Y}_\mathrm{c}^\mathrm{h}\mathrm{Z}_\mathrm{c}^\mathrm{h})$ with origin in a point C and ellipsoidal geodesic coordinates on altitude H, longitude L and latitude B. There are also applied the SC body reference frame (BRF) $\mathbf{B}$ $(\mathrm{O}xyz)$ and the orbit reference frame (ORF) $\mathbf{O}$ $(\mathrm{O}x^\mathrm{o}y^\mathrm{o}z^\mathrm{o})$ with origin in the SC mass center O, the optical telescope (sensor) reference frame (SRF) $\boldsymbol{\mathcal{S}}$ $(\mathrm{O}x^\mathrm{s}y^\mathrm{s}z^\mathrm{s})$ and the image field reference frame (FRF) $\boldsymbol{\mathcal{F}}$ $(\mathrm{O}_i x^i y^i z^i)$ with origin in center O_i of the telescope focal plane $y^i\mathrm{O}_i z^i$.

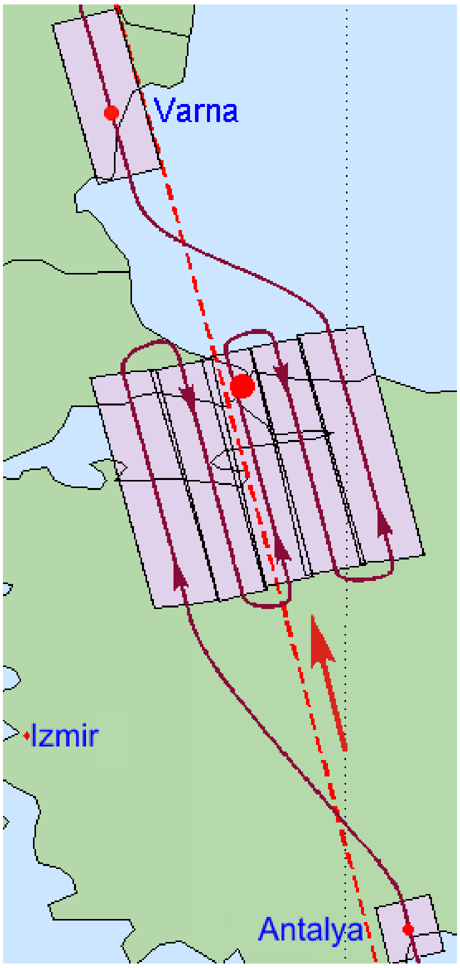

Figure 2. The scanning routes on a map during areal observation

In IRF the BRF orientation is defined by quaternion $\mathbf{\Lambda}_{\mathrm{I}}^{b} \equiv \mathbf{\Lambda} = (\lambda_0, \boldsymbol{\lambda}), \boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ and with respect to the ORF – by column $\boldsymbol{\phi} = \{\phi_i\}$ $(i = 1, 2, 3 \equiv 1 \div 3)$ of Krylov angles ϕ_1 (roll), ϕ_2 (yaw) and ϕ_3 (pitch). Let us vectors $\boldsymbol{\omega}(t)$, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are standard notations of the SC body angular rate, the SC mass center's position and progressive velocity with respect to the IRF. Further symbols $\langle \cdot, \cdot \rangle$, $\times$, $\{\cdot\}$, $[\cdot]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^{\mathrm{t}}$ for matrices are conventional notations. Assume that $\mathbf{\Lambda}^p$, $\boldsymbol{\omega}^p = \{\omega_i^p\}$ and $\boldsymbol{\varepsilon}^p = \dot{\boldsymbol{\omega}}^p$ are the quaternion, angular rate and acceleration vectors of SC body's attitude guidance law in the IRF. The error quaternion is $\mathbf{E} = (e_0, \mathbf{e}) = \tilde{\mathbf{\Lambda}}^p(t) \circ \mathbf{\Lambda}$, Euler parameters' vector is $\boldsymbol{\mathcal{E}} = \{e_0, \mathbf{e}\}$, the attitude error's matrix is $\mathbf{C}^e \equiv \mathbf{C}(\boldsymbol{\mathcal{E}}) = \mathbf{I}_3 - 2[\mathbf{e}\times]\mathbf{Q}_e^{\mathrm{t}}$ with matrix $\mathbf{Q}_e \equiv \mathbf{I}_3 e_0 + [\mathbf{e}\times]$, and angular rate error vector is $\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \mathbf{C}^e \boldsymbol{\omega}^p(t)$. The GMC's angular momentum (AM) vector $\boldsymbol{\mathcal{H}}$ has the form $\boldsymbol{\mathcal{H}}(\boldsymbol{\beta}) = h_g \mathbf{h}(\boldsymbol{\beta}) \equiv h_g \sum \mathbf{h}_p(\beta_p)$, there h_g is a constant own AM value for each GD $p = 1 \div m$ with the GD's AM unit $\mathbf{h}_p(\beta_p)$ and column $\boldsymbol{\beta} = \{\beta_p\}$. Within precession theory of the control moment gyros, for a fixed position of the SC flexible structures with some simplifying assumptions and $t \in \mathrm{T}_{t_0} = [t_0, +\infty)$ a SC angular motion model is appeared as follows

$$\dot{\mathbf{\Lambda}} = \mathbf{\Lambda} \circ \boldsymbol{\omega}/2; \quad \mathbf{A}^o \{\dot{\boldsymbol{\omega}}, \ddot{\mathbf{q}}\} = \{\mathbf{F}^\omega, \mathbf{F}^q\}, \tag{1}$$

