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Experimental Investigating of Non-linear and Chaotic Behavior of a Doubly-Clamped Beam under Electromagnetic Excitation

Zohreh Mohammadi
Department of Mechanical Engineering
Sharif University of Technology
Iran
zohreh_mohammadi@mech.sharif.edu

Azadeh MaroufMashat
Department of Mechanical Engineering
Sharif University of Technology
Iran
azadeh.mashat@gmail.com

Hassan Salarieh
Department of Mechanical Engineering
Sharif University of Technology
Iran
salarieh@mech.sharif.edu

Mohammad Abediny
Department of Mechanical Engineering
Sharif University of Technology
Iran
mabedinyna@yahoo.com

Mehdi Maasoumy
Department of Mechanical Engineering
Sharif University of Technology
Iran
mehdi_maasoumi@yahoo.com

Aria Alasty
Department of Mechanical Engineering
Sharif University of Technology
Iran
aalasti@sharif.edu

Rasool Shabani
Mechanical Engineering Group,
Faculty of Engineering Urmia University,
Iran
r.shabani@mail.urmia.ac.ir

Abstract:

This paper studies the chaotic behavior of a doubly clamped Euler-Bernoulli beam under magnetic excitation via an experimental approach. The Responses of vibrations of the beam under different magnetic excitations are being investigated. The setup includes a doubly-clamped beam which is installed on two static bases and it is excited in the middle by an electromagnetic exciter. The excitation frequency varies from 1 to 50 Hz with different bias and amplitude voltages. Using various numerical methods such as Fast Fourier Transformation, Phase diagrams and Maximum Lyapunov Exponents, the experimental results are examined to find regular and irregular responses. The experimental results show that there exists some harmonic and super harmonic and in some cases chaotic responses in this system.

Key words
Electromagnetic excitation, Doubly-Clamped Beam, Chaos.

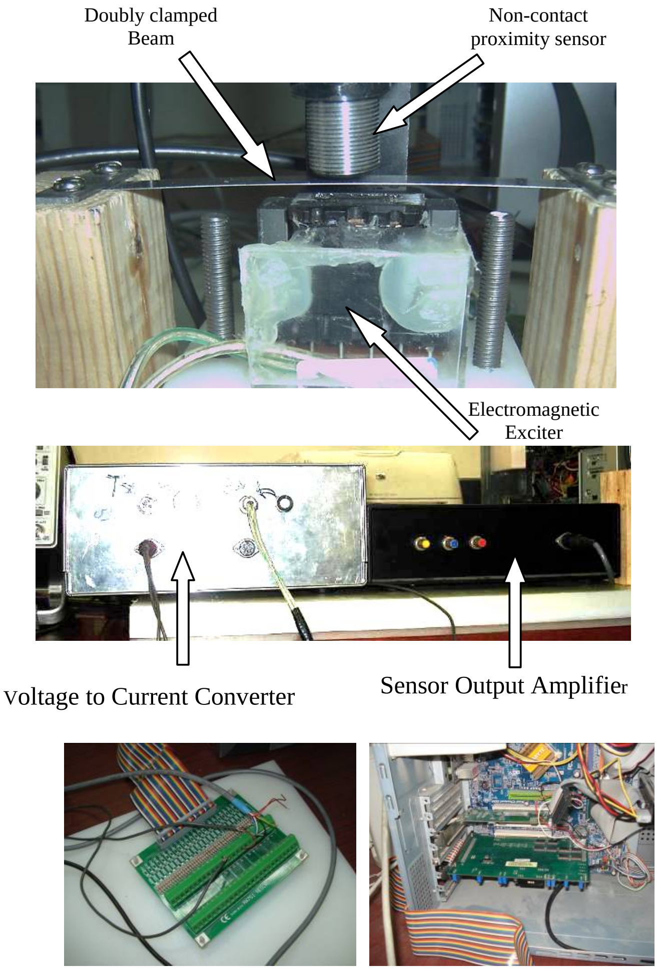
1. Introduction:

Chaos is a dynamic property that appears in nonlinear systems in some special conditions. It can be introduced as a behavior that dose not follow the specific pattern in phase diagrams, like periodic orbits.

The important point is that the differential equations of these systems are non-linear. In addition, these equations have order of 3 or higher in continuous form. In many cases, lower order but discrete and non-linear equations may introduce chaotic behaviors. The most important property of a chaotic system is its intense sensitivity to initial conditions, so a very small difference in initial condition causes big differences in results.

