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OPTIMAL PRODUCTION CONTROL METHOD FOR TANDEM MANUFACTURING LINES

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Abstract

In this paper authors introduce novel results on optimality and performance for surplus-based decentralized production control method. The main objective of this production method is to guarantee that the cumulative number of produced products follows the cumulative production demand on the output of any given network. As an extension of our previous result a general idea of this method is presented for the case of one manufacturing machine, where the implemented control strategy is proven to be optimal. Then a flow model for a line of N machines with bounded buffers is analyzed. Results on performance of this strategy for a line of machines show the uniform ultimate boundedness of production errors of each machine in the network. Performance and robustness issues of the closed-loop flow line model are illustrated in numerical simulations.

Key words

manufacturing systems, discrete-time systems, intelligent control, optimal control

1 Introduction

Production control methods with capabilities of quick responses to rapid changes in the demand and efficient distribution of the raw material throughout the network are of a big importance among leading manufacturers.

Thus, there is a substantial literature on control policies for manufacturing systems as well as many classifications of these policies are introduced by different authors. In this paper we will follow the classification introduced by S.R. Gershwin. In his work (Gershwin (2000)) the author separates the control policies in 3 main streams: token-based, time-based and surplus-based. In token-based approaches so called tokens are generated and utilized in order to trigger certain events occurring in the manufacturing system. The most famous example of such a policy are Kanban (Rees et al. (1987)), Conwip (Spearman et al. (1990)) and Basestock (Silver et al. (1998)). In time-based approaches the control decisions depend on a time when a certain operation should take place, i.e. Material Resource Planning, Least Stack and Earliest Due Date strategies (see, e.g., Burgess and Passino (1997)). In the surplus approach control decisions are made based on the production error which is the difference between the cumulative demand and the cumulative output of the system (see, e.g., Bielecki and Kumar (1988); Gershwin (2000); Nilakantan (2010)).

In this paper we first tackle the problem of optimality analysis for a surplus-based control approach for one manufacturing machine introduced in (Starkov et al. (2010)). The main idea of this approach is based on production managing with a primary goal of tracking demand. Further, our goal is to apply classical tools from control theory in order to evaluate a performance of this technique for a unidirectional manufacturing line of N machines. The proposed methodology is proven to be optimal for one machine and is reformulated for a production line of N machines with limited capacity intermediate buffers. The production flow process is described by means of difference equations and in order to analyse performance Lyapunov theory approach is exploited.

The paper is organized as follows. First, in Section 2 the flow model of one manufacturing machine is presented, where the analysis of optimal control strategy for production error tracking is developed. Then in Section 3 the flow model of a manufacturing line with bounded intermediate buffers is presented. Here the optimal control strategy from previous section is introduced for this model. The result on performance of each machine in the network is also give in this section. Performance and robustness issues of the closed-loop flow model of production line are illustrated in numerical example in Section 4. Finally, Section 5 contains conclusions and future developments.

2 Analysis of one manufacturing machine

2.1 Flow model

In discrete time a cumulative number of produced products in time k for a simple manufacturing machine can be described as a sum of its production rates at each time step till time k. Thus the flow model of one manufacturing machine in discrete time is defined as

$$y(k+1) = y(k) + u(k) + f(k), \tag{1}$$

where $y(k) \in \mathbb{R}$ is the cumulative output of the machine in time k, $u(k) \in \mathbb{R}$ is the control signal, and $f(k) \in \mathbb{R}$ is an unknown external disturbance.

Under the assumption that there is always sufficient quantity of the raw material to feed the machine, the control aim is to track the non-decreasing cumulative production demand. We define the production demand by using $y_d(k) \in \mathbb{R}$ given by

$$y_d(k) = y_{d0} + v_d k + \varphi(k), \tag{2}$$

where y_{d0} is a positive constant that represents the initial production demand, v_d is a positive constant that defines the average desired demand rate, and $\varphi(k) \in \mathbb{R}$ is the bounded fluctuation that is imposed on the linear demand $v_d k$. Specifically, we are going to minimize the output tracking error $\varepsilon(k) = y_d(k) - y(k)$ in the class of control strategies fed by available data:

$$u(k) = U_k(y(0), \ldots, y(k), y_d(0), \ldots, y_d(k)) \in \{0; 1\}\,. \tag{3}$$

Thus, in time step k the control input u is limited to taking the value of 1 when the machine is required to produce and taking the value of 0 when no production is required.

