Nonlinear model following control with parameter identification for a 3 DOF model helicopter

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***Index Terms*— model helicopters, nonlinear control, model following control, parameter identification**

I. Introduction

The interest for designing feedback controllers for helicopters has increased during the last decade due to important potential applications. The main difficulties for designing stable feedback controllers for helicopters arise from their nonlinearities and couplings. To date, various efforts have been directed to development of effective nonlinear control strategies for helicopters. Most of the existing results have been obtained mainly for flight regulation. In this paper, the flight tracking control problem of 3 DOF model helicopter is considered and a nonlinear model following control method with parameter identification is applied. Experimental results are presented to show the performance of the designed controller.

II. System description

Consider a model helicopter of Quanser Consulting, Inc. shown in Fig.1. The helicopter body is mounted at the end of an arm and free to move about the elevation axis, the pitch axis as well as about the horizontal travel axis. In other words, the helicopter has 3 DOF: the elevation ε, the pitch θ and travel ϕ angles. The angles of movement are measured via optical encoders. Two DC motors with propellers generate a driving force proportional to the voltage output of a controller. The system dynamics are expressed by the following highly nonlinear and coupled state variable equations.

$$\dot{\boldsymbol{x}}_p = f(\boldsymbol{x}_p) + [g_1(\boldsymbol{x}_p)\ g_2(\boldsymbol{x}_p)]\boldsymbol{u}_p \tag{1}$$

where

$$\begin{aligned}
\boldsymbol{x}_p &= [\varepsilon\ \dot{\varepsilon}\ \theta\ \dot{\theta}\ \phi\ \dot{\phi}]^T,\ \boldsymbol{u}_p = [u_{p1}\ u_{p2}]^T \\
u_{p1} &= V_f + V_b,\ u_{p2} = V_f - V_b \\
f(\boldsymbol{x}_p) &= \begin{bmatrix} \dot{\varepsilon} \\ p_1\cos\varepsilon + p_2\sin\varepsilon + p_3\dot{\varepsilon} \\ \dot{\theta} \\ p_5\cos\theta + p_6\sin\theta + p_7\dot{\theta} \\ \dot{\phi} \\ p_9\dot{\phi} \end{bmatrix} \\
g_1(\boldsymbol{x}_p) &= [0\quad p_4\cos\theta\quad 0\quad 0\quad 0\quad p_{10}\sin\theta]^T \\
g_2(\boldsymbol{x}_p) &= [0\quad 0\quad 0\quad p_8\quad 0\quad 0]^T
\end{aligned}$$

Here, V_f and V_b are the voltages for the front motor and the rear motor, respectively.

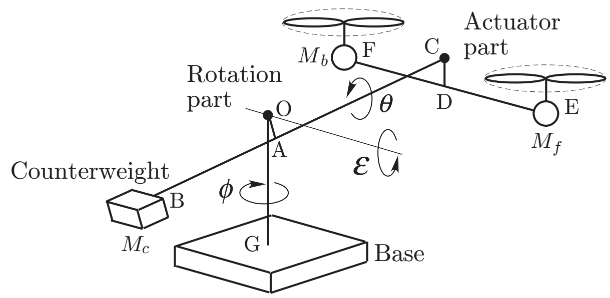

Fig. 1. Overview of a model helicopter.

It is worth noting that all the parameters $p_i \quad (i = 1 \ldots 10)$ of the equations are constant.

For position control of the model helicopter, two angles, the elevation ε and the travel ϕ angles, are selected as the outputs among three detected signals of the angles. Hence, we have

$$\boldsymbol{y}_P = [\varepsilon,\ \phi]^T \tag{2}$$

Then, it is not difficult to verify that the system is input-output linearizable and minimum phase.

III. Control system design

In this section, a nonlinear model reference control system is designed for the 3 DOF model helicopter mentioned in the previous section.

When the outputs are chosen as (2), the decoupling matrix $B(\boldsymbol{x}_P)$ is calculated as

$$B(\boldsymbol{x}_P) = \begin{bmatrix} p_4\cos\theta & 0 \\ p_{10}\sin\theta & 0 \end{bmatrix} \tag{3}$$

and obviously singular. Hence, it is needed to apply the nonlinear structure algorithm for design of a model reference controller.

