Robust Model Predictive Control for Spacecraft Rendezvous with Online Prediction of Disturbance Bounds ⋆

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Abstract: A Model Predictive Controller is introduced to solve the problem of rendezvous of spacecraft, using the HCW model and including additive disturbances and line-of-sight constraints. It is shown that a standard MPC is not able to cope with disturbances. Then a robust Model Predictive Control that introduces the concepts of robust satisfaction of constraints is proposed. The formulation also includes a predictor of the disturbance properties which are needed in the robust algorithm. In simulations it is shown that the robust MPC scheme is able to handle not only additive disturbances (which are the ones used in the formulation) but also large multiplicative disturbances and unmodelled dynamics (due to eccentricity of the orbit of the target spacecraft).

Keywords: Spacecraft autonomy, Space robotics, Predictive control, Robustness, Random perturbations, Prediction methods.

1. INTRODUCTION

Technology enabling simple autonomous spacecraft rendezvous and docking is becoming a growing necessity as access to space continues increasing. After decades of development, many approaches have been proposed and there have been many experiences, positive and negative; see Woffinden and Geller (2008) for an historical account or Fehse (2003) for the basics. However, no approach has emerged as universally successful, and autonomous rendezvous is still a subject open to further investigation. Correspondingly, the field of has become very active in recent years, with an increasingly growing literature; see, for instance, Richards and How (2003), P.K.C. Wang (2003), Geller (2006), or Breger and How (2008).

This work approaches the problem of rendezvous of spacecraft using Model Predictive Control (MPC) [Camacho and Bordons (2004)]. MPC originated in the late seventies and has developed considerably since then. There are many applications of predictive control successfully in use at the current time, not only in the process industry but also applications to the control of other processes ranging from solar technology [Camacho et al. (1994)] to flight control [Breger and How (2006)]. Model Predictive Control is considered to be a mature technique for linear and rather slow systems like the ones usually encountered in the process industry, even though more complex systems, such as nonlinear, hybrid, or very fast processes, are not so amenable to the methods of MPC. Hence MPC is very suitable to deal with the problem of spacecraft rendezvous, which is inherently slow and can be modeled by linear equations (shown in Section 2). See for instance Richards and How (2003), where the use of MPC is analyzed for the rendezvous problem and compared with other methods.

The term Model Predictive Control does not designate a specific control strategy but rather an ample range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function over a finite receding horizon. In MPC the process model is used to predict the future plant outputs, based on past and current values and on the proposed optimal future control actions. These actions are calculated by the optimizer taking into account the cost function (where the future tracking error is considered) as well as the constraints.

One of the advantages of MPC is that robust control ideas can be easily incorporated. The key idea is to take into account disturbances and uncertainties about the process in an explicit manner and to design MPC to optimize the objective function for the worst situation of the disturbances/uncertainties [Camacho and Bordons (2004)]. However it is necessary to obtain an estimate of some of the properties, such as upper and lower bounds or the mean of the disturbances and uncertainties. In Richards (2004), some methods for estimation of uncertainty properties are proposed.

⋆ The authors would like to acknowledge finantial support of the Spanish Ministry of Science and Innovation and of the European Commission for funding part of this work under grants DPI2008-05818 and project FPT ICT HD-MPC.

We use both techniques (robust MPC design and disturbance properties estimation) in this work, and show that they can successfully be applied to solve the problem of spacecraft rendezvous with large, persistent disturbances. It is also shown that our design also allows for a successful rendezvous even in the case that the target orbit is elliptic—implying that the HCW model used for MPC computations is no longer accurate.

The structure of this paper is as follows. In Section 2 we introduce the mathematical model for rendezvous spacecraft used for MPC and the constraints of the rendezvous problem. We follow in Section 3 with a formulation of Model Predictive Control suitable for the problem at hand, introducing the concepts of robust constraint satisfaction and estimation of disturbance properties. In Section 4 we show simulations for the unperturbed case and the perturbed case with large disturbances. We also show that our control design is successful for elliptical target orbits, with the discrepancies due to ellipticity considered as a disturbance. We close the paper with some remarks in Section 5.

2. MODEL OF SPACECRAFT RENDEZVOUS

There are numerous mathematical models for spacecraft rendezvous; which one should be used depends on the parameters of the scenario. In Carter (1998) a survey of numerous mathematical models for spacecraft rendezvous can be found.