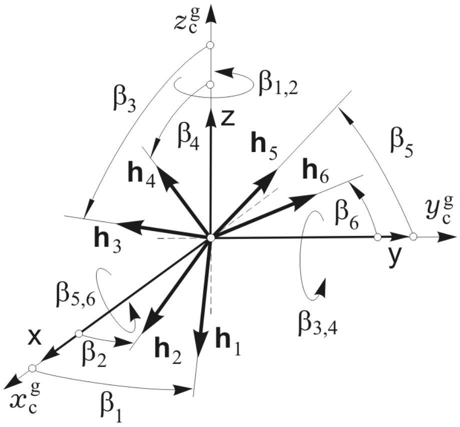

Figure 3. The GMC scheme *3-SPE* based on six gyrodines

where $\boldsymbol{\omega} = \{\omega_i, i = x, y, z \equiv 1 \div 3\}$; $\mathbf{q} = \{q_j\}$;
$\mathbf{F}^\omega = \mathbf{M}^{\mathrm{g}} - \boldsymbol{\omega} \times \mathbf{G} + \mathbf{M}^d(t, \mathbf{\Lambda}, \boldsymbol{\omega}) + \mathbf{Q}^o(\boldsymbol{\omega}, \dot{\mathbf{q}}, \mathbf{q})$;
$\mathbf{F}^q = \{-a_j^q((\delta^q/\pi)\Omega_j^q \dot{q}_j + (\Omega_j^q)^2 q_j) + \mathrm{Q}_j^q(\boldsymbol{\omega}, \dot{q}_j, q_j)\}$;
$\mathbf{A}^o = \begin{bmatrix} \mathbf{J} & \mathbf{D}_q \\ \mathbf{D}_q^{\mathrm{t}} & \mathbf{A}^q \end{bmatrix}$; $\mathbf{G} = \mathbf{G}^o + \mathbf{D}_q \dot{\mathbf{q}}$; $\mathbf{M}^{\mathrm{g}} = -h_g \mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta})\dot{\boldsymbol{\beta}}$; $\mathbf{G}^o = \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta})$; $\mathbf{A}_{\mathrm{h}} = \partial \mathbf{h}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}$;
vector $\mathbf{M}^d(\cdot)$ presents the external disturbance torques, and $\mathbf{Q}^o(\cdot)$, $\mathrm{Q}_j^q(\cdot)$ are nonlinear continuous functions.

The GMC torque vector $\mathbf{M}^{\mathrm{g}}$ is presented as follows

$$\mathbf{M}^{\mathrm{g}} = -\boldsymbol{\mathcal{H}}^* = -h_g \mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta})\, \mathbf{u}^{\mathrm{g}}; \; \dot{\boldsymbol{\beta}} = \mathbf{u}^{\mathrm{g}}, \tag{2}$$

$\mathbf{u}^{\mathrm{g}} = \{\mathrm{u}_p^{\mathrm{g}}\}$, $\mathrm{u}_p^{\mathrm{g}}(t) = \mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u_{pk}^g, d^g), \bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}}), T_u]$ with period $T_u = t_{k+1} - t_k$, $k \in \mathbb{N}_0 \equiv [0, 1, 2, \dots)$; discrete functions $u_{pk}^g \equiv u_p^g(t_k)$ are outputs of nonlinear control law (CL), functions $\mathrm{Sat}(x, a)$ and $\mathrm{Qntr}(x, a)$ are general-usage ones, while the holder model has the form $y(t) = \mathrm{Zh}[x_k, T_u] = x_k \; \forall t \in [t_k, t_{k+1})$.

Collinear pair of two GDs was named as *Scissored Pair Ensemble* (*SPE*) in well-known paper *J.W. Crenshaw* (1973). Redundant multiply scheme, based on six gyrodines in the form of three collinear GD's pairs, has name *3-SPE*. Fig. 3 presents a simplest arrangement of this scheme into canonical orthogonal gyroscopic basis $\mathrm{O}x_{\mathrm{c}}^{\mathrm{g}} y_{\mathrm{c}}^{\mathrm{g}} z_{\mathrm{c}}^{\mathrm{g}}$. By a slope of the GD pairs suspension axes it is possible to change essentially a form of the AM variation domain. Based on four gyrodines the redundant scheme *2-SPE* is obtained from the *3-SPE* scheme – without third pair (GD #5 and GD #6). In park state above GMC schemes have the vector of summary normed AM $\mathbf{h}(\boldsymbol{\beta}) = \mathbf{0}$.

We apply a strapdown inertial navigation system (SINS) with an inertial measurement unit (IMU) based on the gyro sensors corrected by an astronomical system (AS) with the star trackers, the that are fixed to the SC body. We use our approach (Somov *et al.*, 2013) to a signal processing in the SINS applying the following methods: (i) approximation and interpolation of the IMU quasi-coordinate increment vector values into two adjacent sliding windows; (ii) estimating of the IMU bias vector, the matrix of a mutual angular position of the IMU and AS reference frames; (iii) estimating of angular rate vector and IMU scale factor's value, compensation of the IMU bias; (iv) discrete filtering and forming of the SINS coordinated digital output signals on orientation and angular rate in the time moments $t_l, l \in \mathbb{N}_0$ with given period T_p. We assume also that column $\boldsymbol{\beta}_k \equiv \{\beta_{pk}\}$ of the GDs measured angles is accessible for the SC attitude digital control.

For given the SC body angular guidance law during a time interval $t \in \mathrm{T} \equiv [t_\mathrm{i}, t_\mathrm{f}] \subset \mathrm{T}_{t_0}$, $t_\mathrm{f} \equiv t_\mathrm{i} + T$ and a forming of the vector of continuous control torque $\mathbf{M}^\mathrm{g}$ (2), the columns $\dot{\boldsymbol{\beta}} = \{\dot{\beta}_p\}$ and $\ddot{\boldsymbol{\beta}} = \{\ddot{\beta}_p\}$ are component-wise module restricted

$$|\dot{\beta}_p(t)| \leq \bar{\mathrm{u}}_\mathrm{g} < \bar{\mathrm{u}}_\mathrm{g}^\mathrm{m},\ |\ddot{\beta}_p(t)| \leq \bar{\mathrm{v}}_\mathrm{g},\ \forall t \in \mathrm{T}, \qquad (3)$$

where values $\bar{\mathrm{u}}_\mathrm{g}$ and $\bar{\mathrm{v}}_\mathrm{g}$ are some constants.

At simplest modeling of the SC body as a free solid, the ACS AM vector has a constant value $\mathbf{G}^o = \mathbf{G}_0^o$. Assume that the ACS is balanced on its AM with condition $\mathbf{G}_0^o \equiv \mathbf{0}$, moreover model of the SC attitude dynamics has the form $\dot{\boldsymbol{\omega}} = \boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon} = \mathbf{J}^{-1}\mathbf{M}^\mathrm{g}$ is vector of angular acceleration, and model of SC attitude motion has the following kinematic representation

$$\dot{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda} \circ \boldsymbol{\omega}/2;\ \ \dot{\boldsymbol{\omega}} = \boldsymbol{\varepsilon};\ \ \boldsymbol{\varepsilon}^* = \dot{\boldsymbol{\varepsilon}} = \mathbf{v}. \qquad (4)$$

Modules of vectors $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\boldsymbol{\varepsilon}^*(t)$ are restricted, namely $|\boldsymbol{\omega}(t)| \leq \bar{\omega}$, $|\boldsymbol{\varepsilon}(t)| \leq \bar{\varepsilon}$ and $|\boldsymbol{\varepsilon}^*(t)| \leq \bar{\varepsilon}^*$, that is connected with a limited envelop of the variation domains for the GMC vectors of the AM $\boldsymbol{\mathcal{H}}$ and control torque $\mathbf{M}^\mathrm{g}$ with permissible rate of its variation.