First, the reference model is given as

$$\begin{aligned}
&\dot{\boldsymbol{x}}_M = A_M\boldsymbol{x}_M + B_M\boldsymbol{u}_M,\ \boldsymbol{y}_M = C_M\boldsymbol{x}_M \qquad (4) \\
&\boldsymbol{x}_M = [x_{M1},\ x_{M2},\ x_{M3},\ x_{M4},\ x_{M5},\ x_{M6},\ x_{M7},\ x_{M8}]^T \\
&\boldsymbol{y}_M = [\ \varepsilon_M \quad \phi_M\ ]^T, \quad \boldsymbol{u}_M = [\ u_{M1} \quad u_{M2}\ ]^T
\end{aligned}$$

where

$$A_M = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, B_M = \begin{bmatrix} \boldsymbol{i}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{i}_1 \end{bmatrix}, C_M = \begin{bmatrix} \boldsymbol{i}_2{}^T & \mathbf{0}^T \\ \mathbf{0}^T & \boldsymbol{i}_2{}^T \end{bmatrix}$$

$$K_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k_{i1} & k_{i2} & k_{i3} & k_{i4} \end{bmatrix}, \ \boldsymbol{i}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \ \boldsymbol{i}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then, the input vector is given by

$$\boldsymbol{u}_P = R(\boldsymbol{x}) + S(\boldsymbol{x})\,\boldsymbol{u}_M, \ \boldsymbol{x} = [\boldsymbol{x}_P^T \ \ \boldsymbol{x}_M^T]^T \tag{5}$$

$$R(\boldsymbol{x}) = \frac{1}{d_2(\boldsymbol{x})p_4 \cos x_{p3}} \begin{bmatrix} -d_2(\boldsymbol{x}) & 0 \\ d_1(\boldsymbol{x}) & p_4 \cos x_{p3} \end{bmatrix} \begin{bmatrix} \bar{e}_1 - r_1(\boldsymbol{x}) \\ \bar{e}_2 - r_2(\boldsymbol{x}) \end{bmatrix}$$

$$S(\boldsymbol{x}) = \frac{-1}{d_2(\boldsymbol{x})p_4 \cos x_{p3}} \begin{bmatrix} -d_2(\boldsymbol{x}) & 0 \\ d_1(\boldsymbol{x}) & p_4 \cos x_{p3} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ d_3(\boldsymbol{x}) & 1 \end{bmatrix}$$