Here $\varepsilon(k+1)) - \varepsilon(k)$ along the solutions of $\varepsilon(k)$ is given by:

$$\varepsilon(k+1) = \varepsilon(k) - u(k) + v_d + \Delta\varphi(k) - f(k). \tag{4}$$

The external disturbance $f(k)$ and the fluctuation $\varphi(k)$ in (1) and (2) are bounded

$$\alpha_1 < \Delta\varphi(k) - f(k) < \alpha_2 \quad \forall k \in \mathbb{N} \tag{5}$$

where $\Delta\varphi(k) = \varphi(k+1) - \varphi(k)$ and α_1, α_2 are unknown constants that obey the following bounds

$$\alpha_2 < 1 - v_d, \qquad \alpha_1 > -v_d. \tag{6}$$

By first and second inequalities in (6) we state that the machine can never produce products faster than its maximal speed and that considering the presence of perturbations bounded by (α_1, α_2) the demand rate can only de positive, respectively. Thus, from (5) and (6) the following condition (also known as capacity condition) holds

$$0 < \xi(k) < 1 \qquad \forall k, \tag{7}$$

where for the sake of brevity $\xi(k) := v_d + \Delta\varphi(k) - f(k)$.

2.2 Results on performance

In this section, we examine the following two performance criteria:

$$J_T = \sup_{\xi(0),\ldots,\xi(T-1)} \sum_{k=0}^{T} |\varepsilon(k)|^p \to \min_U, \tag{8}$$

$$J_\infty = \limsup_{k\to\infty} \sup_{\xi(\cdot)} |\varepsilon(k)|^p \to \min_U\,. \tag{9}$$

Here sup is taken over all $\xi(\cdot)$ satisfying (7), $U = \{U_k(\cdot)\}_{k=0}^{\infty}$ is the control strategy, formula (8) deals with a finite and given time horizon T of the experiment, whereas it is infinite in (9), and $p \in [1, +\infty)$ is a given parameter. It will be shown, that the optimal control strategy doesn't depend on the choice of p.

To state the main result, we introduce the following notation:

$$\operatorname{sign}_+(\varepsilon) = \begin{cases} 1 & \text{if } \varepsilon > 0 \\ 0 & \text{if } \varepsilon < 0 \\ 0,1 & \text{if } \varepsilon = 0 \end{cases}.$$

The last line means that sign_+ is permitted to take any of the values 0 and 1.

Theorem 1. *The following control strategy*

$$u(k) = sign_+\big(\varepsilon(k)\big) \tag{10}$$

is optimal with respect to the performance index (8) for any given T, as well as with respect to the performance criterion (9). This is true irrespective of the choice of $p \in [1, +\infty)$.

Proof. Based on (4), it is easy to see that without any loss of generality, the class of admissible control strategies (3) can be reduced to those processing only the tracking errors: $u(k) = U_k(\varepsilon(0), \ldots, \varepsilon(k)) \in \{0; 1\}\,.$

We start with the problem (8). The proof is based on the min-max dynamic programming. So we first introduce the cost-to-go:

$$V_\tau(a) = \min_{U_\tau(\cdot),\ldots,U_{T-1}(\cdot)} \sup_{\xi(\cdot)} \sum_{k=\tau}^{T} |\varepsilon(k)|^p, \quad V_T(a) := |a|^p, \tag{11}$$

where the minimum is over all functions $U_k(\varepsilon_k, \ldots, \varepsilon_{T-1}) \in \{0;1\}$, and $\varepsilon(k)$ is obtained from (4), where $k = \tau, \ldots, T-1$ and $\varepsilon(\tau) = a$. This function satisfies the Bellman equation (Bertsekas (2005)):

$$V_{\tau-1}(a) = \min_{u=0;1} \sup_{\xi \in (0;1)} \{|a|^p + V_\tau(a - u + \xi)\}, \tag{12}$$

and the optimal strategy is given by $u(\tau-1) = U^0_{\tau-1}[\varepsilon(\tau-1)]$, where $U^0_{\tau-1}[a]$ is the point furnishing the minimum in (12).

