ON FORMULATION AND SOLUTION OF A CONTROL PROBLEM FOR MINE VENTILATION NETWORKS

Andrey M. Valuev
Department of Management in Mining Industry
Moscow State Mining University
Russia
amvaluev@online.ru

Abstract

The practical problem of effective ventilation in underground mines meeting requirements of production leads to optimization problems having specific features. Recent experience concerned, however, only problems with fixed air flow in branches-consumers and a limited dimension. In the paper a combined method having features of feasible directions and gradient-restoration methods is proposed to overcome these deficiencies. The principal algorithm is formulated and substantiated, some details of its implementation are discussed. Experience of the method application is presented showing its usefulness to meet practical requirements.

Key words

mine ventilation system, control, optimality.

1 Introduction

For underground mining proper ventilation, i.e., supply of required quantity of fresh air to working zones as well as removal of methane and other pollutant gases, is a question of both safety and successful operation of a mine. The cost of ventilation is an important thing too, since energy consumption by fans is great and expensive. Regulation of a mine ventilation network (MVN) consists in determination of aerodynamic resistances of passive regulators (e.g., special doors) alongside with controlled parameters of main fans (MFs). Its 1st aim is to satisfy requirements of a given set of fresh air consumers, the 2nd to minimize energy consumption provided that consumer requirements are satisfied. It is the general treatment of the problem of optimization of air distribution in mine ventilation networks displayed in many papers, including [Kumar et al, 1998; Lilić and Kuzmanović, 1994; Wang, 1999].

The differences between various approaches to the problem in question lie in interrelated topics of representation of consumer requirements and regulators and the choice of optimization techniques. In the above listed papers a consumer demand is treated as a fixed air flow quantity and the action of a passive regulator is described by the loss of pressure in its branch. Both [Kumar et al, 1998; Wang, 1999] rely on the generalized reduced-gradient method [Avriel, 1976], but the 1st assume regulators to be only in fixed-quantity branches and the second enable them to be in an arbitrary branch. As to [Lilić and Kuzmanović, 1994], it is based on sequential unconstrained minimization technique by Fiacco and McCormic [Fiacco and McCormic, 1968]. Although the principal model of MVN in the papers [Kumar et al, 1998; Lilić and Kuzmanović, 1994; Wang, 1999] and many other is the same, it is transformed into the optimization problem in slightly different ways. In the presented examples different approaches were successful to cope with optimization problems in question, but dimension of the problems were rather small and much less than for mines of Donetsk coal basin for which we solved the similar problems.

The model of air distribution problem put forward in [Ushakov, 1999] differs from the mentioned above models in two aspects: 1) the requirement of a consumer is treated as a given direction and an admissible range of air flow quantity; 2) an additional aerodynamic resistance of a branch containing passive regulator instead of pressure loss in it is treated as a controlled variable. For our opinion, both assumptions are more realistic; besides, fixed air quantity is a particular case of the admissible range for which lower and upper bounds coincide.

Besides, the method of hierarchical segmentation of a mine ventilation network from the viewpoint of separation of air intake between groups of consumers put forward in [Ushakov, 1999] gives recommendations where to place passive regulators.

The method proposed to solve the optimization problems based on the proposed model is the combined method having features of feasible directions and gradient-restoration methods. Besides, it takes into account the most general properties of the network model relationships set structure. Instead, the specific technique of feasibility restoration is incorporated in it, namely well known Hardy Cross iterative method [Cross, 1936].

2 The problem statement

2.1 Data and dependences that determine the problem

Let J be the set of the MVN branches, q_j being directed air flow value for the i-th branch, and N the set of ventilation network nodes. For any branch the nominal direction is determined and $q_j > 0$ for flows which direction coincide with it, otherwise $q_j < 0$. We assume that the nominal direction of branches containing fans (W) is the required direction.

For every fan $j \in W$ dependences for depression and energy consumption, i.e., $H_j(q_j,\ u_j)$, $N_j(q_j, u_j)$, must be determined representing respectively depression and energy consumption as functions of air flow q_j and control parameter u_j values . The working zone of a fan in the j-th branch is described in the space (q_j, u_j) with restrictions

$$Q_{j\,\min}(u_j) \le q_j \le Q_{j\,\max}(u_j),$$

$$u_{j\,\min} \le u_j \le u_{j\,\max}.$$

The desired quality of ventilation is described with the set of branches-consumers (P) with given ranges of air flow quantity ($[q_{imin},\ \le\ q_{imax}], i \in P$), its required direction being the nominal direction for each $i \in P$, and the set of so-called likely diagonals (D) for which the nominal direction of air flow in each guarantee the absence of forbidden subsequent ventilation of consumers.