For instance, if the target is orbiting in a *circular* keplerian orbit and approaching vehicle is very close to the target, then the linear Hill-Clohessy-Wiltshire (HCW) equations, as introduced by Hill (1878) and Clohessy and Wiltshire (1960), describe with adequate precision the relative position of the spacecraft. The HCW model is the one we use throughout this paper, even though we include the possibility of disturbances to allow for unmodeled effects.

Remark 1. It must be noted that, in many situations, the HCW equations are not accurate. For instance, if the chaser vehicle is far from the target then linearization is no longer possible. Other situations include the possibility that the target is moving in a keplerian *eccentric* orbit (see Inalhan et al. (2002)) or that some orbital perturbations are taken into account (see for example Humi and Carter (2008)). In Section 4 we consider the target orbiting in an eccentric keplerian trajectory, and show that our control design (based on the HCW equations) still works.

The HCW model assumes that the target vehicle is passive and moving along a circular orbit of radius R. Thus the angular speed of the target through its orbit is $n = \sqrt{\frac{\mu}{R^3}}$, where μ is the gravitation parameter of the Earth, $\mu = 398600.4\ \text{km}^3/\text{s}^2$.

Considering that the control inputs are constant for the sample time T, we can use the following discrete time version of the HCW equations with disturbances:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) + \delta(k). \tag{1}$$

In (1), $\mathbf{x}(k)$, $\mathbf{u}(k)$ and $\delta(k)$ denote respectively the state, input, and disturbance at time k, where

$$\mathbf{x} = [x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}]^T,\ \mathbf{u} = [u_x\ u_y\ u_z]^T, \tag{2}$$

$$\delta = [\delta_x\ \delta_y\ \delta_z\ \delta_{\dot{x}}\ \delta_{\dot{y}}\ \delta_{\dot{z}}]^T. \tag{3}$$

In these definitions, x, y, and z denote the position of the chaser in a local–vertical/local–horizontal (LVLH) frame of reference fixed on the center of gravity of the target vehicle. In the LVLH frame, x refers to the radial position, y to the in-track position, and z to the cross-track position. The velocity of the chaser in the LVLH frame is given by $\dot{x}$, $\dot{y}$, and $\dot{z}$. The variables u_x, u_y, and u_z are the inputs (thrust actuation) acting on the chaser vehicle, whereas δ_x, δ_y, δ_z, $\delta_{\dot{x}}$, $\delta_{\dot{y}}$, and $\delta_{\dot{z}}$ represent the disturbances entering the system. Both are referred to the LVLH axes as indicated by their respective subscripts.

The matrices A and B appearing in (1) are given by

$$A = \begin{bmatrix} 4-3C & 0 & 0 & \frac{S}{n} & \frac{2(1-C)}{n} & 0 \\ 6(S-nT) & 1 & 0 & -\frac{2(1-C)}{n} & \frac{4S-3nT}{n} & 0 \\ 0 & 0 & C & 0 & 0 & \frac{S}{n} \\ 3nS & 0 & 0 & C & 2S & 0 \\ -6n(1-C) & 0 & 0 & -2S & 4C-3 & 0 \\ 0 & 0 & -nS & 0 & 0 & C \end{bmatrix}, \tag{4}$$

$$B = \begin{bmatrix} \frac{1-C}{n^2} & \frac{2nT-2S}{n^2} & 0 \\ \frac{2(S-nT)}{n^2} & -\frac{3T^2}{2} + 4\frac{1-C}{n^2} & 0 \\ 0 & 0 & \frac{1-C}{n^2} \\ \frac{S}{n} & 2\frac{1-C}{n} & 0 \\ \frac{2(C-1)}{n} & -3T + 4\frac{S}{n} & 0 \\ 0 & 0 & \frac{S}{n} \end{bmatrix}, \tag{5}$$

where $S = \sin nT$ and $C = \cos nT$ (the sampling time chosen in this work is 60 s). The disturbances are unknown, so we assume $\delta(k)$ is a random vector, with mean $\bar{\delta}$ and covariance Σ, which are also unknown. These disturbances might arise from errors in the input signals (as thrusters are typically subject to command uncertainties and are never perfectly aligned), or they could also be thought of as unmodeled dynamics (in which case they are not random; however we keep the randomness assumption for convenience). In Section 4 we show some of these disturbance models.