We apply vector $\boldsymbol{\sigma} = \{\sigma_i\} = \mathbf{e}\ \mathrm{tg}(\Phi/4)$ of the modified Rodrigues parameters (MRP) with traditional notations of Euler unit $\mathbf{e}$ and angle Φ of own rotation. Vector $\boldsymbol{\sigma}$ is one-one connected with quaternion $\boldsymbol{\Lambda}$ by straight $\boldsymbol{\sigma} = \boldsymbol{\lambda}/(1+\lambda_0)$ and reverse

$\lambda_0 = (1-\sigma^2)/(1+\sigma^2)$; $\boldsymbol{\lambda} = 2\boldsymbol{\sigma}/(1+\sigma^2)$ relations, its kinematic equations have the form $\dot{\boldsymbol{\sigma}} = \frac{1}{4}(1-\sigma^2)\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{\sigma}\times\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{\sigma}\langle\boldsymbol{\sigma},\boldsymbol{\omega}\rangle$;

$\boldsymbol{\omega} = \frac{4}{(1+\sigma^2)^2}[(1-\sigma^2)\dot{\boldsymbol{\sigma}} - 2(\boldsymbol{\sigma}\times\dot{\boldsymbol{\sigma}}) + 2\boldsymbol{\sigma}\langle\dot{\boldsymbol{\sigma}},\boldsymbol{\sigma}\rangle]$,

and its second derivative is presented as follows

$$\ddot{\boldsymbol{\sigma}} = \tfrac{1}{2}[-\langle\boldsymbol{\sigma},\dot{\boldsymbol{\sigma}}\rangle\boldsymbol{\omega} + \tfrac{1}{2}(1-\sigma^2)\boldsymbol{\varepsilon} + \dot{\boldsymbol{\sigma}}\times\boldsymbol{\omega} + \boldsymbol{\sigma}\times\boldsymbol{\varepsilon} + \dot{\boldsymbol{\sigma}}\langle\boldsymbol{\sigma},\boldsymbol{\omega}\rangle + \boldsymbol{\sigma}\langle\dot{\boldsymbol{\sigma}},\boldsymbol{\omega}\rangle + \boldsymbol{\sigma}\langle\boldsymbol{\sigma},\boldsymbol{\varepsilon}\rangle].$$

The first problem gets up on angular guidance of the SC during its spatial route motion when a space observation is executed at given time interval $t \in \mathrm{T}$ – determination of quaternion $\boldsymbol{\Lambda}(t)$, vectors of angular rate $\boldsymbol{\omega}(t)$ and acceleration $\boldsymbol{\varepsilon}(t)$ in the form of explicit functions, proceed from the main requirement: optical image of the Earth given part must move by desired way in focal plane $y^i\mathrm{O}_i z^i$ of the telescope. The problem consists in analytical representation of the guidance law with given accuracy without any restriction on duration of interval T. This law corresponds to required SR $\boldsymbol{\Lambda}(t), \boldsymbol{\omega}(t)$ by arbitrary type – trace, orthodromic, with optimal alignment of a longitudinal image motion velocity (IMV), areal and stereo observation etc.

If we have two adjacent time intervals of a scanning observation, then for model (4) we obtain the RM boundary conditions on quaternion $\boldsymbol{\Lambda}$, vectors $\boldsymbol{\omega}, \boldsymbol{\varepsilon}$, and also on vector $\boldsymbol{\varepsilon}^*$ in a time moment when second route is beginning. For RM time interval $\mathrm{T}_p \equiv [t_\mathrm{i}^p, t_\mathrm{f}^p]$, $t_\mathrm{f}^p \equiv t_\mathrm{i}^p + T_p$ and boundary conditions

$$\begin{aligned}&\boldsymbol{\Lambda}(t_\mathrm{i}^p) = \boldsymbol{\Lambda}_\mathrm{i};\ \boldsymbol{\omega}(t_\mathrm{i}^p) = \boldsymbol{\omega}_\mathrm{i};\ \boldsymbol{\varepsilon}(t_\mathrm{i}^p) = \boldsymbol{\varepsilon}_\mathrm{i};\\ &\boldsymbol{\Lambda}(t_\mathrm{f}^p) = \boldsymbol{\Lambda}_\mathrm{f};\ \boldsymbol{\omega}(t_\mathrm{f}^p) = \omega_\mathrm{f};\ \boldsymbol{\varepsilon}(t_\mathrm{f}^p) = \boldsymbol{\varepsilon}_\mathrm{f};\ \boldsymbol{\varepsilon}^*(t_\mathrm{f}^p) = \boldsymbol{\varepsilon}_\mathrm{f}^*,\end{aligned} \qquad (5)$$

taking into account given restrictions on modules of vectors $\boldsymbol{\omega}(t), \boldsymbol{\varepsilon}(t)$ and $\boldsymbol{\varepsilon}^*(t)$, the SC spatial RMs are not the only ones. We consider second problem on synthesis of a guidance law at the SC RM using analytic relations only, a planning of land-survey as a sequence of alternating SRs with rotational maneuvers in-between and analytical synthesis of an unified vector spline law for the satellite guidance during standard (Fig. 1) and areal (Fig. 2) land-surveying.

Finally, the third considered problem consists in synthesis of algorithms for discrete filtering of the SINS signals and the GMC nonlinear digital control, analysis of the SC ACS accuracy characteristics during the above types of a scanning land-survey.

3 Spacecraft attitude guidance laws

Analytic matching solution have been obtained for problem of the SC angular guidance during an scanning observation (Somov *et al.*, 2011; Somov, 2016). The solution is based on a vector composition of all elemental motions in GRF $\mathbf{E}_\mathrm{e}$ using the following reference frames: HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, SRF $\boldsymbol{\mathcal{S}}$ and FRF $\boldsymbol{\mathcal{F}}$. For any observed point C the oblique range D is analytically calculated as $\mathrm{D} = |\mathbf{r}_\mathrm{c}^\mathrm{e} - \mathbf{r}^\mathrm{e}|$. If orthogonal matrix $\mathbf{C}_\mathrm{h}^\mathrm{s} \equiv \tilde{\mathbf{C}} = \|\tilde{c}_{ij}\|$ defines SRF $\boldsymbol{\mathcal{S}}$ attitude with respect to HRF $\mathbf{E}_\mathrm{e}^\mathrm{h}$, then for any point $\mathrm{M}(\tilde{y}^i, \tilde{z}^i)$ at the telescope focal plane $y^i\mathrm{O}_i z^i$ the components $\tilde{V}_y^i$ and $\tilde{V}_z^i$ of normed vector by an image motion velocity is appeared as follows

$$\begin{bmatrix}\tilde{V}_y^i\\ \tilde{V}_z^i\end{bmatrix} = \begin{bmatrix}\tilde{y}^i & 1 & 0\\ \tilde{z}^i & 0 & 1\end{bmatrix}\begin{bmatrix}q^i\tilde{\mathrm{v}}_\mathrm{e1}^\mathrm{s} - \tilde{y}^i\ \omega_\mathrm{e3}^\mathrm{s} + \tilde{z}^i\ \omega_\mathrm{e2}^\mathrm{s}\\ q^i\tilde{\mathrm{v}}_\mathrm{e2}^\mathrm{s} - \quad \omega_\mathrm{e3}^\mathrm{s} - \tilde{z}^i\ \omega_\mathrm{e1}^\mathrm{s}\\ q^i\tilde{\mathrm{v}}_\mathrm{e3}^\mathrm{s} + \quad \omega_\mathrm{e2}^\mathrm{s} + \tilde{y}^i\ \omega_\mathrm{e1}^\mathrm{s}\end{bmatrix}. \qquad (6)$$