where

$$\begin{aligned}
\bar{e}_1 &= -\sigma_{12}\dot{e}_1 - \sigma_{11}e_1 \\
\bar{e}_2 &= -\sigma_{24}e_2^{(3)} - \sigma_{23}\ddot{e}_2 - \sigma_{22}\dot{e}_2 - \sigma_{21}e_2 \\
r_1(\boldsymbol{x}) &= -p_1 \cos x_{p1} - p_2 \sin x_{p1} - p_3 x_{p2} + x_{M3} \\
r_2(\boldsymbol{x}) &= \{-d_5(\boldsymbol{x})\,(p_9 p_{11} \tan x_{p3} + x_{p4} d_4(\boldsymbol{x})) \\
&\quad -p_{11} x_{p2} \tan x_{p3}\,(p_1 \cos x_{p1} + p_2 \sin x_{p1})\}\, x_{p2} \\
&\quad + \{p_3 x_{p4} d_4(\boldsymbol{x}) + p_{11} \tan x_{p3} (p_3 p_9 - d_5(\boldsymbol{x}))\} \\
&\quad (x_{M3} - r_1(\boldsymbol{x})) + \{p_3\,(x_{M3} - r_1(\boldsymbol{x})) \\
&\quad +(2x_{p4} \tan x_{p3} - p_{12})\,(\ddot{e}_1 - r_1(\boldsymbol{x})) - x_{M4} \\
&\quad \left. +e_1^{(3)} - x_{p2} d_5(\boldsymbol{x}) \right\} x_{p4} d_4(\boldsymbol{x}) + d_4(\boldsymbol{x}) \\
&\quad (\ddot{e}_1 - r_1(\boldsymbol{x}))\,(p_5 \cos x_{p3} + p_6 \sin x_{p3} + p_7 x_{p4}) \\
&\quad -(p_9)^3 x_{p6} + (x_{p4} d_4(\boldsymbol{x}) - p_{11} p_{12} \tan x_{p3})\, e_1^{(3)} \\
&\quad +p_{11} \tan x_{p3}\,(p_{12} x_{M4} - k_1 x_{M1} - k_2 x_{M2} \\
&\quad -k_3 x_{M3} - k_4 x_{M4}) - x_{p4} x_{M4} d_4(\boldsymbol{x}) \\
&\quad +k_5 x_{M5} + k_6 x_{M6} + k_7 x_{M7} + k_8 x_{M8} \\
&\quad +p_{11} e_1^{(4)} \tan x_{p3} \\
d_1(\boldsymbol{x}) &= \left(p_3 p_9 - d_5(\boldsymbol{x}) - (p_9)^2\right) p_{10} \sin x_{p3} \\
&\quad +p_3 p_4 x_{p4} d_4(\boldsymbol{x}) \cos x_{p3} \\
d_2(\boldsymbol{x}) &= p_8 d_4(\boldsymbol{x})\,(\ddot{e}_1 - r_1(\boldsymbol{x})) \\
d_3(\boldsymbol{x}) &= -p_{11} \tan x_{p3}, \quad e_1 = x_{M1} - x_{p1} \\
\dot{e}_1 &= x_{M2} - x_{p2}, \quad \ddot{e}_1 = -\sigma_{12}\dot{e}_1 - \sigma_{11}e_1 \\
e_1^{(3)} &= (\sigma_{12}^2 - \sigma_{11})\dot{e}_1 + \sigma_{12}\sigma_{11}e_1 \\
e_1^{(4)} &= (-\sigma_{12}^3 + 2\sigma_{12}\sigma_{11})\dot{e}_1 - \sigma_{11}(\sigma_{12}^2 - \sigma_{11})e_1 \\
e_2 &= x_{M5} - x_{p5}, \quad \dot{e}_2 = x_{M6} - x_{p6} \\
\ddot{e}_2 &= p_{11} \tan x_{p3}\,(\ddot{e}_1 - r_1(\boldsymbol{x})) - p_9 x_{p6} + x_{M7} \\
e_2^{(3)} &= p_{11} \tan x_{p3}\,\{p_3\,(x_{M3} - r_1(\boldsymbol{x})) - x_{p2} d_5(\boldsymbol{x}) \\
&\quad \left. +e_1^{(3)} + p_{12}\,(r_1(\boldsymbol{x}) - \ddot{e}_1) - x_{M4} \right\} + x_{M8} \\
&\quad +x_{p4} d_4(\boldsymbol{x})\,(\ddot{e}_1 - r_1(\boldsymbol{x})) - (p_9)^2 x_{p6} \\
p_{11} &= \frac{p_{10}}{p_4}, \quad d_4(\boldsymbol{x}) = \frac{p_{11}}{\cos^2 x_{p3}} \\
p_{12} &= p_3 - p_9, \quad d_5(\boldsymbol{x}) = p_1 \sin x_{p1} - p_2 \cos x_{p1}
\end{aligned}$$

The design parameters $\sigma_{ij} \quad (i = 1, 2, j = 1, \cdots, 4)$ are selected so that the characteristic equations $\lambda^2 + \sigma_{12}\lambda + \sigma_{11} = 0$ and $\lambda^4 + \sigma_{24}\lambda^3 + \sigma_{23}\lambda^2 + \sigma_{22}\lambda + \sigma_{21} = 0$ are stable. Then, the closed-loop system has the following error equations

$$\ddot{e}_1 + \sigma_{12}\dot{e}_1 + \sigma_{11}e_1 = 0 \tag{6}$$

$$e_2^{(4)} + \sigma_{24}e_2^{(3)} + \sigma_{23}\ddot{e}_2 + \sigma_{22}\dot{e}_2 + \sigma_{21}e_2 = 0 \tag{7}$$

and the plant outputs converge to the reference outputs.

