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# DYNAMICAL ANALYSIS OF THE ITALIAN HIGH-VOLTAGE POWER GRID

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## Abstract

In this work, the Italian high-voltage power grid has been analysed by using a dynamical model based on Kuramoto-like oscillators. Synchronization in the network represents the normal working operating regime, after which the effect of perturbations has been studied to investigate the dynamical robustness of the network to faults. The analysis allows to define several dynamical parameters whose relationship with the topological ones is not-trivial. The results obtained have been then compared with those obtained on a surrogate random network.

## Key words

Complex networks, power grid, Kuramoto model.

## 1 Introduction

Many real systems may be represented in the form of networks of nodes joined together by links: technological networks such as the Internet or power grids, transportation networks such as airline routes or roads, distribution networks such as the movements of delivery trucks or the blood vessels of the body, biological networks such as the metabolic networks or food chain, social networks such as the co-authors, actors, friends networks. In the last years there has been within the scientific community an increasing interest in the study of complex networks because it is possible to describe, using a common paradigm, different kinds of systems [Boccaletti et al., 2006], [Buscarino et al., 2010]. The rapid development of complex network theory provides new research tools for complex power grids e. g. to analyze error and attack resilience of both artificially generated topologies and real world networks where nodes are generators, substations and transformers and edges are high-voltage (220-380-400 kV) transmission lines.

Power grids are one of the most attractive case studies of complex networks, together with transportation networks and Internet so that many works [Crucitti et al., 2004a], [Crucitti et al., 2005] focused on them, although often without considering the specific nature and characteristics of nodes and links, but working on a higher level of abstraction.

The analysis of the topology of power grids of the major european countries carried out in [Solé et al., 2008] and [Rosas-Casals, 2010] allows to reveal some common characteristics of these networks: (a) most of them are small world; (b) they are very sparse; (c) the link distribution is exponential; (d) these networks are weakly or not correlated.

Another interesting topic, specially when the analysis of blackouts is dealt with, is identification of critical lines and modeling of cascading failures [Rosas-Casals, 2010], [Rosato et al., 2007], [Crucitti et al., 2004a], [Crucitti et al., 2005]. Such phenomena are often explained by focusing on the topological properties of the network. In fact, in most of the above mentioned works, the approach is essentially static, and the dynamical characteristics of the nodes are not considered. However, recent works [Filatrella et al., 2008], [Dörfler and Bullo, 2010], [Fioriti et al., 2009], have removed this hypothesis by applying to the power grids analysis the Kuramoto model of coupled oscillators [Acebrón et al., 2005] and studied the power grid behavior in terms of synchronization.

Power systems depend on synchronous machines for electricity generation and so the synchronism of the machines that form the system is a necessary condition for the whole network to operate in a proper way. The concept of stability of a power system is therefore closely linked to that of synchronism. An important form of stability for an electrical network is the so-called transient stability [Dörfler and Bullo, 2010], which is the ability of the network to maintain synchronism when it is subjected to transient disturbances such as faults in transmission systems or problems with generators or heavy loads. If the perturbations cause a limited angular separation between the components of the system, the system maintains synchronism.

The response of the system to these perturbations involves large ranges of machine rotor angles values, power flows, voltages at the nodes, and other variables of the system. In this case it is possible that the automatic security devices with which the nodes are equiped isolate parts of the system to prevent damages. For example, the Italian blackout of 2003 was caused by the fault on a line and caused a series of failures that led to loss of synchronism of the Italian power system with respect to the rest of Europe [Buzna et al., 2009].

In this paper the complex networks theory tools are used for the analysis of the Italian high-voltage (380 kV) power grid. The topological properties of the network are investigated and, starting from the model introduced in [Filatrella et al., 2008], the synchronization of Kuramoto-like oscillators in the network is analyzed. To evaluate the transient stability, perturbations have been applied to the nodes and the minimum value of perturbation leading to instability for each node has been calculated, then the relationship between threshold and topological properties and the time to obtain complete loss of synchronization have been investigated. The network behavior has been then compared with that exhibited by a surrogate network.

The paper is organized in the following way: Section 2 discusses the Kuramoto-like model used for our analysis and the topological characteristics of the Italian high-voltage power grid. Section 3 shows the results obtained. Section 4 reports the comparison with surrogate network. In the last section the conclusions of the work are drawn.