Here $\tilde{y}^i = y^i/f_e$, $\tilde{z}^i = z^i/f_e$ are normed focal coordinates where f_e is the telescope equivalent focal distance; function $q^i \equiv 1 - (\tilde{c}_{21}\tilde{y}^i + \tilde{c}_{31}\tilde{z}^i)/\tilde{c}_{11}$, and vector of normed SC's mass center velocity has components $\tilde{\mathrm{v}}_{ei}^\mathrm{s} = \mathrm{v}_{ei}^\mathrm{s}/\mathrm{D}, i = 1 \div 3$. Assume that taking into account (6) and some kinematic requirements we computed the SC guidance attitude law by numerical integrating of the nonlinear quaternion kinematic equation in (1) for any time interval $t \in \mathrm{T} \equiv [0, T]$. We consider the interval T with the following notations for its four points τ_p, $p = 1 \div 4 : \tau_1 = 0$, $\tau_2 = T/3$, $\tau_3 = 2T/3$ and $\tau_4 = T$. For six values $\boldsymbol{\omega}_l$, $l \in \tilde{\mathbb{N}} \subset \mathbb{N}_0$ nearby points $\tau_1 = 0$ and $\tau_4 = T$ standard interpolation is carried out by the vector spline of degree five, that allows to calculate values $\boldsymbol{\varepsilon}_1 = \dot{\boldsymbol{\omega}}(\tau_1)$ and $\boldsymbol{\varepsilon}_4 = \dot{\boldsymbol{\omega}}(\tau_4)$

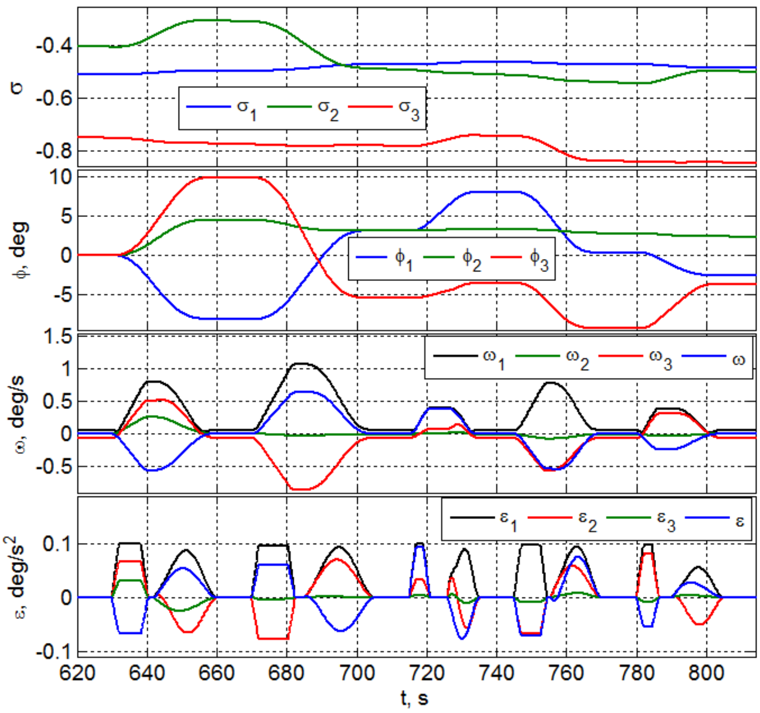

Figure 4. The SC vector spline guidance law for a scanning land-survey of Benevento, Rome, Florence, Padua and Munich

of angular acceleration vector. For four points $\tau_p \in \mathrm{T}$ values $\boldsymbol{\sigma}_p$, $p = 1 \div 4$ are computed, also values $\dot{\boldsymbol{\sigma}}_p$ and $\ddot{\boldsymbol{\sigma}}_p$, $p = 1, 4$ for two boundary points are calculated.

Interpolation of the MRP vector $\boldsymbol{\sigma}(t)$ $\forall t \in \mathrm{T}$ is carried out by the vector spline of 7 degree $\boldsymbol{\sigma}_a(t) = \sum_0^7 \mathbf{a}_s t^s$ with 8 columns $\mathbf{a}_s \in \mathbb{R}^3$, $s = 0 \div 7$ of unknown coefficients. Eight columns $\mathbf{a}_s$ are defined for spline $\boldsymbol{\sigma}_a(t)$ on the basis of (i) three boundary conditions $\boldsymbol{\sigma}_a(0) = \boldsymbol{\sigma}_1; \dot{\boldsymbol{\sigma}}_a(0) = \dot{\boldsymbol{\sigma}}_1; \ddot{\boldsymbol{\sigma}}_a(0) = \ddot{\boldsymbol{\sigma}}_1$ on the left end of interval T, that results in $\mathbf{a}_0 = \boldsymbol{\sigma}_1$, $\mathbf{a}_1 = \dot{\boldsymbol{\sigma}}_1$ and $\mathbf{a}_2 = \ddot{\boldsymbol{\sigma}}_1/2$; (ii) two conditions $\boldsymbol{\sigma}_a(\tau_2) = \boldsymbol{\sigma}_2; \boldsymbol{\sigma}_a(\tau_3) = \boldsymbol{\sigma}_3$; (iii) three boundary conditions $\boldsymbol{\sigma}_a(T) = \boldsymbol{\sigma}_4$; $\dot{\boldsymbol{\sigma}}_a(T) = \dot{\boldsymbol{\sigma}}_4$; $\ddot{\boldsymbol{\sigma}}_a(T) = \ddot{\boldsymbol{\sigma}}_4$. The matrix relation is formed for computing columns $\mathbf{a}_s$, $s = 3 \div 7$ and it is applied for simultaneous analytic computation of these five sought columns. Verification of the proposed method was carried out, we have obtained that the errors are $\delta\phi^{\mathrm{m}} = \max|\delta\boldsymbol{\phi}| = 0.03$ arc sec and $\delta\omega^{\mathrm{m}} \equiv \max|\delta\boldsymbol{\omega}| = 0.04$ arc sec/sec for arbitrary type of scanning observation at a route duration $T \leq 40$ s. The applied interpolation method allows a smooth conjugation for adjacent parts of a guidance law on values of vectors $\boldsymbol{\sigma}, \boldsymbol{\omega}$ and $\boldsymbol{\varepsilon}$ during a scanning observation with arbitrary duration (Somova, 2016*a*; Somova, 2016*b*).