Vibrations under electro-magnetic excitation may introduce nonlinear phenomena such as sub-harmonic, super-harmonic oscillations and chaos. In using extended Kalman filtering method, the nonlinear force of an electromagnet on a single clamped beam is identified. In that work, it is shown that harmonic excitation on a clamped elastic beam may result in super-harmonic behavior and ir-regular response. Electro-magnetic excitation have many applications in active magnetic bearing systems. In using a nonlinear model of electromagnetic force which can justify the chaotic response of one-dimensional magnetic levitation systems, has been proposed. Using an experimental setup, Chang and Tung showed the chaotic response for magnetic bearing system in high frequency range (30 to 40 Hz). Instead of a multiplicative form they have identified a superlative form for electromagnetic force model. In experimental and analytical studies are performed on chaotic behavior of active magnetic bearings. Investigation of chaos in magnetically levitated doubly clamped beams is examined via analytical and experimental methods in.

In this paper, using an experimental setup the chaotic behavior of a doubly-clamped beam under electromagnetic excitation is investigated. It is shown that the nonlinear response of the system consists of the n^{th} order super-harmonic responses of the excitation frequency.



Mediator Board A/D Card in Computer

Fig.1. Prepared setup for experiments

2. Experimental Setup:

The prepared setup (Fig. 1) includes an elastic doubly-clamped beam which is excited by an E-type electromagnetic exciter with ferrite core. A non-contact proximity displacement sensor with characteristic curve of Fig. 2 and sensitivity of 0.1% is used to measure displacement of the mid-point of the beam. The head diameter of this sensor is 15mm.

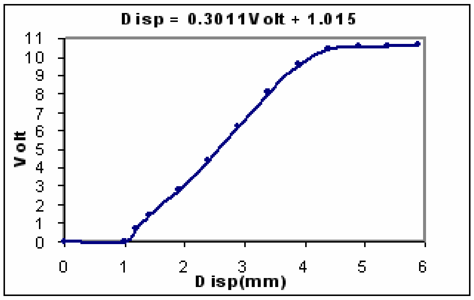


Fig.2. Characteristic curve of the non-contact proximity sensor

A code was written in Visual Basic that produces the exciting voltage and saves the output data in computer. By using this code, the amplitude and DC magnitude of voltage, and also exciting frequency and time interval of excitations can be set. After running the program and exciting the beam, the program saves displacement data taken by non-contact proximity displacement sensor. Also the excitation current which is related proportionally to the computer voltage is saved. The time interval for data acquisition is 0.004 sec.

First, the output of displacement sensor enters an amplifier, and then through a mediator board the analogue signal enters to an I/O card to prepare digital data needed for computer analysis. On the other hand, the output voltage of the computer enters to the mediator board, and then it enters a linear voltage-current transducer. After that voltage is converted to current and this current in the electromagnetic exciter coil, causes the beam to vibrate.

Fig.3. shows a schematic of the setup. The bases of the beam are installed on a polyamide flat plate, to avoid vibrations of the bases. These bases are glued on the polyamide plate. The electromagnetic exciter is glued to other poly-amid plate too. To have better calibration of the sensor and also changing the distance between the exciter and the beam, the setup is prepared such that the vertical position of the electromagnetic exciter stand can be changed easily.

3. Experimental Results:

For investigating the non-linear behavior of the beam, under electromagnetic exciting, the required voltage after passes through I/O card, enters to the voltage-current transducer, in the form of the $V_0 + V\sin(\omega t)$. (V is the amplitude of the voltage and V_0 is the bias voltage.) The current in the magnetic coil causes the oscillations of the beam in the midpoint. The required data are saved in the computer and the required diagrams are obtained via these data. The experiments are done for different ratio of amplitude and bias voltages, and different exciting frequency.

Some of the results are noted here:

First, V and V_0 are set to be 5 a 4volt respectively. We will see an increase in non-linear and unstable behavior and also a decrease in periodic and harmonic behavior of the system while increasing exciting frequency. In this case the periodic and quasi periodic behaviors in the system are observed for different exciting frequencies. Existence of the 2nd, the third and the 4th and higher order super-harmonic responses is the most important phenomena in this system. In addition, maximum Lyapunov exponent of the time series is negative or very near to 0. If the exciting frequency is set to 5Hz, the behavior of the system will be similar to the one observed in Fig. 4

If the exciting frequency is 20Hz, the behavior of the system will be almost quasi periodic and not periodic like the previous case and the magnitude of maximum Lyapunov exponent is positive but very close to 0 (Fig 5).