Remark 2. Even though we are modelling the disturbances as additive, in Section 4 we show that our control scheme works for other kind of disturbances such as multiplicative disturbance or modelling errors.

2.1 Constraints on the problem

For sensing purposes (see Breger and How (2008)), during rendezvous it is required that the chaser vehicle remains inside a line of sight (LOS) area. This LOS region is the intersection of a cone (given by the equations $-x_0 - c_{LOS}y \le x \le x_0 + c_{LOS}y$) and the region $y \ge 0$, as shown in Figure 1.

The LOS constraint is formulated as $A_{LOS}\mathbf{x}(k) \le b_{LOS}$, where

$$A_{LOS} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & -c_{LOS} & 0 & 0 & 0 & 0 \\ -1 & c_{LOS} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad b_{LOS} = \begin{bmatrix} 0 \\ x_0 \\ x_0 \end{bmatrix}. \tag{6}$$

We also assume that the control inputs are bounded above and below:

$$\mathbf{u}_{min} \le \mathbf{u}(k) \le \mathbf{u}_{max}. \tag{7}$$

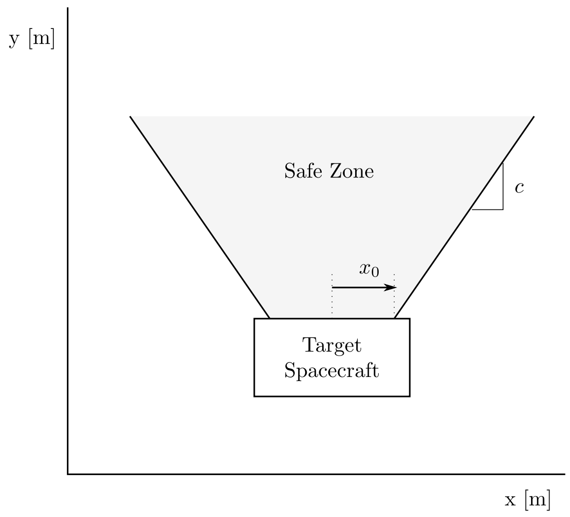

Fig. 1. Line of Sight region.

3. MPC FORMULATION

We next formulate our robust MPC scheme; first we develop some notation and formulate the general problem, and afterwards we explain how to tackle the disturbances appearing in (1).

3.1 Prediction of the state

The state at time $k+j$, given the state at time k, and the input signals and disturbances from time k to time $k+j-1$, is computed by applying recursively Equation (1):

$$\mathbf{x}(k+j) = A^j\mathbf{x}(k) + \sum_{i=0}^{j-1} A^{j-i-1}B\mathbf{u}(k+i) + \sum_{i=0}^{j-1} A^{j-i-1}\delta(k+i). \tag{8}$$

Define now $\mathbf{x_S}(k)$, $\mathbf{u_S}(k)$, $\delta_\mathbf{S}(k)$ as a stack of N_p states, input signals, and disturbances beginning at time k, where N_p is the prediction horizon.

$$\mathbf{x_S}(k) = \begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}(k+2) \\ \vdots \\ \mathbf{x}(k+N_p) \end{bmatrix}, \mathbf{u_S}(k) = \begin{bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \vdots \\ \mathbf{u}(k+N_p-1) \end{bmatrix},$$
$$\delta_\mathbf{S}(k) = \begin{bmatrix} \delta(k) \\ \delta(k+1) \\ \vdots \\ \delta(k+N_p-1) \end{bmatrix}. \tag{9}$$

Then, we have

$$\mathbf{x_S}(k) = \begin{bmatrix} A\mathbf{x}(k) + B\mathbf{u}(k) + \delta(k) \\ A^2\mathbf{x}(k) + \sum_{i=0}^{1} A^{1-i}\left(B\mathbf{u}(k+i) + \delta(k+i)\right) \\ \vdots \\ A^{N_p}\mathbf{x}(k) + \sum_{i=0}^{N_p-1} A^{N_p-1-i}\left(B\mathbf{u}(k+i) + \delta(k+i)\right) \end{bmatrix}, \tag{10}$$

which can be written as

$$\mathbf{x_S}(k) = \mathbf{F}\mathbf{x}(k) + \mathbf{G_u}\mathbf{u_S}(k) + \mathbf{G}_\delta\delta_\mathbf{S}, \tag{11}$$

where $\mathbf{G_u}$ and $\mathbf{G}_\delta$ are block lower triangular matrix with its non-null elements defined by $(\mathbf{G_u})_{ij} = A^{i-j}B$ and $(\mathbf{G}_\delta)_{ij} = A^{i-j}$, and the matrix $\mathbf{F}$ is defined as:

$$\mathbf{F} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_p} \end{bmatrix}. \tag{12}$$