## 2 The Model

Following [Filatrella et al., 2008], a Kuramoto-like second-order model of electric systems can be obtained using a power balance equation to describe each generator or machine. A generator converts some source of energy into electrical power, while the reverse is true for a machine. The turbine of the generic generator $i$ produces electrical power with a frequency that is close to the standard frequency $\Omega$ of the electric system (50 or 60 Hz):

$$\theta_i = \Omega t + \tilde{\theta}_i, \tag{1}$$

where $\theta_i$ is the phase angle at the output generator $i$ and $\tilde{\theta}_i$ is the deviation from the uniform rotation. During the rotation the turbine dissipates energy at a rate proportional to the square of the angular velocity $\dot{\theta}_i$:

$$P_{diss} = K_D \dot{\theta}_i^{\,2} \tag{2}$$

or it accumulates kinetic energy

$$P_{acc} = \frac{1}{2} I \frac{d}{dt} (\dot{\theta}_i)^2, \tag{3}$$

where $I$ is the moment of inertia.

The condition for the power transmission is that devices do not operate in phase, being the mismatch between the rotators of two of them (devices $i$ and $j$) indicated by:

$$\Delta\theta = \theta_j - \theta_i = \tilde{\theta}_j - \tilde{\theta}_i, \tag{4}$$

considering that all the oscillators share the same common frequency $\Omega$. As a function of this phase difference a power is transmitted:

$$P_{transmitted} = -P^{MAX} \sin \Delta\theta. \tag{5}$$

Each generator or machine is described by a power balance equation of the type:

$$P_{source} = P_{diss} + P_{acc} + P_{transmitted}. \tag{6}$$

Substituting expressions (2), (3) and (5) in equation (6) and assuming that dissipation is the same for all sources, it is possible to obtain a Kuramoto-like equation for the node $i$:

$$\ddot{\tilde{\theta}}_i = -\alpha \dot{\tilde{\theta}}_i + P_i + P^{MAX} \sum_{j \neq i} a_{j,i} \sin(\tilde{\theta}_j - \tilde{\theta}_i), \tag{7}$$

where $\alpha$ is the dissipation parameter, $P_i$ is the power generated or absorbed and contains informations on the nature of the device and it is positive for a generator that is a source of power while negative for an absorbing machine, $a_{j,i}$ is the element of the adjacency matrix and accounts for the topology of the power grid.

The Italian high-voltage (380 kV) power grid is taken into account in this work. It counts 127 nodes, divided into 34 sources (hydroelectric and thermal power stations) and 93 substations, and 342 edges. Informations on the location of generating plants and substations have been obtained from the UCTE map [UCTE] and the data used in [Crucitti et al., 2004a], [Crucitti et al., 2004b], [Crucitti et al., 2005] and have been used to obtain the elements of the adjacency matrix.

For this network some significant topological parameters such as degree distribution, clustering and betweenness have been calculated. Average values for these three parameters are $\langle k \rangle = 2.6850$, $\langle c \rangle = 0.1561$, $\langle b \rangle = 0.2032$. In Fig. 1 and Fig. 2 the degree and betweenness distributions are shown respectively. It is possible to observe that there are a lot of nodes characterized by a low degree and a low betweenness, a characteristic of the Italian power grid network that it is possible to correlate to the streched shape of the Italian peninsula (and consequently on the related power grid). In [Fioriti et al., 2009] the topological vulnerability and improvability of the Italian high-voltage (380 kV) power grid have been analyzed. The removal of a

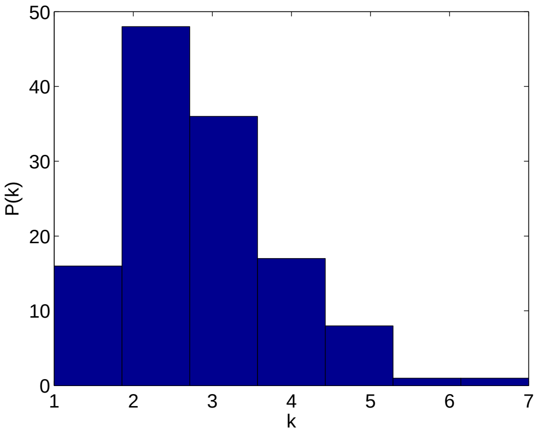

Figure 1. Degree distribution of the high-voltage Italian power grid network.

single edge, as the line connecting Laino and Rossano, is sufficient to isolate seven nodes from the rest of the network and Italian power grid, compared with spanish and french ones, is the most vulnerable but also the most improvable network.