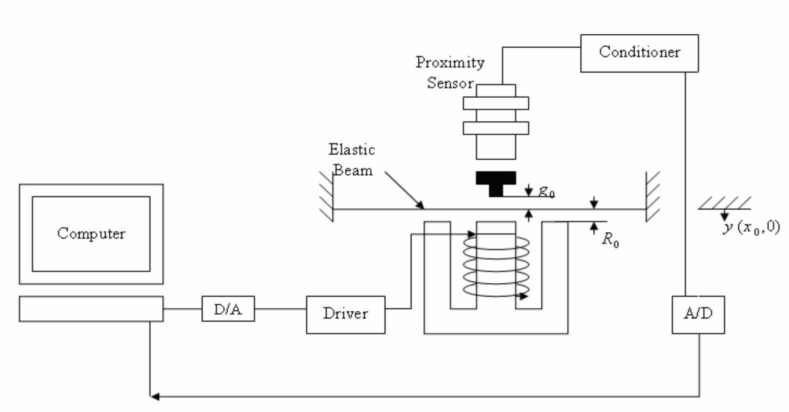
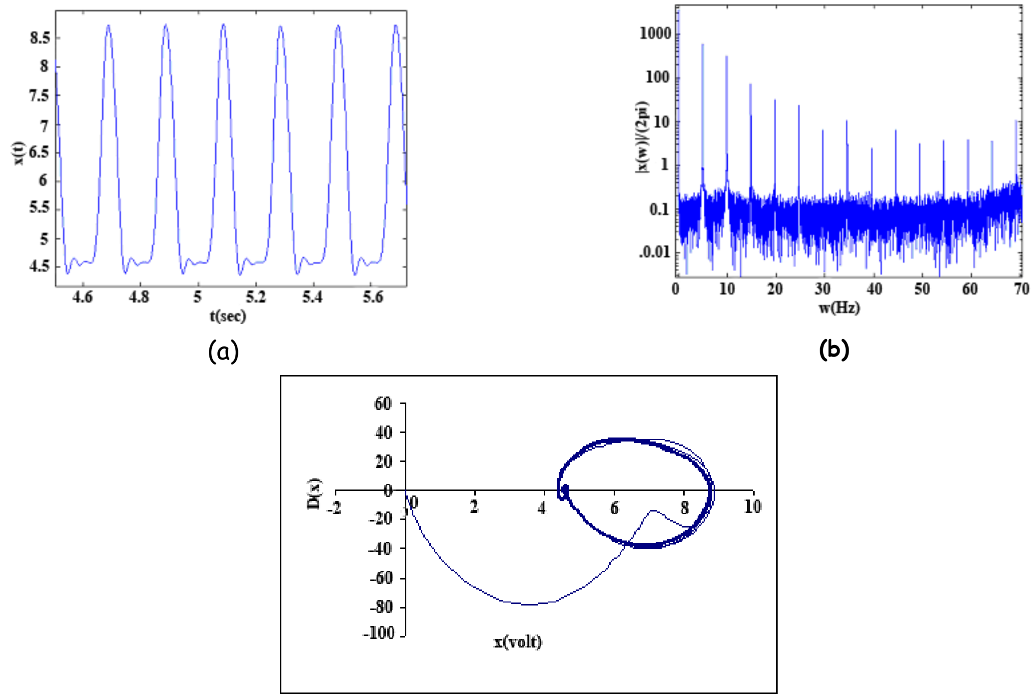


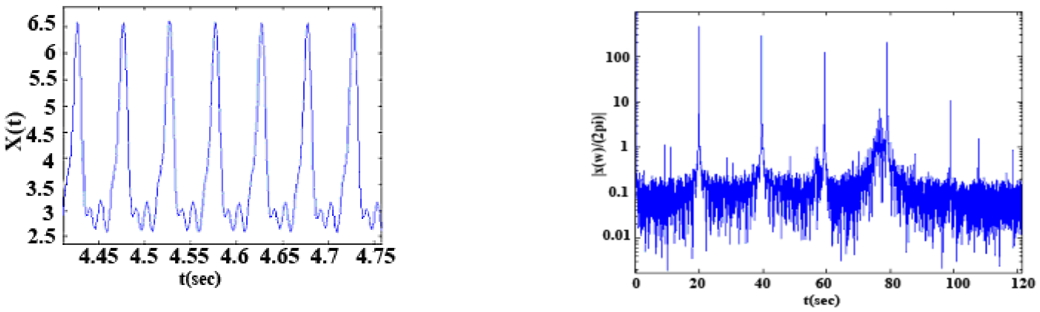
Fig.3. Schematic of the prepared setup for experiments