Remark 3. Note that we assume that we have perfect knowledge of the state vector $\mathbf{x}(k)$. If it were not accessible, a Kalman filter [Camacho and Bordons (2004)] would be required.

3.2 Objective Funtion

Taking mathematical expectation, we define $\hat{\mathbf{x}}(k+j|k) = E[\mathbf{x}(k+j)]$, the expected value of $\mathbf{x}(k+j)$ given $\mathbf{x}(k)$. Similarly define $\hat{\mathbf{x}}_\mathbf{S}(k+j|k) = E[\mathbf{x_S}(k+j)]$. For the MPC formulation we use the following cost function:

$$J(k) = \sum_{i=1}^{N_p}\left[\hat{\mathbf{x}}^T(k+i|k)R(k+i)\hat{\mathbf{x}}(k+i|k)\right] + \sum_{i=1}^{N_p}\left[\mathbf{u}^T(k+i-1)Q\mathbf{u}(k+i-1)\right], \tag{13}$$

where $Q = \mathrm{Id}_{3\times3}$ and where the matrix $R(k)$ is defined as

$$R(k) = \gamma h(k-k_a)\begin{bmatrix} \mathrm{Id}_{3\times3} & \Theta_{3\times3} \\ \Theta_{3\times3} & \Theta_{3\times3} \end{bmatrix}. \tag{14}$$

In (14), h is the step function, k_a is the desired arrival time for docking, γ a large number, and $\mathrm{Id}_{3\times3}$, $\Theta_{3\times3}$ are respectively the identity matrix and a matrix full of zeroes, both of order 3 by 3.

The reason for choosing (13) is that we wish to arrive at the origin at time k_a (and *remain* there) and at the same time minimize the control effort.

Using (11), and since $E[\delta(k+i)] = \bar{\delta}$, Equation (13) can be rewritten as:

$$J(k) = (\mathbf{G_u}\mathbf{u_S}(k) + \mathbf{F}\mathbf{x}(k) + \mathbf{G}_\delta\bar{\delta}_\mathbf{S})^T\mathbf{R_S} \times(\mathbf{G_u}\mathbf{u_S}(k) + \mathbf{F}\mathbf{x}(k) + \mathbf{G}_\delta\bar{\delta}_\mathbf{S}) + \mathbf{u_S}^T\mathbf{Q_S}\mathbf{u_S}, \tag{15}$$

where $\bar{\delta}_\mathbf{S}$ is a stack vector with $\bar{\delta}$ repeated N_p times, $\mathbf{Q_S} = \mathrm{Id}$ and $\mathbf{R_S}$ is a block diagonal matrix defined by:

$$\mathbf{R_S} = \begin{bmatrix} R(k+1) & & \\ & \ddots & \\ & & R(k+N_p) \end{bmatrix}. \tag{16}$$

Using the notation above developed with the constraints formulated in Section 2.1, the constraints equations for the state can be rewritten as:

$$\mathbf{A}_c \mathbf{x}_\mathbf{S} \leq \mathbf{b}_c, \tag{17}$$

where $\overline{A}_c$ and $\overline{b}_c$ are given by:

$$\mathbf{A}_c = \begin{bmatrix} A_{LOS} & & & \\ & A_{LOS} & & \\ & & A_{LOS} & \\ & & & A_{LOS} \end{bmatrix}, \mathbf{b}_c = \begin{bmatrix} b_{LOS} \\ b_{LOS} \\ \vdots \\ b_{LOS} \end{bmatrix}. \tag{18}$$