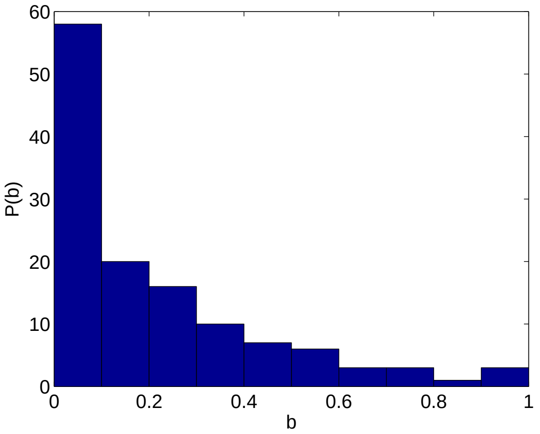

Figure 2. Betweenness distribution of the high-voltage Italian power grid network.

## 3 Analysis of the Italian high-voltage power grid

In this Section we discuss the results of the Kuramoto-like second-order model of the Italian high-voltage (380 kV) power grid. There are two kinds of network nodes: generators and substations. The system has been simulated using equal parameters for all the nodes and links. It has been considered unitary absorbed power for substations (1 pu) and, in order to respect the equality between generated and absorbed power, the power supplied by generators are all been put equal to 2.7353 pu. The dissipation parameter $\alpha$ is the same for all the nodes and its value is $\alpha = 0.1$. Concerning the coupling parameter $P^{MAX}$, computer simulations were carried out to find the value that allows to obtain complete synchronization. It was found that for values less than 5 it is not possible to obtain complete synchronization. In fact, the difference between two phases is subjected to fluctuations that persist over time. For values greater or equal to 5, the network reaches synchronism: differences between two phases, apart from the initial transient, stabilize at a value that remains constant over the time, that means that the units have the same frequency.

We have then studied transient stability of the network applying disturbances $\Delta P_i$ to the nodes. This extra energy is taken from the kinetic energy of the rotators that after few time units restore normal operation. This type of perturbation constitutes a realistic model of an unbalanced power due to faults in transmission systems or problems with generators or heavy loads.

When a perturbation $\Delta P_i$ is applied to node $i$ equation (7) becomes:

$$\ddot{\tilde{\theta}}_i = -\alpha\dot{\tilde{\theta}}_i + P_i + \Delta P_i + P^{MAX}\sum_{j\neq i} a_{j,i}\sin(\tilde{\theta}_j - \tilde{\theta}_i),$$

while the other dynamics remain unchanged.

Two outcomes are possible:

1. the network is able to return to synchronism condition, despite the initial fluctuations that affect the transmitted power;

2. the network is not able to restore the synchronism and fluctuations in the phases difference persist over time. In this case the system loses its stability even when perturbations end.

To evaluate the perturbation response of each node, once synchronization between nodes has been established, increasing values of perturbations have been applied for 50 seconds. In this way a threshold $\tilde{P}_i$ has been defined for each node, representing the minimum value of node perturbation that causes loss of synchronization in the network:

$$\tilde{P}_i = (\Delta P_i)_{MIN}. \qquad (8)$$

The treshold distribution is shown in Fig. 3. The different threshold values indicate that not all nodes respond in the same way. An analysis to investigate the correspondence between threshold and topological properties of the node has been carried out. In Fig. 4 the trend of the threshold with nodes degree is shown. The value of the threshold tends to increase with increasing value of the degree. It is sufficient to apply a lower disturbance to nodes with few links to lose the network synchronism.

To fully investigate the response of the network, and the failure propagation, an high perturbation (20 pu) has been applied to each of the nodes.