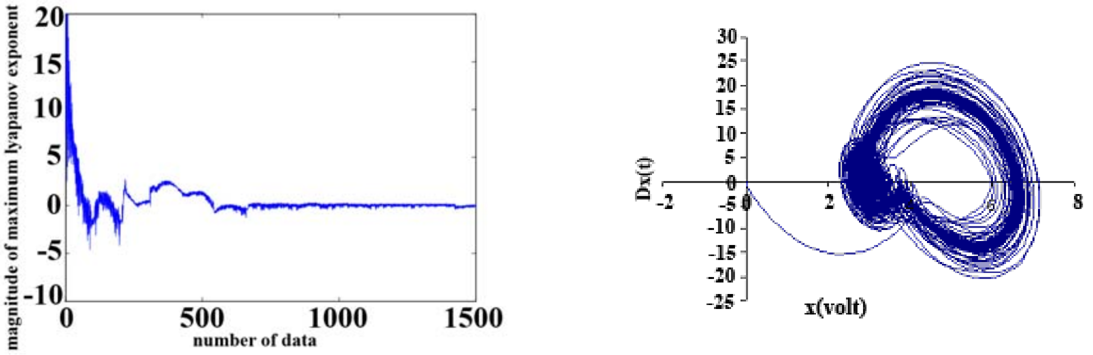
(a) **(b)**

(c)

Fig.4. Experimental results for sinusoidal excitation of $U = 4 + 5\sin(10\pi t)$, (a) time history, (b) Fourier transform, (c phase plane

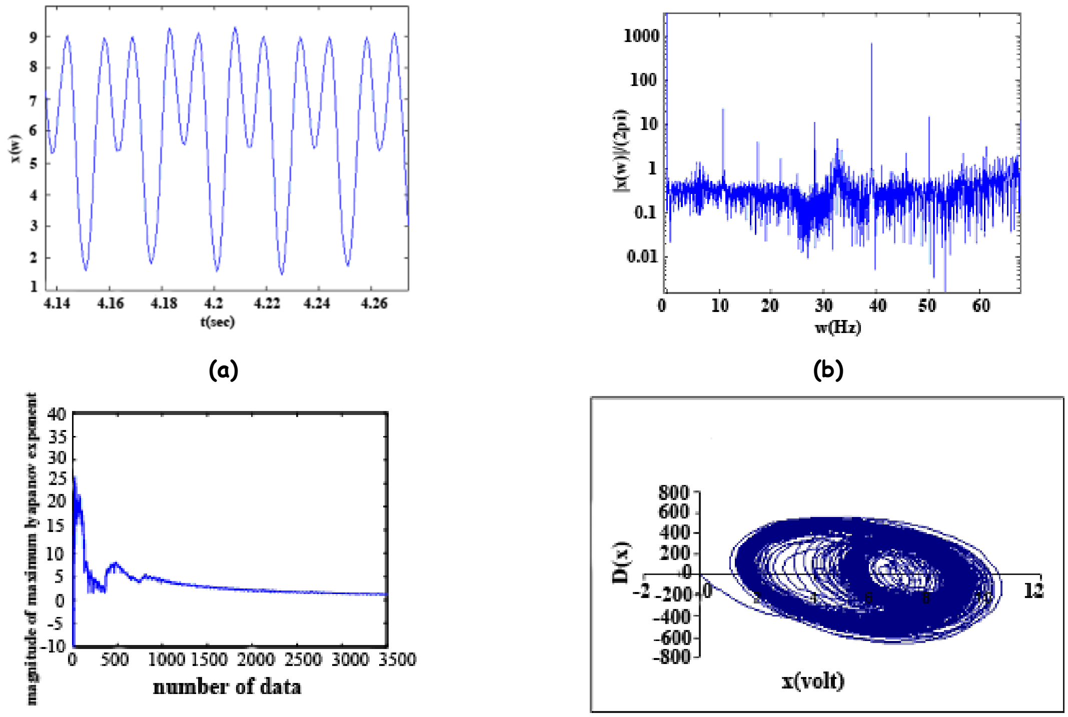


(a) (b)