Using equation (11), one can reformulate the LOS constraints as constraints for the control signals in the following way:

$$\mathbf{A}_c \mathbf{G_u u_S} \leq \mathbf{b}_c - \mathbf{A}_c \mathbf{F} x(k) - \mathbf{A}_c \mathbf{G}_\delta \delta_\mathbf{S}, \tag{19}$$

and similarly we can write (7) as:

$$\mathbf{u}_{min} \leq \mathbf{u_S} \leq \mathbf{u}_{max}. \tag{20}$$

3.3 Computation of control input

To compute the control input at time k, one seeks the control signal that minimizes the cost function over the prediction horizon, satisfying at the same time the constraints:

$$\begin{aligned} \min_{\mathbf{u_S}} \quad & J(x(k), \mathbf{u_S}, \overline{\delta}_\mathbf{S}) \qquad (21)\\ \text{subject to: } & \mathbf{A}_c \mathbf{G_u u_S} \leq \mathbf{b}_c - \mathbf{A}_c \mathbf{F} x(k) - \mathbf{A}_c \mathbf{G}_\delta \delta_\mathbf{S}, \; \forall \delta_\mathbf{S} \\ \mathbf{u}_{min} \quad & \leq \mathbf{u_S} \leq \mathbf{u}_{max}. \end{aligned}$$

Since the cost function is quadratic and the constraints are linear, if the future δ is perfectly known (for example, the undisturbed case) then (21) can be solved; the control $\mathbf{u}(k)$ is set to the first three components of $\mathbf{u_S}$, and the computation is repeated for every time step.

However, if the disturbances are not known but rather we know the mean and some bounds on them, it is necessary to modify (21), as we explain next.

3.4 Robust satisfaction of constraints

Assume now that we know bounds for the disturbances δ given as $(\delta_x)_{\min} \leq \delta_x \leq (\delta_x)_{\max}$ and similarly for the rest of the components of δ, which we summarize as $\mathbf{A}_\delta \delta_\mathbf{S} \leq \mathbf{c}_\delta$; we assume the region defined by this constraint is enclosed by a polytope. Then, we are able to eliminate the disturbances from (19) by bounding the term $-\mathbf{A}_c \mathbf{G}_\delta \delta_\mathbf{S}$. This would give us

$$\mathbf{A}_c \mathbf{G_u u_S} \leq \mathbf{b}_c - \mathbf{A}_c \mathbf{F} x(k) + \mathbf{b}_\delta, \tag{22}$$

where $\mathbf{b}_\delta$ is column vector, whose i-th term $(\mathbf{b}_\delta)_i$ is given by

$$(\mathbf{b}_\delta)_i = \min_{\text{s.t.} \mathbf{A}_\delta \delta_\mathbf{S} \leq \mathbf{c}_\delta} a_i \delta_\mathbf{S}, \tag{23}$$

where a_i is the i-th *row* of the matrix $-\mathbf{A}_c \mathbf{G}_\delta$. Since the function to minimize is linear and the feasible region is enclosed by a polytope, this minimization can be rapidly solved.

Equation (20) represents the constraints computed for the *worst-case* disturbances. Hence, enforcing (20) we are robustly satisfying the constraints (17), i.e., satisfying them for *any possible disturbance*.

The control input at time k is now computed from

$$\begin{aligned} \min_{\mathbf{u_S}} \quad & J(x(k), \mathbf{u_S}, \overline{\delta}_\mathbf{S}) \qquad (24)\\ \text{subject to: } & \mathbf{A}_c \mathbf{G_u u_S} \leq \mathbf{b}_c - \mathbf{A}_c \mathbf{F} x(k) + \mathbf{b}_\delta, \\ \mathbf{u}_{min} \quad & \leq \mathbf{u_S} \leq \mathbf{u}_{max}, \end{aligned}$$

where now everything is known except for the control inputs to be computed. Note that we need to calculate (23) at every time step and for every row of the $-\mathbf{A}_c \mathbf{G}_\delta$ matrix. However it is a linear optimization with linear constraints which can be implemented very efficiently.

3.5 Disturbance estimation algorithm

The robust satisfaction of constraints presented in Section 3.4 requires knowledge of disturbance bounds. However, it is often the case that disturbance are completely unknown and such bounds have to obtained online.

To do so, we make the assumption that the disturbances are normally distributed with mean $\bar{\delta}$ and variance Σ, i.e., $\delta \sim N_6(\bar{\delta}, \sqrt{\Sigma})$.