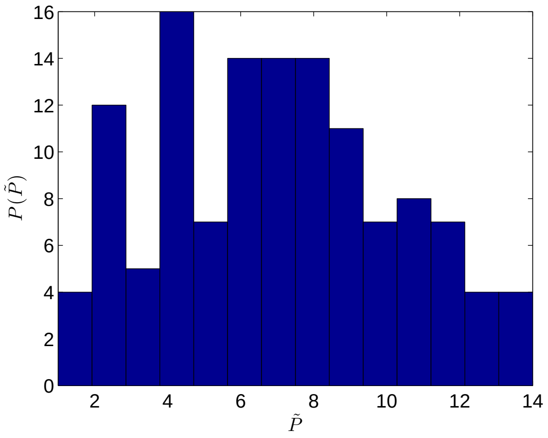

Figure 3. Threshold distribution of the Italian high-voltage power grid.

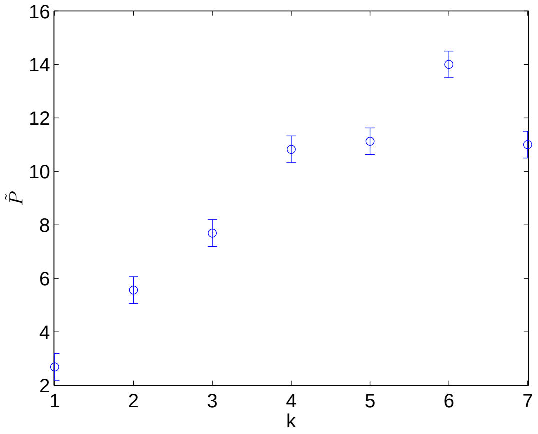

Figure 4. Threshold $\tilde{P}$ with respect to node degree.

Two different responses have been observed:

1. cascading failure: the perturbed node fails (loss of synchronism) and the failure involves first the nearby elements and then propagates to other (more for) nodes;

2. fast failure: all the nodes fail in a short time range.

These two kinds of response can be distinguished by comparing for each node the time $\tilde{t}$ defined as the time from the application of the perturbation to the complete loss of the network synchronism.

Fig. 5 shows the behaviours of nodes 1 (in blue) and 68 (in red) that may be considered as examples of the two cases. In fact, these two nodes are characterized respectively by degree 1 and 7, betweenness 0 and 0.5385 and threshold values of 4 and 11, so node 1 is more peripheral than node 68. There is a graduality in the failure propagation of node 68 while an immediate propagation is obtained in node 1. The two graphs represent the trend of $\theta_1$ and $\theta_{68}$ respectively showing the different propagation delay of perturbation. Time for complete desyncronization is respectively and $\tilde{t}_1 = 13.4947s$ and $\tilde{t}_{68} = 105.8578s$ for node 1 and node 68.

In Fig. 6 the parameter $\tilde{t}$ is shown for all the network nodes, while in Fig. 7 the parameter $\tilde{t}$ with respect to node degree is shown. The bigger is the degree of a node the bigger is the time interval $\tilde{t}$ to lose synchronization. Nodes with high degree tend to cascading failures.

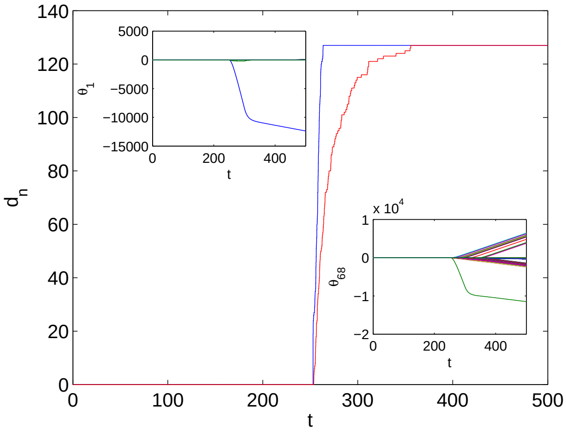

Figure 5. Time to obtain complete loss of desynchronization (desynchronized nodes $d_n = 127$) for nodes 1 (blue) and 68 (red). The beahaviour of $\theta_1$ and $\theta_{68}$ are showed respectively on the top and down of the picture.