2.2 The problem formulation

Air flows distribution in a mine ventilation network (MVN), i.e., directed air flows quantities in network branches (q_j, $j \in J$) with aerodynamic resistances r_j, is subject to Kirchhoff equations

$$\textstyle\sum_{i\in I^+(j)} q_i - \sum_{i\in I^-(j)} q_i = 0,\ \ j \in N\backslash\{0\}, \quad (1)$$

$$-\textstyle\sum_{i\in I(k)} r_i q_i|q_i| + \sum_{i\in I(k)\cap W} H_i(q_i, u_i) = 0,\ k \in K, \quad (2)$$

where N and K are the sets of MVN nodes and independent contours respectively [Abramov, Tyan and Potyomkin, 1978], $I^-(j)$ and $I^+(j)$ being the sets of branches quitting and entering the j-th node. In the formulation of the problem it is convenient to represent the set of Kirchhoff equations (1) with the use of incidence matrix of the ventilation network [Abramov, Tyan and Potyomkin, 1978] as

$$\sum_{j\in J} I_{ij} q_j = 0,\ \ i \in N\backslash\{0\}, \quad (3)$$

(where I_{ij}=1 if the j-th branch enters to the i-th node, $I_{ij} = -1$ if the j-th branch quits the i-th node, otherwise I_{ij}=0), and to represent equations (2) with the use of generalized incidence matrix as

$$\sum_{j\in J} (I_{m+ij} r_j q_j|q_j| + H_j(q_j, u_j)) = 0,\ \ i \in K, \quad (4)$$

where $H_j(q_j,\ u_j)$=0 if the j-th branch has no fan, m is the number of nodes, I_{m+ij}=-1 if the j-th branch belongs to the i-th contour and the contour is passed round in the nominal branch direction, $I_{m+ij} = 1$ if the directions of passing round the contour and the branch are opposite and I_{m+ij}=0 if the j-th branch does not belong to the i-th contour.

Controlled variables are: 1) aerodynamic resistances of branches belonging to the set $R \subset J$ of branches containing air flow regulators and 2) controlled parameters of MFs ($u_i, i \in W$) subject to corresponding constraints

$$0 < r_{j\,\min} \le r_j \le r_{j\,\max}, \quad j \in R, \quad (5)$$

$$u_{j\,\min} \le u_j \le u_{j\,\max}, \quad j \in W, \quad (6)$$

$$Q_{j\,\min}(u_j) \le q_j \le Q_{j\,\max}(u_j), \quad j \in W. \quad (7)$$

Restrictions expressing demands for ventilation quality are

$$q_{j\,\min} \le q_j \le q_{j\,\max}, \quad j \in P, \quad (8)$$

$$0 \le q_j, \quad j \in D. \quad (9)$$

Restrictions (4)–(6) are easy to satisfy. Due to the above notation the 1$^{\text{st}}$ problem (to minimize the maximum residual in the constraints) has the form

$$z \quad \to \min, \quad (10)$$

$$q_{i\min} - q_j \le z, \quad q_j - q_{i\max} \le z, \quad j \in P, \quad (11)$$

$$q_j \ge -z, \quad j \in D, \quad (12)$$

$$Q_{j\min}(u_j) - q_j \le z, \quad (13)$$
$$q_j - Q_{j\max}(u_j) \le z, j \in W,$$

under restrictions (3)–(6) and the 2nd one (to minimize total energy consumption by MFs) — as

$$\sum\nolimits_{i\in W} N_i(q_i, u_i) \to \min \quad (14)$$

under restrictions (3)–(9).

3 Computational method and algorithm

3.1 A combined optimization method

The proposed method of the problem solution may be treated as a specific variant of a feasible directions method for a problem with inequality constraints. For this representation Kirchhoff equations are treated as means of representation of dependences that determine these constraints as implicit functions of controlled variables r_j, $j \in R$, u_i, $i \in W$. So we must treat the operation of the method as taking place in admissible domain X of controlled variables space. According to the general scheme of the method descent directions are (for a certain vector norm) the direction of steepest descent when the control vector x lies inside X and not close to its boundary ∂X. In the vicinity of ∂X the descent directions deviate towards the tangent plane in the nearest point of the boundary and tend to the projection of the steepest descent direction to the tangent plane.