At each time k we estimate $\bar{\delta}$ and variance Σ taking into account all past disturbances, which can be computed a posteriori as

$$\delta(i) = \mathbf{x}(i+1) - A\mathbf{x}(i) - B\mathbf{u}(i), \tag{25}$$

for $i = 1, \ldots, k-1$.

Then $\hat{\delta}(k)$ and $\hat{\Sigma}(k)$, the estimates of $\bar{\delta}$ and variance Σ at time k, are given by

$$\hat{\delta}(k) = \frac{\sum_{i=0}^{k-1} \mathrm{e}^{-\lambda(k-i)} \delta(i)}{\sum_{i=0}^{k-1} \mathrm{e}^{-\lambda(k-i)}}, \tag{26}$$

$$\hat{\Sigma}(k) = \frac{\sum_{i=0}^{k-1} \mathrm{e}^{-\lambda(k-i)} \left(\delta(i) - \hat{\delta}(i)\right) \left(\delta(i) - \hat{\delta}(i)\right)^T}{\sum_{i=0}^{k-1} \mathrm{e}^{-\lambda(k-i)}}, \tag{27}$$

where $\lambda > 0$ is a *forgetting factor*. Even though we have assumed the disturbances are just a random variable, this would help accommodate the case in which they are a *random process*, i.e., their statistical properties change with time.

Define $\gamma_k = \sum_{i=0}^{k-1} \mathrm{e}^{-\lambda(k-i)}$. Using the sum of a geometric progression, we have that

$$\gamma_k = \frac{\mathrm{e}^{-\lambda}\left(1 - \mathrm{e}^{-\lambda}\right)}{1 - \mathrm{e}^{-\lambda k}}. \tag{28}$$

We can define recursive formulas for (26)–(27) as follows:

$$\hat{\delta}(k) = \frac{\mathrm{e}^{-\lambda}}{\gamma_k} \left(\gamma_{k-1} \hat{\delta}(k-1) + \delta(k)\right), \tag{29}$$

$$\hat{\Sigma}(k) = \frac{\mathrm{e}^{-\lambda}}{\gamma_k} \Bigg(\gamma_{k-1} \hat{\Sigma}(k-1) + \left(\delta(k) - \hat{\delta}(k)\right) \left(\delta(k) - \hat{\delta}(k)\right)^T \Bigg). \tag{30}$$

These formulas allow us to save memory, only needing to store the last estimate of the mean and covariance.

Once the mean and covariance is known, we can obtain a "confidence interval" (whose statistical validity will depend on the number of disturbance samples that have been processed and on how approximate the normality assumption is). This interval is used for the computations

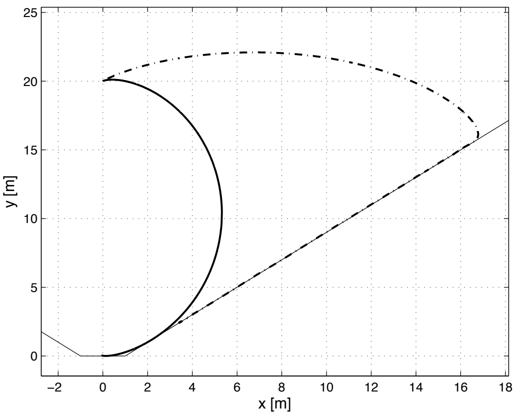

Fig. 2. Non-robust MPC without disturbances (solid line), and with disturbances (dashed line)

of Section 3.4. For instance, for the bounds $(\delta_x)_{\min} \leq \delta_x(k) \leq (\delta_x)_{\max}$ at time k we can take $\hat{\delta}_x(k) \pm \alpha\sqrt{\hat{\Sigma}_{xx}(k)}$, where α depends on how tight we wish to make the confidence interval (for a few samples α should be large, while if many are available it can be set to 3 or 4).

4. SIMULATION RESULTS

Using the model of Section 2, different scenarios are considered. Even though the disturbances in (1) are additive, we consider two types of disturbances: those coming from thruster errors and those due to unmodelled dynamics.