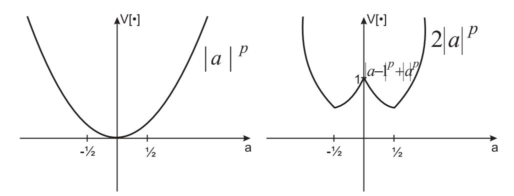

Figure 1. (Left):The graph of V_T; (Right): The graph of V_{T-1}

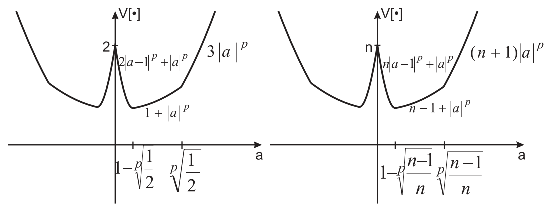

Figure 2. (Left):The graph of V_{T-2}; (Right): The graph of V_{T-n} with $n \geq 3$

Lemma 1. *The cost-to-go (11) is the piece-wise smooth even function depicted in Figures* 1 *and* 2*, and*

$$U^0_\tau[a] = sign_+(a) \quad for \quad \tau = 0, \ldots, T-1. \tag{13}$$

Proof. We first note that (12) can be shaped into

$$V_{\tau-1}(a) = \min \left\{ \begin{array}{c} \overbrace{\sup_{\xi \in (0;1)} V_\tau(a+\xi)}^{S_0}, \\ \overbrace{\sup_{\xi \in (0;1)} V_\tau(a-\xi)}^{S_1} \end{array} \right\} + |a|^p. \tag{14}$$

Here S_0 and S_1 correspond to $u = 0$ and $u = 1$, respectively. So $U^0_{\tau-1}(a) = \sigma_{\min}$, where $\sigma_{\min} = 0, 1$ is the index of the term S_σ furnishing the minimum in (14). We also note that since the function $a \mapsto |a|^p$ is even, simple induction on $\tau = T, \ldots, 0$ and the last equation from (11) show that $V_\tau(\cdot)$ is even for any τ. With this in mind, it becomes clear that firstly, $\sigma_{\max} = 0, 1$ for $a = 0$ and secondly, substitution $a := -a$ in (14) switches $\sigma_{\min}$ to the alternative value. This permits us to focus on $a > 0$ in the subsequent proof. For $a > 0$, formula (13) (to be justified) takes the form $U^0_\tau[a] = 1$.

We proceed with immediate proof of the lemma, arguing by induction on $\tau = T - n, n = 0, 1, \ldots$.

$\boldsymbol{n = 0}$. The claim is immediate from the last equation in (11).

$\boldsymbol{n = 1}$. $\boldsymbol{a \geq \frac{1}{2}}$ **:** Then evidently, $S_1 = |a|^p$, and $S_0 = |a+1|^p > S_1$. So, due to (14), $V_{T-1}(a) = 2|a|^p$, as is depicted in Fig. 1(right), and $U^0_\tau(a) = 1$. $\boldsymbol{0 < a < \frac{1}{2}}$ **:** Since $V_T(\cdot)$ is even, $S_1 = |a-1|^p < |a+1|^p = S_0$. So $V_{T-1} = |a-1|^p + |a|^p$, as is depicted in Fig. 1(Right), and $U^0_\tau(a) = 1$.

$\boldsymbol{n = 2}$ $\boldsymbol{a \geq 1}$ **:** Similarly, in (14), the supremum S_0 is equal to $2|a+1|^p$, whereas $S_1 = 2|a|^p < S_0$. $\boldsymbol{\frac{1}{2} \leq a < 1}$ **:** $S_1 = \left\{ \begin{array}{ll} 2|a|^p & a > \sqrt[p]{\frac{1}{2}} \\ 1 & a < \sqrt[p]{\frac{1}{2}} \end{array} \right\} < 2|a+1|^p = S_0$. $\boldsymbol{0 \leq a < \frac{1}{2}}$ **:**

$$S_1 = \left\{ \begin{array}{ll} 2|a-1|^p & a < 1 - \sqrt[p]{\frac{1}{2}} \\ 1 & a > 1 - \sqrt[p]{\frac{1}{2}} \end{array} \right\} < 2|a+1|^p = S_0.$$

Thus

$$V_{T-2}(a) = \begin{cases} 3|a|^p & a \geq \sqrt[p]{\frac{1}{2}} \\ 1 + |a|^p & 1 - \sqrt[p]{\frac{1}{2}} \leq a < \sqrt[p]{\frac{1}{2}} \\ 2|a-1|^p + |a|^p & a < 1 - \sqrt[p]{\frac{1}{2}} \end{cases},$$

as depicted in Figure 2(Left), and $U^0_\tau(a) = 1$.