For detailed description it is more convenient to regard it as a hybrid method that combine features of feasible directions and gradient-restoration methods. One of the preferences of this representation is that variants of the method with regulated accuracy of the projection phase depending on how near the obtained point is to the optimum point.

In this representation the set of controlled variables consists of r_j, $j \in R$, u_i, $i \in W$, and q_j, $j \in J$. Kirchhoff equations determine the surface S in the domain Y of respective controlled variables vectors. The problem I of finding the descent direction for a known $y \in Y$ is being sought in the tangent hyperplane to S in y and then a step is done in this direction. It is analogous to gradient-restoration problem [Bazaraa, Sherall and Shetty, 1993], but the problem I is more complicated. After that the obtained point y' of the tangent hyperplane $T_S(y)$ is being projected to S (the problem *II*) analogously to classical gradient-restoration methods. Each (r-th) iteration of the method consists of solutions of problems I and *II* for the corresponding $y^{(r)}$. The principal construction of the method resembles that of [Valuev, 1990].

Let us determine x as a vector which components are r_j, $j \in R$, u_i, $i \in W$, and q as a vector with components q_j, $j \in J$, and their dimensions as N_X and N_Q. Both the 1st and the 2nd problems may be represented in the form

$$G_0(x, q) \quad \to \min, \quad (15)$$

$$G_i(x, q) \le 0, \; i = 1, ..., M, \quad (16)$$

$$x_{j\min} \le x_j \le x_{j\max}, \; j = 1, ..., N_X, \quad (17)$$

$$F_i(x, q) = 0, \; i = 1, ..., N_Q, \quad (18)$$

where M is the total amount of constraints-inequalities (7).

The descent direction in $T_S(y)$ is represented with vectors Δx, Δq, Δx being determined from the special optimization problem and the corresponding Δq from the system of linear equations

$$(\nabla_x F_i(x, q), \; \Delta x) + (\nabla_q F_i(x, q), \; \Delta q) = 0,$$
$$i = 1, ..., N_Q.$$

The principal requirements a descent direction is reducing the value of the linear approximation of the target function, i.e.,

$$(\nabla_x G_0(x, q), \; \Delta x) + (\nabla_q G_0(x, q), \; \Delta q) < 0,$$

safeguarding the inequality constraints satisfaction what is expressed by the condition

$$G_i(x, q) + (\nabla_x G_i(x, q), \; \Delta x) +$$
$$(\nabla_q G_i(x, q), \; \Delta q) < 0, \; i = 1, ..., M,$$

and in the formal setup of the problem I both requirements are balanced. The value of Δq satisfying the equations set (18) may be expressed as $C(x, q)\Delta x$ where the $N_Q \times N_X$ matrix $C(x, q)$ which elements are partial derivatives of $q_k(x)$ with respect to x_j gives the solution of the equations system

$$(\nabla_x F_i(x, q))^T + (\nabla_q F_i(x, q))^T C(x, q) = 0, \quad (19)$$
$$i = 1, ..., N_Q.$$

Taking into account the dependence $\Delta q = C(x,q)\Delta x$, we define the problem I like in [Zukhovitskii, Polyak and Primak, 1963] (using as well recommendations of [Pshenichnyi and Danilin, 1975][p. 246] for linear inequalities)

$$s \equiv \max\{m_i + [(\nabla_x G_i(x,q))^T + (\nabla_q G_i(x,q))^T \times C(x,q)]\Delta x \mid i = 0, ..., M\} \to \min, \tag{20}$$

$$x_j \le x_j + \Delta x_j \le x_j; \quad -1 \le \Delta x_j \le 1, \quad j = 1, ..., N_X, \tag{21}$$

where m_0=0, $m_i = G_i(x,q)$, i=1,...,M.