For the first type of disturbance, we considered that the real output u coming from the thrusters is not exact, but rather takes the form:

$$\mathbf{u}_{\text{real}} = T(\delta\theta)\mathbf{u}(1 + \delta_1) + \delta_2, \tag{31}$$

where $\mathbf{u}_{\text{real}}$ is the output coming from the thrusters, $\mathbf{u}$ is the commanded output obtained in the control laws, δ_1 and δ_2 are random variables, and $T(\delta\theta)$ is a rotation matrix where $\delta\theta$ is a vector of small, random angles modelling imperfect alignment. Hence in this case $\delta = B\left((T(\delta\theta) - \text{Id})\,\mathbf{u}(1 + \delta_1) + \delta_2\right)$, which is not strictly an additive disturbance.

The results obtained by the non-robust controller (21), where the disturbances are just ignored, are shown in Fig. 2, both for the nominal system ($\mathbf{u}_{\text{real}} = \mathbf{u}$) and the perturbed system. The non-robust controller achieves perfect rendezvous for the nominal case satisfying the constraints, whereas in the perturbed case the controller violates the constraints and is not able to reach the target.

Fig. 3 shows the result obtained when a robust MPC is applied to the perturbed case. The spacecraft evolution is shown in Fig. 3(a). The spacecraft achieves rendezvous without violating the constraints. The corresponding control efforts are shown in Fig. 3(b). The solid line shows the commanded signal $\mathbf{u}$ and the dotted line shows $\mathbf{u}_{\text{real}}$.

Figure 4 shows the result obtained with rather high perturbations applied. As can be seen the controller is able to achieve rendezvous and constraint satisfaction.

Unmodelled dynamics due by eccentricity ($e = 0.2$) in the target orbit are considered next. The corresponding spacecraft path is plotted in Fig. 5. The controller is able to arrive at the origin in spite of using an incorrect model and with disturbances in the thrusters.

5. CONCLUDING REMARKS

In this paper we have presented a robust MPC controller to solve the problem of rendezvous of spacecraft, using the HCW model with disturbances and line-of-sight constraints, We show that standard MPC is not able to cope with disturbances. Thus we formulated a robust Model Predictive Control introducing robust constraint satisfaction and estimation of disturbance properties. In simulations it is shown that the robust MPC scheme is able to handle not only additive disturbances but also large multiplicative disturbances and unmodelled dynamics (due to eccentricity of the orbit of the target spacecraft).

Future work might include the addition of safety constraints [Breger and How (2008)] in the robust scheme, more sophisticated disturbance estimation models, and considering eccentricity in the target's orbit, which makes the model time varying [Inalhan et al. (2002)].

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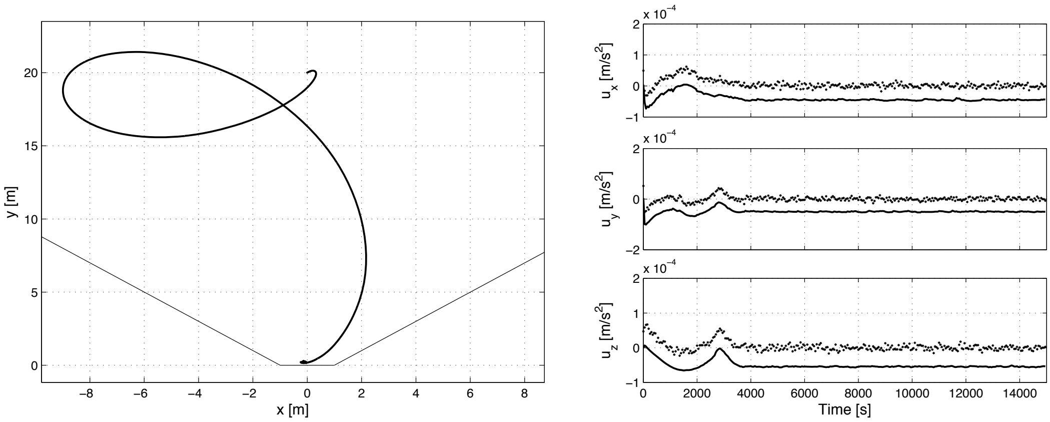

Fig. 3. Robust MPC with thruster disturbance ($\delta_1 = 0$, $\delta_2 \sim N_3(5\cdot10^{-5}, 0.5\cdot10^{-5})$, $\delta\theta \sim N_3(0.0436, 0.0436)$). Controller parameters are set to: $N_p = 60$, $\gamma = 1000$, $k_a = 60$, $\alpha = 2$, $\lambda = 0.23$