Figure 2(Left) is a particular case of Figure 2(Right). So to complete the proof, it suffices to show that

C) Figure 2(Right) is correct and $U^0_{T-n}(a) = 1$

for $n = 2, 3, \ldots$, arguing by induction on n.

Suppose that **C)** is true for some $n \geq 2$. To compute $V_{T-n-1}(a)$, we consider separately several cases.

- $\boldsymbol{a \geq \sqrt[p]{\frac{n}{n+1}}}$ **:** Here $\sqrt[p]{\frac{n}{n+1}} > \sqrt[p]{\frac{n-1}{n}}$. It follows that in (14), the supremum S_1 is attained at $\xi = 0$ and thus equals $(n+1)|a|^p$, whereas $S_0 = (n+1)|a+1|^p > S_1$. Thus **C)** does hold for $n := n+1$.
- $\boldsymbol{\sqrt[p]{\frac{n-1}{n}} \leq a \leq \sqrt[p]{\frac{n}{n+1}}}$ **:** Then evidently $S_1 = n$, whereas $S_0 = (n+1)|a+1|^p > S_1$. Thus **C)** does hold for $n := n+1$.
- $\boldsymbol{1 - \sqrt[p]{\frac{n-1}{n}} \leq a \leq \sqrt[p]{\frac{n-1}{n}}}$ **:** Since the left end $a-1$ of the interval $[a-1, a]$ is still to the right of the first fracture point of the graph from Figure 2(Right), the situation replicates the previous one.
- $\boldsymbol{1 - \sqrt[p]{\frac{n}{n+1}} \leq a \leq 1 - \sqrt[p]{\frac{n-1}{n}}}$ **:** That end is to the left of the first fracture point. So either $S_1 = n$ (and is attained at the third fracture point) or $S_1 = (n+1)|a-1|^p$ (and is attained at $\xi = 1$). Elementary comparison shows that in fact $S_1 = n$, and so the situation still replicates the previous two ones.
- $\boldsymbol{0 \leq a \leq 1 - \sqrt[p]{\frac{n}{n+1}}}$ **:** Then conversely, $S_1 = (n+1)|a-1|^p$, whereas $S_0 = (n+1)|a+1|^p > S_1$.

Thus **C)** does hold for $n := n+1$, which completes the proof.

For the performance index (8), Theorem 1 is straightforward from Lemma 1 and the dynamic programming principle (Bertsekas (2005)).

To deal with (9), we introduce the following intermediate performance criterion

$$J_{\text{av}} = \limsup_{T\to\infty} \sup_{\xi(0),\dots,\xi(T-1)} \frac{1}{T}\sum_{k=0}^{T} |\varepsilon(k)|^p. \tag{15}$$

It is clear that

$$\inf_U J_{\text{av}} \geq \limsup_{T\to\infty} \frac{1}{T}\min_U J_T \overset{(11)}{=\!=} \limsup_{T\to\infty} \frac{V_0^T[a]}{T},$$

where the upper index ^{T} in V_τ^T underscores that the cost-to-go is computed for the time horizon $[0:T]$. As a result, Lemma 1 and the evident inequality $J_\infty \geq J_{\text{av}}$ imply the following lower estimates

$$\inf_U J_\infty \geq \inf_U J_{\text{av}} \geq \begin{cases} |a|^p & \text{if } |a| \geq 1 \\ 1 & \text{otherwise} \end{cases}.$$

Now we are going to show that this lower estimate of J_∞ is attained at the control strategy (10), which will complete the proof.