(c) (d)

Fig.5. Experimental results for sinusoidal excitation of and $U = 4 + 5\sin(40\pi t)$, (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = -0.25$



(a) (b)

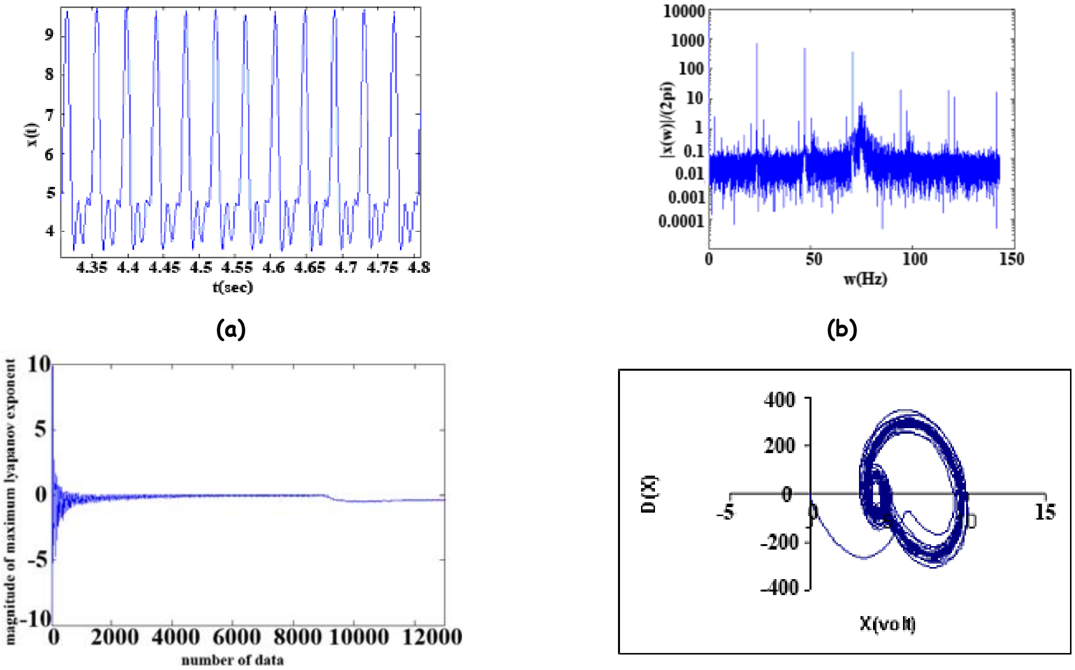
(c) (d)

Fig.6. Experimental results for sinusoidal excitation of $U = 4 + 5\sin(80\pi t)$, (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = 0.204$

If the exciting frequency is 40Hz, the stability of the system behavior decreases and the phase diagram is not regular according to Fig6.

For amplitude and bias voltage equal to 5 and 3 volt respectively we will see instability and non-linearity in the system while exciting frequency up to 20Hz. After that, there is a periodic and harmonic behavior in this system while increasing the exciting frequency up to 50Hz and maximum Lyapunov exponent is negative here. If the exciting frequency is 24Hz it is clear from FFT diagram in Fig 7 that the response frequencies are integer factors of exciting frequency.