For problem on synthesis of SC guidance law during its RM we have developed analytical method based on necessary and sufficient condition for solvability of Darboux problem. Here solution is obtained as the result of composition by three simultaneously derived rotations of "embedded" bases $\mathbf{E}_k$ about units $\mathbf{e}_k$, $k = 1 \div 3$ of Euler axes, quaternion $\boldsymbol{\Lambda}$ is defined as $\boldsymbol{\Lambda}(t) = \boldsymbol{\Lambda}_{\mathrm{i}} \circ \boldsymbol{\Lambda}_1(t) \circ \boldsymbol{\Lambda}_2(t) \circ \boldsymbol{\Lambda}_3(t)$, where $\boldsymbol{\Lambda}_k(t) = (\cos(\varphi_k(t)/2), \mathbf{e}_k \sin(\varphi_k(t)/2))$ and $\varphi_k(t)$ is angle of k's elementary rotation (Somov, 2016). Unit $\mathbf{e}_k$ is fixed in base $\mathbf{E}_{k-1}$, therefore the vectors are $\boldsymbol{\omega}_k(t) = \dot{\varphi}_k(t)\mathbf{e}_k$, $\boldsymbol{\varepsilon}_k(t) = \ddot{\varphi}_k(t)\mathbf{e}_k$, $\boldsymbol{\varepsilon}_k^*(t) \equiv \dot{\boldsymbol{\varepsilon}}_k(t) = \dddot{\varphi}_k(t)\mathbf{e}_k$. We use notations $\boldsymbol{\omega}^{(k)}, \boldsymbol{\varepsilon}^{(k)}, \dot{\boldsymbol{\varepsilon}}^{(k)}$ with $k = 1 \div 3$ for vectors $\boldsymbol{\omega}$, $\boldsymbol{\varepsilon}$ and $\dot{\boldsymbol{\varepsilon}}$ in base $\mathbf{E}_k$ and the vector operator $\mathbf{a}_{k-1}^{(k)} = \boldsymbol{\Phi}(\mathbf{a}_{k-1}, \boldsymbol{\Lambda}_k) \equiv \tilde{\boldsymbol{\Lambda}}_k \circ \mathbf{a}_{k-1} \circ \boldsymbol{\Lambda}_k$ for conversion from basis $\mathbf{E}_{k-1}$ to basis $\mathbf{E}_k$. Assume that we assigned vectors $\boldsymbol{\omega}_1(t) = \dot{\varphi}_1(t)\mathbf{e}_1$, $\boldsymbol{\varepsilon}_1(t) = \ddot{\varphi}_1(t)\mathbf{e}_1$ and $\dot{\boldsymbol{\varepsilon}}_1(t) = \dddot{\varphi}_1(t)\mathbf{e}_1$. Then vectors $\boldsymbol{\omega}(t)$,$\boldsymbol{\varepsilon}(t)$,$\dot{\boldsymbol{\varepsilon}}(t)$ in BRF are computed by the reccurrent formulas with $k = 2, 3$:

$$
\begin{aligned}
&\boldsymbol{\omega}_{k-1}^{(k)} = \boldsymbol{\Phi}(\boldsymbol{\omega}_{k-1}, \boldsymbol{\Lambda}_k); \boldsymbol{\varepsilon}_{k-1}^{(k)} = \boldsymbol{\Phi}(\boldsymbol{\varepsilon}_{k-1}, \boldsymbol{\Lambda}_k);\\
&\dot{\boldsymbol{\varepsilon}}_{k-1}^{(k)} = \boldsymbol{\Phi}(\dot{\boldsymbol{\varepsilon}}_{k-1}, \boldsymbol{\Lambda}_k);\\
&\boldsymbol{\omega}^{(k)} = \boldsymbol{\omega}_{k-1}^{(k)} + \boldsymbol{\omega}_k;\ \boldsymbol{\varepsilon}^{(k)} = \boldsymbol{\varepsilon}_{k-1}^{(k)} + \boldsymbol{\varepsilon}_k + \boldsymbol{\omega}_{k-1}^{(k)} \times \boldsymbol{\omega}_k;\\
&\dot{\boldsymbol{\varepsilon}}^{(k)} = \dot{\boldsymbol{\varepsilon}}_{k-1}^{(k)} + \dot{\boldsymbol{\varepsilon}}_k + \boldsymbol{\omega}_{k-1}^{(k)} \times \boldsymbol{\varepsilon}_k\\
&\qquad + (2\boldsymbol{\varepsilon}_{k-1}^{(k)} + \boldsymbol{\omega}_{k-1}^{(k)} \times \boldsymbol{\omega}_k) \times \boldsymbol{\omega}_k.
\end{aligned}
$$

In result we obtain vector functions $\boldsymbol{\omega}(t) = \boldsymbol{\omega}^{(3)}(t)$, $\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}^{(3)}(t)$ and $\boldsymbol{\varepsilon}^*(t) = \dot{\boldsymbol{\varepsilon}}(t) = \dot{\boldsymbol{\varepsilon}}^{(3)}(t)$ by explicit analytic relations.

Assume that quaternion $\boldsymbol{\Lambda}^* \equiv (\lambda_0^*, \boldsymbol{\lambda}^*) = \tilde{\boldsymbol{\Lambda}}_{\mathrm{i}} \circ \boldsymbol{\Lambda}_{\mathrm{f}}$ has unit $\mathbf{e}_3 = \boldsymbol{\lambda}^*/\sin(\varphi^*/2)$ for 3rd rotation on angle $\varphi^* = 2\arccos(\lambda_0^*)$. For quaternions of the 1st and 2nd rotations the boundary conditions $\boldsymbol{\Lambda}_1(t_{\mathrm{i}}^p) = \boldsymbol{\Lambda}_1(t_{\mathrm{f}}^p) = \boldsymbol{\Lambda}_2(t_{\mathrm{i}}^p) = \boldsymbol{\Lambda}_2(t_{\mathrm{f}}^p) = \mathbf{1}$ are applied, and for 3rd rotation – conditions $\boldsymbol{\Lambda}_3(t_{\mathrm{i}}^p) = \mathbf{1}$, $\boldsymbol{\Lambda}_3(t_{\mathrm{f}}^p) = (\cos(\varphi_3^{\mathrm{f}}/2), \mathbf{e}_3 \sin(\varphi_3^{\mathrm{f}}/2))$, where $\varphi_3^{\mathrm{f}} = \varphi^*$ and $\mathbf{1}$ is a unit quaternion. The Euler axis unit $\mathbf{e}_1$ of 1st rotation is assigned from the condition of its orthogonality to unit $\mathbf{e}_3$, unit $\mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1$. Vectors $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\boldsymbol{\varepsilon}^*(t)$ are presented in analytic form at assigning splines $\varphi_k(t)$ by different degrees, using three parts of given RM time interval T_p in general case :

1) initial part of the time-optimized acceleration under

constraints when the SC moves to its attitude motion with angular rate on fixed unit $\mathbf{e}_3$;

2) SC motion with a constant angular rate on unit $\mathbf{e}_3$;

3) final part to guarantee the specified boundary conditions on the RM right end when the sixth order scalar splines $\varphi_k(t)$ are applied, moreover all parameters of these splines are computed by explicit relations.