## 4 Comparison with surrogate network

We have then compared the characteristics of the high-voltage Italian power grid with a surrogate random network in order to understand which features are peculiar of the Italian high-voltage power grid and which, on the contrary, are common features of these network. The surrogate data has been generated by considering the same number of nodes, the same number of links (and so the same degree) and a different arrangement of the links. In particular, in the surrogate data the links are randomly established. Therefore, an Erdos-Renyi network with 127 nodes and 342 links has been built up. This network has then been used to perform a dynamical simulation with the Kuramoto-like model discussed in Section 2.

We first show the topological characteristics of the surrogate random network. The average degree is $\langle k \rangle = 2.6772$; the clustering coefficient is $\langle c \rangle = 0.0198$ and the betweenness is $\langle b \rangle = 0.1981$. Compared with the values of the Italian high-voltage power grid, it can be observed that the clustering coefficient is lower. It should be also noted that this random network differs from the Italian high-voltage power grid for the fact that it does not take into account physical geographic constraints which, on the contrary, are a key factor of the Italian high-voltage power grid.

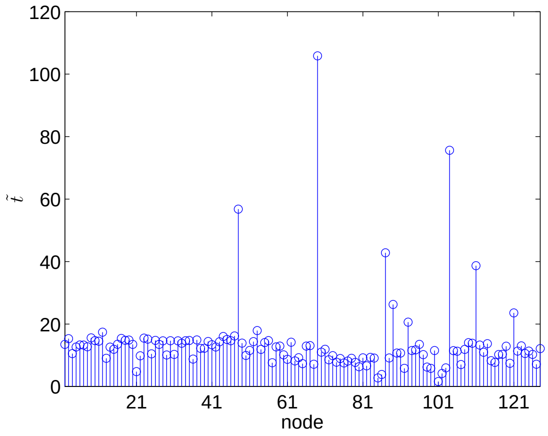

Figure 6. Time for the complete loss of the synchronization for the Italian power grid when a perturbation $\Delta P = 20pu$ is applied to the node $i$.

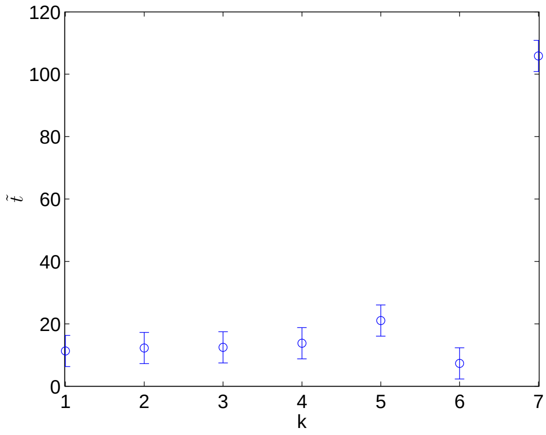

Figure 7. $\tilde{t}$ with respect to node degree when a perturbation $\Delta P = 20pu$ is applied.

The degree distribution and the betwenness distribution are reported in Figs. 8 and 9, respectively. No significant differences with the corresponding distributions observed in the Italian high-voltage power grid emerge.

The Kuramoto-like model in equation (7) has been then simulated on the surrogate network. The simulations were first devoted to derive the minimum value of $P^{MAX}$ leading to synchronization. It has been obtained that $P^{MAX} = 13$ is needed to obtain complete synchronization. Interestingly, this values is significantly larger than the value $P^{MAX} = 5$ of the real network. This means that the surrogate network is more difficult to be synchronized. Then, the transient stability analysis of the surrogate network has been carried out. The threshold distribution obtained for the surrogate network is shown in Fig. 10. It can be observed that on average the threshold values are also on average larger than the corresponding values in the real network. The conclusion is that the surrogate network seems to be more robust.

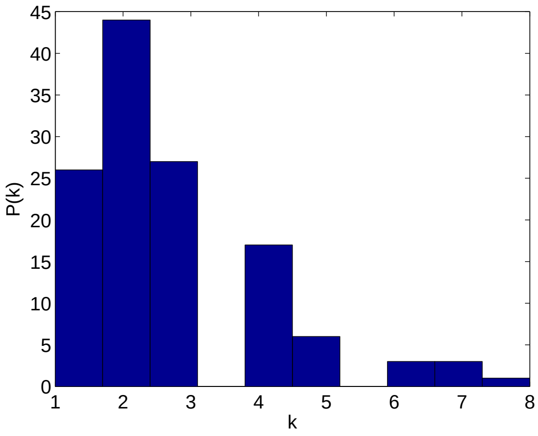

Figure 8. Degree distribution of the surrogate network.