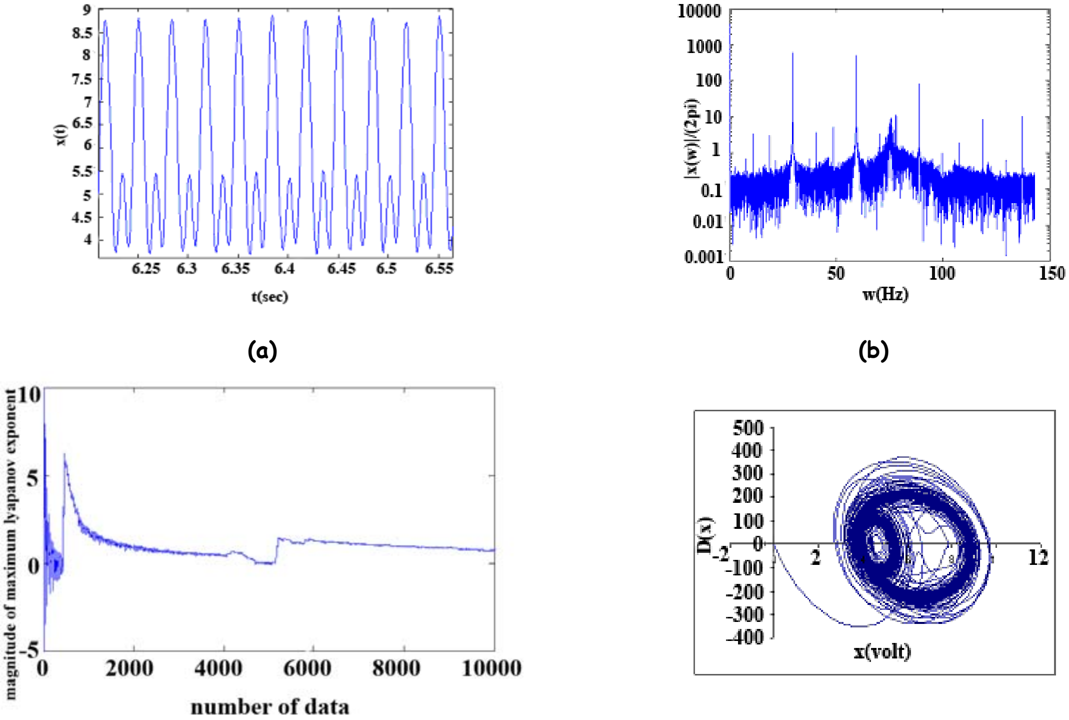
If the exciting frequency is 30Hz (Fig 8) it is clear from time history of the system that the behavior is not periodic and the phase diagram is nearly chaotic and maximum Lyapunov exponent is positive.



(a) (b)

(c) (d)

Fig.7. Experimental results for sinusoidal excitation of $U = 3 + 5\sin(48\pi t)$, (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = -0.27$

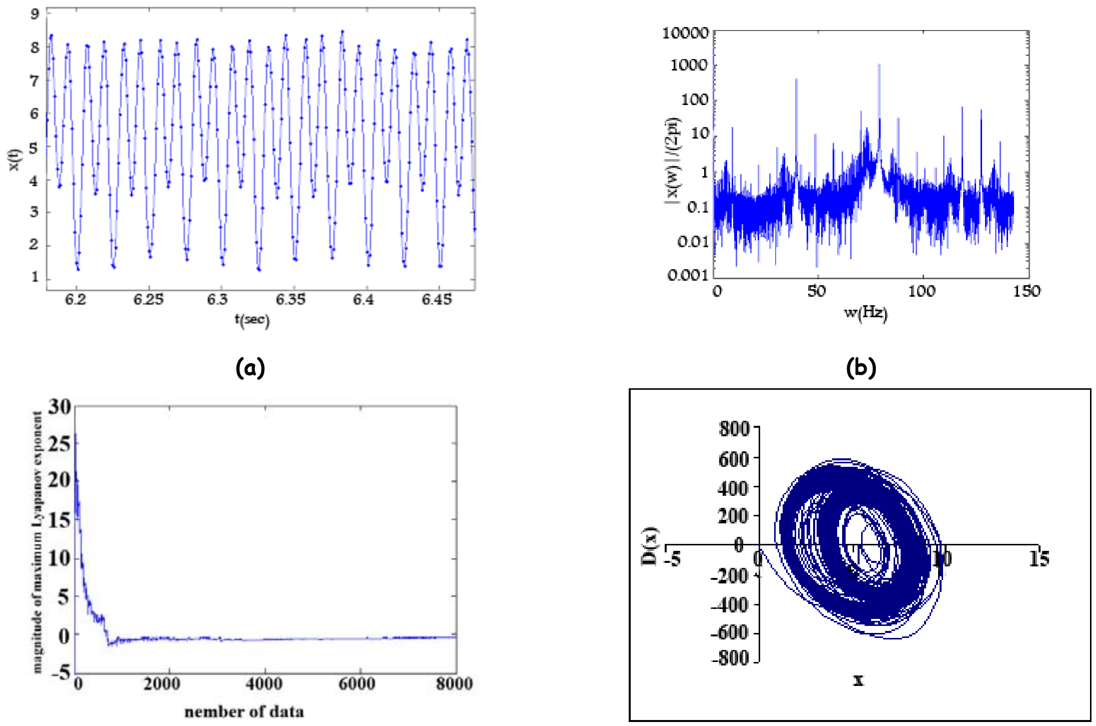


(a) (b)