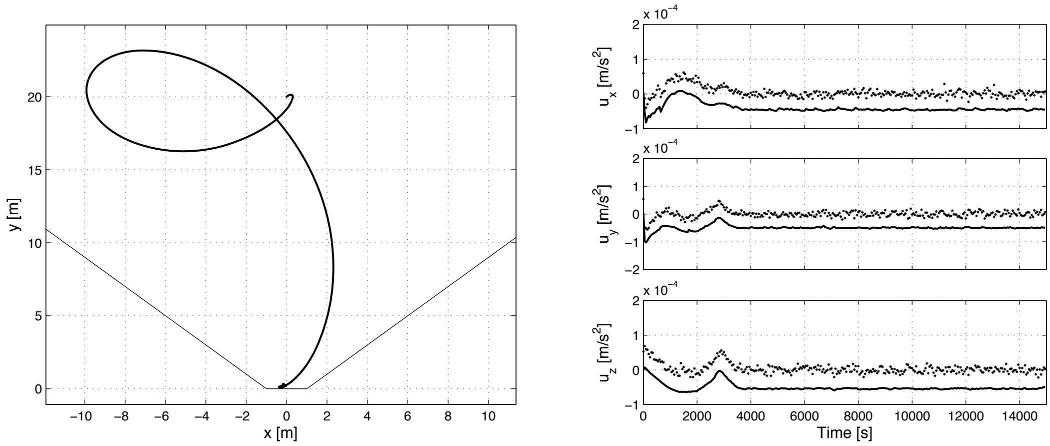

Fig. 4. Robust MPC with large thruster disturbance ($\delta_1 \sim N_3(0, 0.1)$, $\delta_2 \sim N_3(5 \cdot 10^{-5}, 0.5 \cdot 10^{-5})$, $\delta\theta \sim N_3(0.0436, 0.0436)$). Controller parameters are set to: $N_p = 60$, $\gamma = 1000$, $k_a = 60$, $\alpha = 3$, $\lambda = 0.23$.

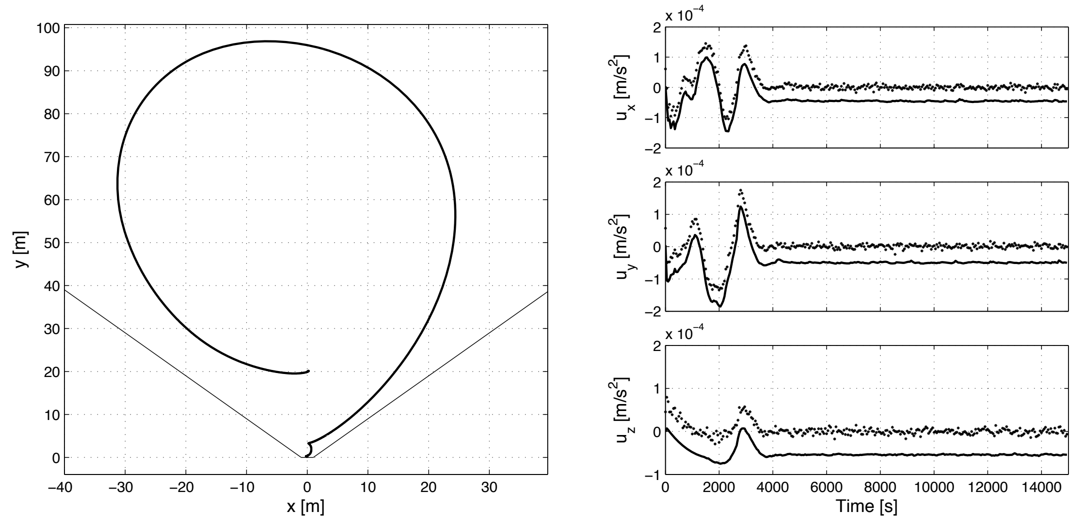

Fig. 5. Robust MPC with unmodeled dynamics and thruster disturbance ($\delta_1 = 0$, $\delta_2 \sim N_3(5 \cdot 10^{-5}, 0.5 \cdot 10^{-5})$, $\delta\theta \sim N_3(0.0436, 0.0436)$). Controller parameters are set to: $N_p = 60$, $\gamma = 1000$, $k_a = 60$, $\alpha = 5$, $\lambda = 0.23$.