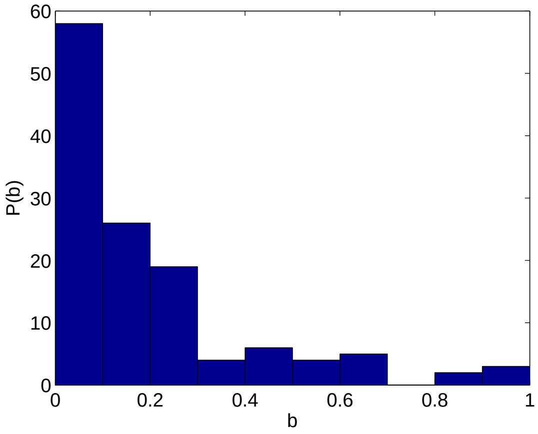

Figure 9. Betweenness distribution of the surrogate network.

The analysis of the threshold with respect to the node degree has been repeated for the surrogate network. The results are shown in Fig. 11, where the same tendency of the threshold to increase with the node degree can be observed.

As concerns the failure analysis, also in this case both cascading failures and fast failures have been observed. Also for the surrogate network cascading failures are associated with high degree nodes.

## 5 Conclusions

In this paper a study of the Italian high-voltage power grid has been proposed. A Kuramoto-like second-order model has been taken into account to model the node dynamics. It is interesting to note that the mapping between oscillators and power grid nodes can be made quantitative and under some approximations the class of Kuramoto-like models with bimodal distribution of the frequencies is the most appropriate choice. In fact

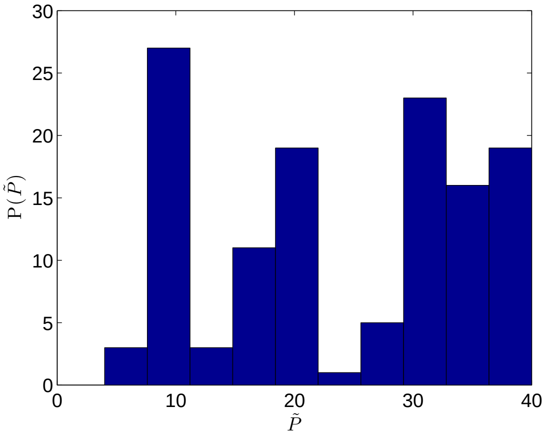

Figure 10. Threshold distribution of the surrogate network.

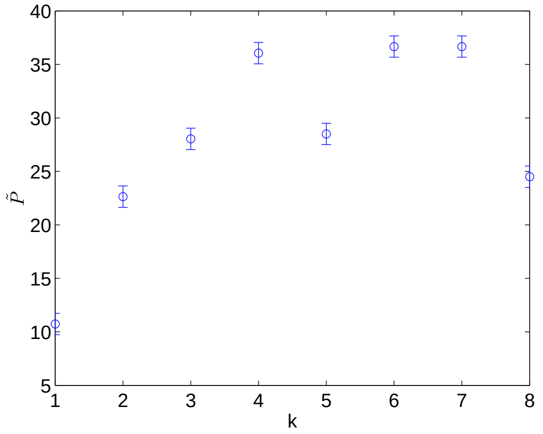

Figure 11. Threshold $\tilde{P}$ with respect to node degree for the surrogate network.

in the power grid there are two kinds of oscillators: "sources" and "consumers". Dynamical parameters such as the minimum value of perturbation leading to desynchronization and the time to reach the complete loss of synchronism have been introduced. A non-trivial relationship between dynamical and topological parameters of the network has been observed. In general the higher is node degree the higher is the minimum absorbable perturbation and the bigger is the time interval to lose synchronization with cascading failure, but it can be concluded that the dynamical parameters studied are not a function of a single topological parameter.

The analysis has been then repeated for a surrogate network with the same number of nodes and links, but with random links in order to understand which specific features are related to the particular geographical shape underlying the Italian high-voltage power grid. The conclusions that can be drawn is that the surrogate network is more difficult to synchronize but also more robust which can be explained with the particular geographic configuration of the Italian peninsula.

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