In result for sequence of the SRs and RMs at land-survey from current orbit we obtain the uniform vector spline attitude guidance law which is a vector command signal for the satellite ACS.

We assume that a land-survey SC is moving on sun-synchronous orbit with altitude 720 km, longitude of the ascending node (AN) 23.5 deg and it fulfills the task for the trace scanning optoelectronic observations of Benevento, Rome, Florence, Padua and Munich with duration of 10 sec each route, Fig. 1. At referencing a time t from the AN flyby moment, the guidance law synthesis was carried out for the following data:

the SC body orientation in ORF $\forall t \in [0, 630)$ s;

RM 1 $\forall t \in [630, 660)$ s with duration 30 s;

SR 1 (Benevento) $\forall t \in [660, 670)$s;

RM 2 $\forall t \in [670, 705)$s;

SR 2 (Rome) $\forall t \in [705, 715)$ s;

RM 3 $\forall t \in [715, 735)$ s;

SR 3 (Florence) $\forall t \in [735, 745)$ s;

RM 4 $\forall t \in [745, 770)$ s;

SR 4 (Padua) $\forall t \in [770, 780)$ s;

RM 5 $\forall t \in [780, 805)$ s;

SR 5 (Munich) $\forall t \in [805, 815)$ s.

Obtained results are presented in Fig. 4, where components of vectors $\boldsymbol{\sigma}(t)$, $\boldsymbol{\omega}(t)$ and $\boldsymbol{\varepsilon}(t)$ are marked by different colors – blue color on roll, green on yaw and red color on pitch, modules of vectors $\boldsymbol{\omega}(t)$ and $\boldsymbol{\varepsilon}(t)$ are marked by black color.

The aim of an area land-survey is to cover a given area on the Earth's surface with geographical center C by a sequence of partly overlapping scanning routes (OSRs, scans). The initial data for planning such a land-survey are the size of the area, parameters of the SC orbital motion, characteristics of the telescope and OECs with the possibility of reverse, restrictions on the kinematic parameters of satellite angular motion. Allowed values of azimuth deviation of orthodromic OSRs from the route are up to $\pm\pi/9$ and $(1 \pm 1/9)\pi$. The main steps in solving this problem are the following: determining required number of scans N and longitudinal IMV in the telescope focal plane during the OSR performing; synthesis of the SC guidance laws for runnig the central and side scans.

Fig. 2 represents the map with projections of scans and of telescope target line trace obtained in planning two single SRs (the first SM Antalya and the final SM Varna) and area land-surveying of neighborhoods of Istanbul for the SC on sun-synchronous orbit with altitude of 720 km and inclination of 98.27 deg, when the allowed deviation of the target line from Nadir is within the cone with semi-angle of 40 deg.

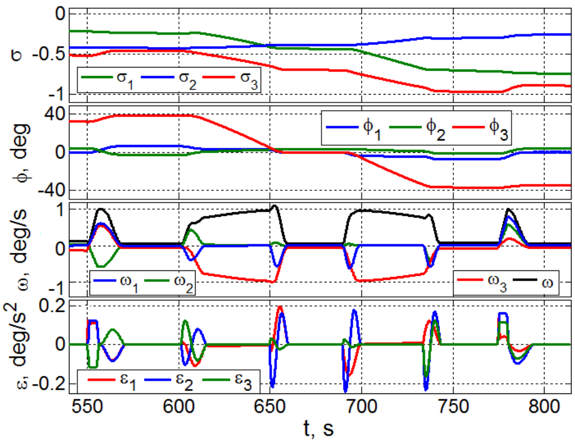

Figure 5. The SC vector spline guidance law for areal land-survey

In Fig. 5 we present the unified vector spline SC guidance law corresponding to the developed plan for the area land-survey. Here angles ϕ_i of the BRF orientation in the ORF, components of vectors $\boldsymbol{\sigma}(t)$, $\boldsymbol{\omega}(t)$ and $\boldsymbol{\varepsilon}(t)$ are marked by the same colors.

4 Control of the Gyro Moment Cluster

At the SC attitude gyromoment control the problem is appeared – a possibility of the GMC singular states. Into orthogonal canonical basis Oxyz (Fig. 3) the GD's AM units have the following projections:

$$x_1 = C_1; x_2 = C_2; y_1 = S_1; y_2 = S_2;$$
$$x_3 = S_3; x_4 = S_4; z_3 = C_3; z_4 = C_4;$$
$$y_5 = C_5; y_6 = C_6; z_5 = S_5; z_6 = S_6,$$

where $S_p \equiv \sin\beta_p$ and $C_p \equiv \cos\beta_p$. Then column $\mathbf{h}(\boldsymbol{\beta}) = \{\mathsf{x}, \mathsf{y}, \mathsf{z}\}$ of normed GMC's AM vector and matrix $\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \partial\mathbf{h}/\partial\boldsymbol{\beta}$ have the form

$$\mathbf{h}(\boldsymbol{\beta}) = \{\Sigma x_p, \Sigma y_p, \Sigma z_p\};$$

$$\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta}) = \begin{bmatrix} -y_1 & -y_2 & z_3 & z_4 & 0 & 0 \\ x_1 & x_2 & 0 & 0 & -z_5 & -z_6 \\ 0 & 0 & -x_3 & -x_4 & y_5 & y_6 \end{bmatrix}.$$