Let the system (4) be driven by the control law (10). By invoking (7), we conclude that

$$\varepsilon(k+1) \in \begin{cases} \big(\varepsilon(k)-1, \varepsilon(k)\big) & \text{if } \varepsilon(k) > 0 \\ \big(\varepsilon(k), \varepsilon(k)+1\big) & \text{if } \varepsilon(k) < 0 \\ \big(\varepsilon(k)-1, \varepsilon(k)+1\big) & \text{if } \varepsilon(k) = 0 \end{cases}.$$

Hence $f_-(\varepsilon(k)) \leq \varepsilon(k+1) \leq f_+(\varepsilon(k))$, where $f_-(\varepsilon) := \min\{\varepsilon; -1\}$, $f_+(\varepsilon) := \max\{\varepsilon; 1\}$. It follows that $\varepsilon_-(k) \leq \varepsilon(k) \leq \varepsilon_+(k)\ \forall k$, where $\varepsilon_-(k)$ and $\varepsilon_+(k)$ are the solutions of the following recursions $\varepsilon_\pm(k+1) = f_\pm(\varepsilon_\pm(k)), \varepsilon_\pm(0) = a$. It is evident that $\varepsilon_\pm(k) \in \Big[\min\{-|a|, -1\}; \max\{|a|; 1\}\Big]$, which completes the proof.

Now that for one machine the optimal tracking controller is derived we extend our analysis of this strategy applied to a line of N manufacturing machines with bounded intermediate buffers.

3 A line of machines with bounded buffers

3.1 Flow model

The flow model of a manufacturing line is presented in this section. Figure 3 presents a schematics of a line of N manufacturing machines with machines M_j, buffers B_j, and infinite product supply. Here the optimal control strategy from previous section is modified with respect to the number of buffers and machines present in the line. New limitations such as desired buffer content and buffer capacity restriction are considered in the model.

The flow model of the manufacturing line is defined as

$$\begin{aligned} \Delta y_1(k) &= \beta_1(k)\text{sign}_-(w_2(k)-\gamma_2), \\ \Delta y_j(k) &= \beta_j(k)\text{sign}_{\textit{Buff}}(w_j(k)-\beta_j(k)) \\ &\quad\times \text{sign}_-(w_{j+1}(k)-\gamma_{j+1}),\ j = 2,\dots,N-1, \\ \Delta y_N(k) &= \beta_N(k)\text{sign}_{\textit{Buff}}(w_N(k)-\beta_N(k)), \end{aligned} \tag{16}$$

where $\Delta y_j(k) = y_j(k+1) - y_j(k)$, $y_j(k)$ is the cumulative output of machine M_j in time k, $w_j(k) = y_{j-1}(k) - y_j(k)$ is the buffer content of buffer B_j, $\beta_j(k) = u_j(k) + f_j(k), \forall j = 1,\dots,N$, f_j is the external disturbance affecting machine M_j (e.g. production speed variations, undesired delay or setup time), u_j is the control input of machine M_j,

$$\text{sign}_{\textit{Buff}}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}, \text{sign}_-(z) = \begin{cases} 1 & \text{if } z \leq 0 \\ 0 & \text{if } z > 0 \end{cases},$$

and γ_{j+1} is the threshold value of the buffer content w_{j+1}. Basically for each machine we introduce an extra restriction on production which is based on the buffer content of its upstream and downstream buffer. Any machine M_j, with $j = 2,\dots,N-1$, is activated only if three authorizations are given. The first authorization comes from control input $u_j(k)$ of M_j. The second authorization comes from the restriction on the upstream buffer content ($\text{sign}_{\textit{Buff}}(\cdot)$), which is granted if the buffer contains at least the minimal number of products required ($\beta_j(k)$) in order for the machine M_j to start its work. The third authorization ($\text{sign}_-(\cdot)$) comes from the downstream buffer of given machine. This authorization is possible only if the downstream buffer have sufficient storage in order to accept incoming production.

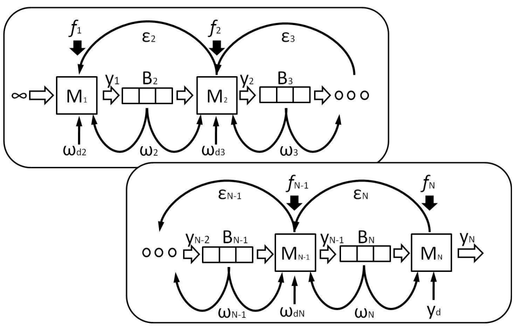

Figure 3. Flow model diagram for a line of N machines.

In order to give a solution to the demand tracking problem we propose the following control inputs:

$$u_j(k) = \mu_j \text{sign}_+(\varepsilon_{j+1}(k) + w_{d_{j+1}} - w_{j+1}(k)) \quad \forall j = 1, \dots, N-1, \tag{17}$$

$$u_N(k) = \mu_N \text{sign}_+(y_d(k) - y_N(k)), \tag{18}$$

where μ_j is the processing speed of machine j, $w_{d_{j+1}}$ is the desirable buffer level of buffer B_{j+1} and ε_{j+1} is the tracking error of machine M_{j+1}. Here for simplicity we restrict the value of sign_+ function, which was defined in the previous chapter, to $\text{sign}_+(x)$ =(1, if $x > 0|0$, otherwise).