(c) (d)

Fig.8. Experimental results for sinusoidal excitation of $U = 3 + 5\sin(60\pi t)$ (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = 0.27$

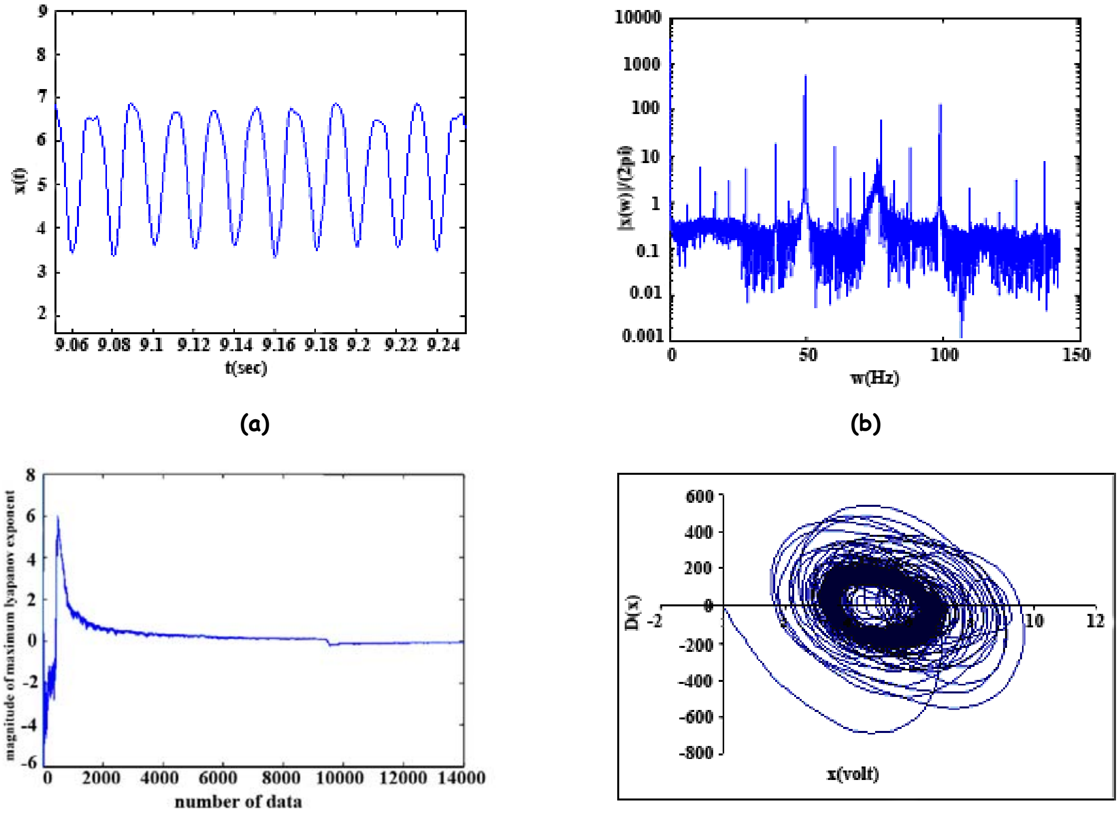
After increasing the exciting frequency to 40Hz, again it is observed that the response frequencies are integer coefficients of excitation frequency and the system behavior is periodic (Fig 9). In this case maximum Lyapunov exponent is negative and one can not have a deterministic decision about the chaotic behavior of the system. Fig 10 shows the behavior of the system with exciting frequency of 50Hz.



(a) (b)

(c) (d)

Fig.9. Experimental results for sinusoidal excitation of $U = 3 + 5\sin(80\pi t)$ (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = -0.48$



(a) (b)

(c) (d)

Fig.10. Experimental results for sinusoidal excitation of $U = 3 + 5\sin(100\pi t)$ (a) time history, (b) Fourier transform, (c) maximum Lyapunov exponent, (d) phase plane, the stability is at $\lambda = -0.03$

Conclusion:

In this paper, experimental researches in existence of chaos in a doubly-clamped beam which are excited by an electromagnetic force are performed. After preparing setup and data acquisition, nonlinear behavior and chaos in the system was studied by plotting the Phase Plane and Lyapunov Exponent. It seems that the super-harmonic responses rout the system toward chaos.

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