For *3-SPE* scheme singular state is appeared when the matrix Gramme $\mathbf{G}(\boldsymbol{\beta}) = \mathbf{A}_h(\boldsymbol{\beta})\mathbf{A}_h^{\mathrm{t}}(\boldsymbol{\beta})$ loses its full rang, e.g. when $\mathsf{G} \equiv \det\mathbf{G}(\boldsymbol{\beta}) = 0$. At introducing the notations

$$x_{12} = x_1 + x_2; \quad x_{34} = x_3 + x_4; \quad y_{12} = y_1 + y_2;$$
$$y_{56} = y_5 + y_6; \quad z_{34} = z_3 + z_4; \quad z_{56} = z_5 + z_6;$$
$$\tilde{x}_{12} = x_{12}/\sqrt{4 - y_{12}^2}\,; \; \tilde{x}_{34} = x_{34}/\sqrt{4 - z_{34}^2};$$
$$\tilde{y}_{12} = y_{12}/\sqrt{4 - x_{12}^2}\,; \; \tilde{y}_{56} = y_{56}/\sqrt{4 - z_{56}^2};$$
$$\tilde{z}_{34} = z_{34}/\sqrt{4 - x_{34}^2}\,; \; \tilde{z}_{56} = z_{56}/\sqrt{4 - y_{56}^2}$$

components of the GMC explicit vector tuning law

$$\mathbf{f}_\rho(\boldsymbol{\beta}) \equiv \{f_{\rho 1}(\boldsymbol{\beta}), f_{\rho 2}(\boldsymbol{\beta}), f_{\rho 3}(\boldsymbol{\beta})\} = \mathbf{0} \qquad (7)$$

are applied in the form

$$f_{\rho 1}(\boldsymbol{\beta}) \equiv \tilde{x}_{12} - \tilde{x}_{34} + \rho\,(\tilde{x}_{12}\,\tilde{x}_{34} - 1);$$
$$f_{\rho 2}(\boldsymbol{\beta}) \equiv \tilde{y}_{56} - \tilde{y}_{12} + \rho\,(\tilde{y}_{56}\,\tilde{y}_{12} - 1);$$
$$f_{\rho 3}(\boldsymbol{\beta}) \equiv \tilde{z}_{34} - \tilde{z}_{56} + \rho\,(\tilde{z}_{34}\,\tilde{z}_{56} - 1).$$

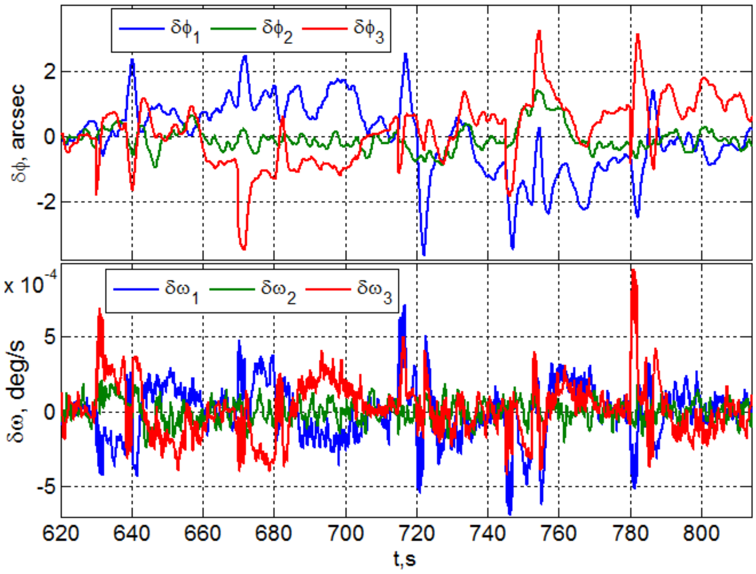

Figure 6. The SC attitude and angular rate errors during fulfillment of a space scanning land-survey with high spatial resolution

The analytical proof has been elaborated that vector tuning law (7) ensures absent of singular states in this GMC scheme for all values of the GMC AM vector inside its variation domain. For notations

$$x_{12} = (\mathsf{x}+\Delta_x)/2; \quad x_{34} = (\mathsf{x}-\Delta_x)/2;$$
$$y_{56} = (\mathsf{y}+\Delta_y)/2; \quad y_{12} = (\mathsf{y}-\Delta_y)/2;$$
$$z_{34} = (\mathsf{z}+\Delta_z)/2; \quad z_{56} = (\mathsf{z}-\Delta_z)/2$$

and $\mathbf{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ one can obtain the nonlinear vector equation $\mathbf{\Delta}(t) = \mathbf{\Phi}(\mathbf{h}(t), \mathbf{\Delta}(t))$. At a known vector $\mathbf{h}(t)$ this equation has single solution $\mathbf{\Delta}(t)$, which is readily computed by method of a simple iteration. Further units $\mathbf{h}_p(\beta_p(t))$ and vector columns $\boldsymbol{\beta}(t)$, $\dot{\boldsymbol{\beta}}(t)$, $\ddot{\boldsymbol{\beta}}(t)$ are calculated by explicit analytical relations.

For the *2-SPE* scheme such evaluation is carried out by the explicit analytical formulas only.

5 Filtering and Robust Digital Control

For continuous forming the control torque $\mathbf{M}^{\mathrm{g}}$ (2) and the SC model as a free rigid body the simplified controlled plant is as follows

$$\dot{\mathbf{\Lambda}} = \mathbf{\Lambda}\circ\boldsymbol{\omega}/2; \;\; \mathbf{J}\dot{\boldsymbol{\omega}} + [\boldsymbol{\omega}\times]\mathbf{G}^o = \mathbf{M}^{\mathrm{g}}; \;\; \dot{\boldsymbol{\beta}} = \mathbf{u}^{\mathrm{g}}(t)$$

and we suggest to form the GMC's control torque vector $\mathbf{M}^{\mathrm{g}} = \boldsymbol{\omega}\times\mathbf{G}^o + \mathbf{J}(\mathbf{C}^{\mathrm{e}}\varepsilon^p(t) - [\boldsymbol{\omega}\times]\mathbf{C}^{\mathrm{e}}\boldsymbol{\omega}^p(t) + \tilde{\mathbf{m}})$, where vector $\tilde{\mathbf{m}}$ is a stabilizing component.

For period T_p applied filter has the discrete transfer function $\mathrm{W_f(z_p)} = (1+\mathrm{b}_1)/(1+\mathrm{b}_1\mathrm{z}_\mathrm{p}^{-1})$, where coefficient $\mathrm{b}_1 = -\exp(-T_p/T_{\mathrm{f}})$ with a time constant T_{f} and $\mathrm{z_p} = \exp(\mathrm{s}T_p)$ with a complex variable s of Laplace transformation. For $l \in \mathbb{N}_0$ and the MRP vector error $\boldsymbol{\sigma}_l^{\mathrm{e}} = \mathbf{e}_l/(1+e_{0l})$ a filtering is executed by relations $\tilde{\mathbf{x}}_{l+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_l + \tilde{\mathbf{B}}\boldsymbol{\sigma}_l^{\mathrm{e}}$ and $\boldsymbol{\sigma}_l^{\mathrm{ef}} = \tilde{\mathbf{C}}\tilde{\mathbf{x}}_l + \tilde{\mathbf{D}}\boldsymbol{\sigma}_l^{\mathrm{e}}$, where matrices $\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ have diagonal form with $\tilde{a}_i = -\mathrm{b}_1^{\mathrm{f}}$; $\tilde{b}_i = \mathrm{b}_1^{\mathrm{f}}$; $\tilde{c}_i = -(1+\mathrm{b}_1^{\mathrm{f}})$ and $\tilde{d}_i = 1+\mathrm{b}_1^{\mathrm{f}}$.