The tracking error of each machine is given by:

$$\varepsilon_j(k) = \varepsilon_{j+1}(k) + (w_{d_{j+1}} - w_{j+1}(k)), \quad \forall j = 1, \dots, N-2 \tag{19}$$

$$\varepsilon_{N-1}(k) = \varepsilon_N(k) + (w_{d_N} - w_N(k)), \tag{20}$$

$$\varepsilon_N(k) = y_d(k) - y_N(k). \tag{21}$$

It follows from (21) that the error of machine M_N is defined exactly as for the single machine case. The buffer restriction, as seen from (16), is the only difference in the flow model of machine M_N with the flow model of (1). For (19), (20) new considerations are applied for the tracking error of each machine M_j, where $j = 1, ..., N-1$. Here tracking error $\varepsilon_j(k)$ depends on number of produced products $y_j(k)$ with respect to current demand $y_d(k)$ and desired buffer content $w_{d_{j+1}}$ of each downstream buffer. This means that every upstream machine needs to supply $w_{d_{j+1}}$ lots more than the downstream one. Constant parameter w_d is introduced in order to prevent downstream machines from lot starvation, e.g. in case of a sudden growth of the product demand.

It is important to take into account that the control actions are decentralized throughout the network. In other words the control action of each machine in the line depends only on the tracking error of its neighboring downstream machine (except for machine M_N, which depends directly on cumulative demand input) and the current buffer content of its upstream and downstream buffer (Fig.3). This gives our flow model an extra robustness with respect to the undesired events such as temporal machine setup or breakdown.

For further analysis, let us rewrite flow model (16) in a closed-loop with (17), (18) as

$$\begin{aligned}
\Delta\varepsilon_1(k) &= v_d + \Delta\varphi(k) \\
&\quad - \beta_1(k)\text{sign}_(w_2(k) - \gamma_2), \\
\Delta\varepsilon_j(k) &= v_d + \Delta\varphi(k) - \beta_j(k)\text{sign}_{\mathit{Buff}}(w_j(k) \\
&\quad - \beta_j(k))\text{sign}_(w_{j+1}(k) - \gamma_{j+1}), \\
\Delta\varepsilon_N(k) &= v_d + \Delta\varphi(k) - \beta_N(k) \\
&\quad \times \text{sign}_(w_N(k) - \beta_N(k)).
\end{aligned} \tag{22}$$

Here we consider that system (22) satisfies the following assumptions.

Assumption 1 (Boundedness of perturbations). *There are constants α_1, α_2 and α_3 such that $W_j(k) = \Delta\varphi(k) - f_j(k)$ satisfies*

$$\alpha_1 < W_j(k) < \alpha_2, \ \forall k \in \mathbb{N}, \tag{23}$$

and $f_j(k)$ satisfies

$$f_j(k) \le \alpha_3, \ \forall k \in \mathbb{N}. \tag{24}$$

Assumption 2 (Capacity condition). *Constants α_1, α_2 satisfy the following inequalities*

$$\alpha_2 < \mu_j - v_d, \tag{25}$$

$$\alpha_1 > -v_d. \tag{26}$$

Thus, from (23), (25), and (26) the following condition holds

$$0 < v_d + W_j(k) < \mu_j, \ \forall j = 1, \dots, N. \tag{27}$$

It is important to notice that each M_j machine in the line has a processing speed of μ_j lots per time unit, which can differ from the rest of the machines and the buffer content condition is considered as

$$\beta_j(k) \le w_j(k) < \gamma_j, \ \forall j = 2, \dots, N. \tag{28}$$

Note that the physical restriction on buffer content is given as

$$0 \le w_j(k) < \gamma_j + \mu_{j-1} + \alpha_3, \ \forall j = 2, \dots, N. \tag{29}$$

Here $\gamma_j = \mu_j + \alpha_2 - \alpha_1 + w_{d_j}$ where w_{d_j} satisfies the following

Assumption 3 (Desired buffer content condition). *The constants w_{d_j} comply with the following inequality $w_{d_j} \ge \mu_j + \mu_{j-1} + \alpha_3 + \alpha_2 - \alpha_1$, from where it follows that*

$$w_{d_j} \ge \beta_j(k) + \mu_{j-1} + \alpha_2 - \alpha_1, \ j = 2, \dots, N. \tag{30}$$

3.2 Results on performance

The obtained results on the production error trajectories behavior of flow model (22) can be formulated through the following theorem.