Since the controller requires the angular velocity signals $\dot{\varepsilon}$, $\dot{\theta}$ and $\dot{\phi}$, in the experiment they are calculated numerically from the measured angular positions by a discretized differentiator with the first order filter

$$H_l(z) = \frac{\alpha\left(1 - z^{-1}\right)}{1 - z^{-1} + \alpha T_s}$$

which is derived by substituting $s = (1 - z^{-1})/T_s$ into the differentiator $G_l(s) = \alpha s/(s+\alpha)$ where z^{-1} is a one step delay operator, T_s is the sampling period and the design parameter α is a positive constant. Hence, for example, we have

$$\dot{\varepsilon}(k) \approx \frac{1}{\alpha T_s + 1}\left[\dot{\varepsilon}(k-1) + \alpha\left\{\varepsilon(k) - \varepsilon(k-1)\right\}\right]$$

IV. Parameter identification

It is difficult to obtain the desired control performance by applying the above algorithm directly to the experimental system since there are parameter uncertainties in the model dynamics. Here, it is easy to understand that the system dynamics (1) are linear with respect to unknown parameters though the equations are nonlinear. Therefore, it is possible to introduce the parameter identification scheme in the feedback control loop.

In this paper, the parameter identification scheme is designed in the discrete-time form using measured discrete-time signals. Hence, the estimated parameters are calculated recursively at every instant kT where T is an updating period of the parameters and k is a nonnegative integer. In the following, T is omitted because of the simplicity of description. Then, the dynamics of the model helicopter are rewritten from (1) as

$$w_1(k) \equiv \ddot{\varepsilon}(k) = \boldsymbol{\zeta}_1^T \boldsymbol{v}_1(k) \tag{8}$$

$$w_2(k) \equiv \ddot{\theta}(k) = \boldsymbol{\zeta}_2^T \boldsymbol{v}_2(k) \tag{9}$$

$$w_3(k) \equiv \ddot{\phi}(k) = \boldsymbol{\zeta}_3^T \boldsymbol{v}_3(k) \tag{10}$$

where

$$\begin{aligned}
\boldsymbol{\zeta}_1 &= \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}^T \\
\boldsymbol{\zeta}_2 &= \begin{bmatrix} p_5 & p_6 & p_7 & p_8 \end{bmatrix}^T, \quad \boldsymbol{\zeta}_3 = \begin{bmatrix} p_9 & p_{10} \end{bmatrix}^T \\
\boldsymbol{v}_1(k) &= \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & \dot{\varepsilon} & u_1 \cos\theta \end{bmatrix}^T \\
\boldsymbol{v}_2(k) &= \begin{bmatrix} \cos\theta & \sin\theta & \dot{\theta} & u_2 \end{bmatrix}^T \\
\boldsymbol{v}_3(k) &= \begin{bmatrix} \dot{\phi} & u_1 \sin\theta \end{bmatrix}^T
\end{aligned}$$

The parameter vectors $\boldsymbol{\zeta}_1$, $\boldsymbol{\zeta}_2$, $\boldsymbol{\zeta}_3$ are identified by the recursive least squares algorithm.

Then, the tracking of the two outputs is achieved under the persistent excitation of the signals $\boldsymbol{v} = [\boldsymbol{v}_1^T, \ \boldsymbol{v}_2^T, \ \boldsymbol{v}_3^T]^T$.