Applied stabilizing component $\tilde{\mathbf{m}}_k$ is formed as follows: $\boldsymbol{\epsilon}_k = -\boldsymbol{\sigma}_k^{\mathrm{ef}}$;

$$\mathbf{g}_{k+1} = \mathbf{B}\mathbf{g}_k + \mathbf{C}\boldsymbol{\epsilon}_k; \, \tilde{\mathbf{m}}_k = \mathbf{K}(\mathbf{g}_k + \mathbf{P}\boldsymbol{\epsilon}_k). \quad (8)$$

Here matrices $\mathbf{B}, \mathbf{C}, \mathbf{P}$ and $\mathbf{K}$ also have diagonal form with $p_i = (1-b_i)/(1-a_i)$; $c_i = p_i(b_i - a_i)$ and $a_i = (d_u\tau_{1i}-1)/(d_u\tau_{1i}+1)$; $b_i = (d_u\tau_{2i}-1)/(d_u\tau_{2i}+1)$, where $d_u = 2/\mathrm{T}_u$, and τ_{1i}, τ_{2i}, k_i are the tuning parameters for ensuring the robust properties of the SC gyromoment ACS. Moreover only the SC attitude *filtered* MRP vector error $\boldsymbol{\sigma}_k^{\mathrm{ef}}$ is applied for forming the stabilizing component $\tilde{\mathbf{m}}_k$ (8).

The GMC's control torque is digitally formed by the analytic relations

$$\hat{\mathbf{G}}_k^o = \mathbf{J}\hat{\boldsymbol{\omega}}_k + \boldsymbol{\mathcal{H}}_k; \mathbf{C}_k^{\mathrm{e}} = \mathbf{C}(\boldsymbol{\mathcal{E}}_k^{\mathrm{f}}); \;\; \mathbf{M}_k^{\mathbf{g}} = \hat{\boldsymbol{\omega}}_k\times\hat{\mathbf{G}}_k^o + \mathbf{J}(\mathbf{C}_k^{\mathrm{e}}\boldsymbol{\varepsilon}_k^p - [\hat{\boldsymbol{\omega}}_k\times]\mathbf{C}_k^{\mathrm{e}}\boldsymbol{\omega}_k^p + \tilde{\mathbf{m}}_k). \quad (9)$$

The continuous tuning law $\mathbf{f}_\rho(\boldsymbol{\beta}(t)) = \mathbf{0}$ (7) bijectively connects continuous vector $\mathbf{M}^{\mathrm{g}}(t)$ with vectors $\boldsymbol{\beta}(t)$ and $\dot{\boldsymbol{\beta}}(t) = \mathbf{u}^{\mathrm{g}}(t)$.

Taking into account restrictions (3) the GDs' control vector $\mathbf{u}^{\mathrm{g}}(t)$ is formed by the analytic relations

$$\mathbf{A}_{\mathrm{h}}(\boldsymbol{\beta})\,\mathbf{u}^{\mathrm{g}}(t) = -\mathbf{M}^{\mathrm{g}}(t)/\mathrm{h_g}; \; \dot{\boldsymbol{\beta}}(t) = \mathbf{u}^{\mathrm{g}}(t);$$
$$(\partial\mathbf{f}_\rho(\boldsymbol{\beta})/\partial\boldsymbol{\beta})\,\mathbf{u}^{\mathrm{g}}(t) = -\mathbf{Sat}(\mu\,\mathbf{f}_\rho(\boldsymbol{\beta}), \Delta\bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}}) \quad (10)$$

with $\Delta\bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}} = \bar{\mathrm{u}}_{\mathrm{g}}^{\mathrm{m}} - \bar{\mathrm{u}}_{\mathrm{g}}$, $\mathbf{Sat}(\mu\,\mathbf{x}, a) = \{\mathrm{Sat}(\mu\,x_i, a)\}$, where μ is a constant parameter.

The GDs' digital control vector $\mathbf{u}_k^{\mathrm{g}}$ is carried out by relations (9) and (10) presented in discrete form (Somov, 2016).

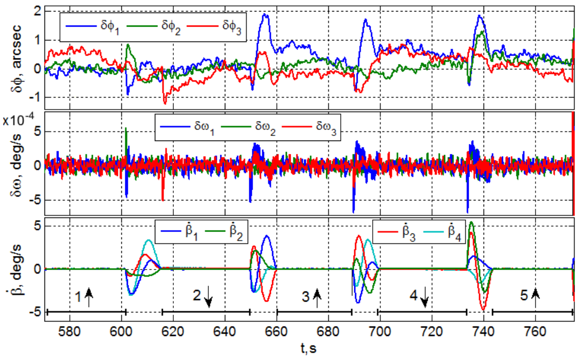

Figure 7. Errors during areal land-survey and the GD angular rates

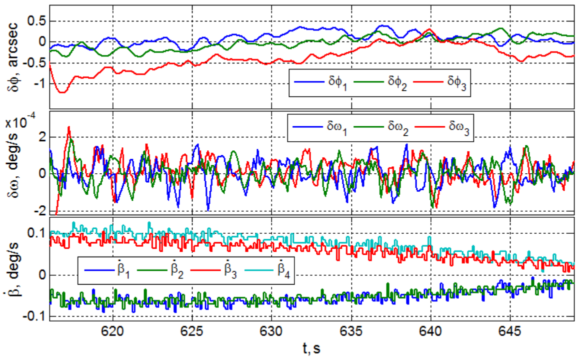

Figure 8. Errors during second OSR and the GD angular rates

6 Simulation of the ACS Operation

Developed algorithms for discrete filtering and digital control of GMC were simulated taking into account the vector spline guidance laws at different sequences of the SRs and RMs.

In Fig. 6 the SC attitude and angular rate errors are presented during fulfillment of a land-surveying Benevento, Rome, Florence, Padua and Munich.

Figs. 7 & 8 present errors on stabilization of the SC angular motion and the GD angular rates (*2-SPE* scheme) during the areal land-survey. In the lower part of Fig. 7 we have pointed the time intervals of the OSMs with their indexes and scanning directions. .

Conclusions

We have considered problems on attitude guidance and robust gyromoment control of agile land-survey spacecraft. We briefly have presented methods for synthesis of nonlinear guidance and attitude control laws, and also results on dynamic research of the spacecraft attitude control system under disturbances and digital control of the gyro moment clusters.

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