Theorem 2. *Assume that the discrete time system defined by (22) satisfies Assumptions 1, 2, and 3. Then all solutions of (22) are uniformly ultimately bounded by*

$$\limsup_{k\to\infty} \varepsilon_j(k) \le v_d + \alpha_2, \tag{31}$$

$$\liminf_{k\to\infty} \varepsilon_j(k) \ge v_d + \alpha_1 - \mu_j. \tag{32}$$

Proof. The proof of Theorem 2 will be given in the forth coming paper.

Now, in order to support the present development let us extend our analysis to simulation example.

4 Simulation example

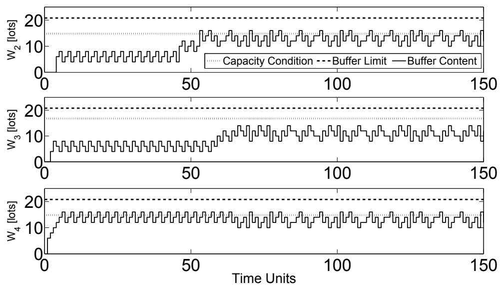

Figure 4. Buffers Content $w_j(k)$.

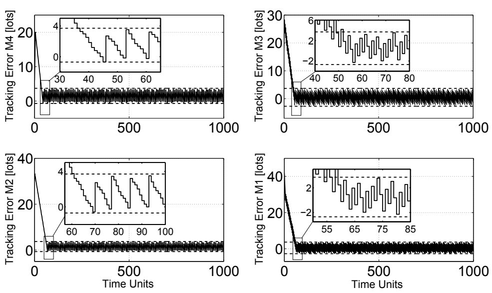

Figure 5. Tracking Errors $\varepsilon_j(k)$.

Consider a following example of a production line that consists of 4 manufacturing machines operating under variable structure regulators (17), and (18). The processing speed for each machine is set to $(\mu_1, \ldots, \mu_4) = (6, 4, 6, 4)$ (lots per time unit), with $j = 1, .., 4$, the desired buffer content of each buffer is selected considering (30) as $(w_{d_2}, w_{d_3}, w_{d_4}) = (12, 12, 12)$ (lots), with $j = 2, .., 4$ and the mean demand rate $v_d = 3.5$ (lots per time unit) with fluctuation rate of $\Delta\varphi(k) = 0.2\sin(5k)$. The tracking error of each machine in the line is depicted in Figure 5. Here the initial conditions (y_{d0}, $y_1(0)$, $y_2(0)$, $y_3(0)$, $y_4(0)$) were set to the zero value. After the first 60 time steps, as it is shown in Figures 5, the system reaches its steady state. Tracking errors are maintained inside [-2.7,3.7] lots for machine M_1, [-0.7,3.7] lots for machine M_2, [-2.7,3.7] lots for machine M_3, and [-0.7,3.7] lots for machine M_4, which satisfy (31) and (32). From Figure 4 it can be observed that the inventory level of each buffer satisfies the buffer limit given by the second part of inequality (29) and the capacity condition (28) is sometimes violated due to the discrete nature of the model. Here $(\gamma_2, \gamma_3, \gamma_4) = (14.8, 16.8, 14.8)$ (lots). In conclusion, presented simulation results reflect the desired flow model behavior. All technical conditions proposed in this section correspond to analytical results described in Section 3.

5 Conclusion

The variable structure controller implemented in (Starkov et al. (2010)), in order to give a solution to the demand trajectory tracking problem for one manufacturing machine, is proven to be optimal. By extending this control strategy to a line of machines the results on uniform ultimate boundedness for tracking error trajectories of each machine were obtained. Here assumption on variable processing speed of machine and restriction on intermediate buffer capacity were considered. Presented simulation example reflects effectiveness and robustness of the flow model. Furthermore, studies on variable structure control policy's application to re-entrant network, multiple part type production systems, and performance analysis with the presence of production delays and setup times will be pursued in our future research.

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