V. EXPERIMENTAL RESULTS

The estimation and control algorithm mentioned above were applied to the experimental system. The design parameters are given as follows. The sampling period T_s is $T_s = 2$ [ms], the updating period of the parameters T is also $T = 2$ [ms] and the filter parameter α for estimation of velocities and accelerations is $\alpha = 100$. The inputs u_{M1} and u_{M2} of the reference model are given by

$$\begin{aligned} u_{M1} &= \begin{cases} 0.3 & 45k - 52.5 \le t < 45k - 30 \\ -0.1 & 45k - 30 \le t < 45k - 7.5 \end{cases} \\ u_{M2} &= \begin{cases} 0 & 0 \le t < 7.5 \\ 0.4 & 45k - 37.5 \le t < 45k - 22.5 \\ -0.4 & 45k - 22.5 \le t < 45k \end{cases} \\ & \qquad\qquad k = 1, 2, 3, \cdots \end{aligned} \tag{11}$$

The eigenvalues of the matrices K_1 and K_2 are all -1, and the characteristic roots of the error equations (6) and (7) are specified as $(-2.0, \ -3.0)$ and $(-2.0, \ -2.2, \ -2.4, \ -2.6)$, respectively.

The outputs are shown in Fig. 2 and Fig. 3. The tracking performance of the both outputs ε and ϕ is achieved. Two estimated parameters are depicted in Fig. 4 and Fig. 5.

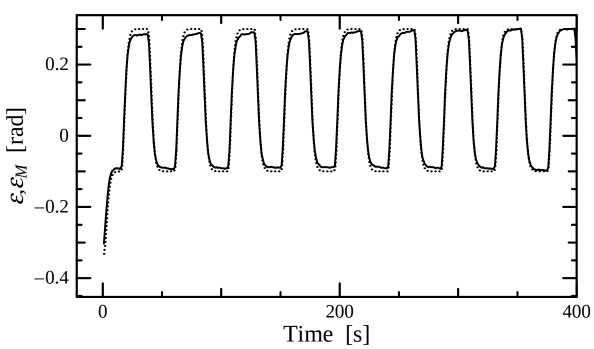

Fig. 2. Outputs

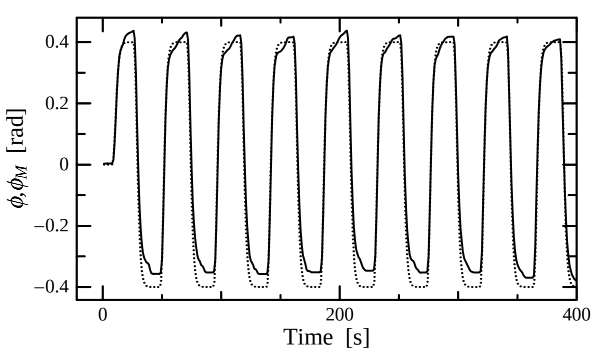

Fig. 3. Outputs;

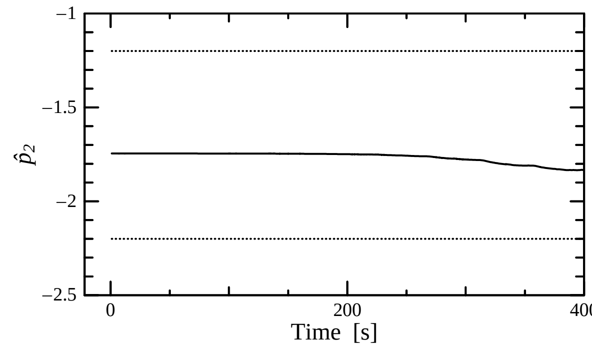

Fig. 4. Estimated parameter $\widehat{p}_2$

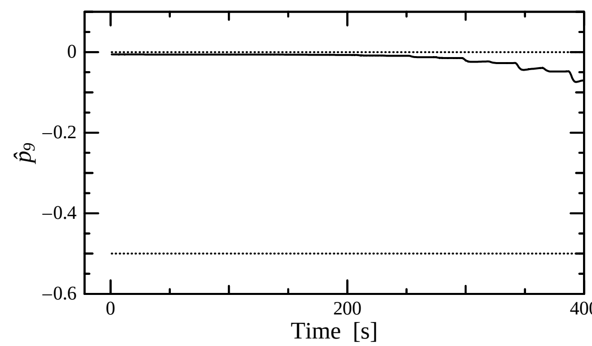

Fig. 5. Estimated parameter $\widehat{p}_9$

VI. A CONCLUSION

This paper considers nonlinear model following control with parameter identification of a 3 DOF model helicopter. The experimental results are shown.