After regrouping equations (19) we have a number of similar equations sets for columns c_{col_j} of the matrix $C(x,q)$, all system having the same matrix on the left side. General form of these systems is:

$$A(x,q)c_{\text{col}_j} = b_j(x,q),\ j = 1, ..., N_X. \tag{22}$$

The set of equations system (22) is being solved in the following way [Demmel, 1997]. By the direct phase of Gauss elimination method (*LU*-decomposition) upper and lower triangular matrices $U(x,q)$ and $L(x,q)$ are determined which product is the matrix obtained from $A(x,q)$ by permutation of columns. After that the inverse phase of the elimination method is performed for each $c_j(x,q)$, that consists in the subsequent solution of equations systems

$$\begin{aligned} U(x,q)y_j &= b_j(x,q), \\ L(x,q)c_{\text{col}_j}(x,q) &= y_j. \end{aligned} \tag{23}$$

Computation of $c_{\text{col}_j}(x,q)$ requires $(N_Q)^2$ multiplicative and $(N_Q)^2$ additive operations because of properties of triangular matrices, while *LU*-decomposition demands $(N_Q)^3$/3 operations of both types [Demmel, 1997]. Thus the total amount of calculations for the whole set of $N_X << N_Q$ systems (23) is not much greater than for the unique one.

If fact, matrices $A(x,q)$ generated by differentiation of left sides of equations (3), (4) are sparse as each equation contains much less variables than the number of branches. Practical experience show that after all transformations they stay sparse and the total ratio of non-zero elements is not more than 15%. Efficient usage of sparse matrices techniques lead to greater efficiency of the whole optimization method.

3.2 The algorithm of the proposed optimization method

The proposed principal algorithm of the problem (15)–(18) solution is stationary, i.e. consists of a succession of the same iteration algorithms that calculate the pair $x^{(l+1)}$, $q^{(l+1)}$ from the pair $x^{(l+1)}$, $q^{(l+1)}$ using the same vector of parameters.

Algorithm Ω consists in the following.

Step 0. The vector $x^{(0)}$ is given that satisfies (17). Solving the system (18) by Hardy Cross iterative method, compute the corresponding $q^{(0)}$. Set l=0.

Step 1 (the l-th algorithm iteration). Solve the problem (19)–(20) with $x = x^{(l)}$, $q = q^{(l)}$. If its solution yields s=0, halt (the optimum vector $x^{(l)}$ is found). Otherwise do the following.

Set a=1. Compute $x = x^{(l)} + a\Delta x$ and find q from solution of equation set (18) by Hardy Cross iterative method. If constraints (16) are satisfied and

$$G_0(x,q) \le G_0(x^{(l)},\ q^{(l)}) + as/2,$$

set $x^{(l+1)} = x$, $q^{(l+1)} = q$ and proceed to Step 2, otherwise set $a = a/2$ and repeat computations of Step 1.

Step 2. Set $l = l + 1$ and proceed to Step 1.

The substantiation of the method may be done analogously to the paper [Valuev, 1990]. According to [Valuev, 1990], we will consider two conditions on properties of the problem (15)–(18).

Condition 1. There is $\Delta > 0$ and Y_Δ is the set of approximately admissible in (15)–(18) pairs $y = (x,q)$ satisfying the conditions

$$G_i(x,q) \le \Delta,\ i = 1, ..., M,$$

$$x_{j\,\min} - \Delta \le x_j \le x_{j\,\max} + \Delta,\ j = 1, ..., N_X,$$

$$|F_i(x,q)| \le \Delta,\ i = 1, ..., N_Q,$$

Y_Δ is bounded, functions $G_i(y)$, i=0,…, M, and $F_i(y)$, i=0,…, N_Q, are defined, continuous and Lipschipz-continuously differentiable on Y_Δ.

Practically dependences $H_j(q_j,\ \ u_j)$, $N_j(q_j,u_j)$, $Q_{j\,\min}(u_j)$, $Q_{j\,\max}(u_j)$ are presented by manufactures of fans in graphic form that show curves representing them are smooth as well as boundedness of Y which bounds are presented with (6)–(7). Hence Y_Δ for small $\Delta > 0$ is bounded too. Smoothness of graphs of these functions shows the possibility to describe them as at least twice continuously differentiable and therefore Lipschipz-continuously differentiable on the closed bounded set Y_Δ. Other functions are linear, except $|q|q$ which 1st derivative is equal to $2|q|$ and so Lipschipz-continuous for any q=0 with Lipschipz constant 2. So Condition 1 is valid for problems in question.

Let us define for arbitrary $\varepsilon > 0$ and $(x,q) \in Y_\Delta$ sets of ε-active restrictions of types (16)–(17) respectively as $I_{2\varepsilon}(x,q)$=$\{i \quad |G_i(x,q) \ \ \ge\varepsilon\}$ for (16),

$I_{3L\varepsilon}(x)$={$j \quad | \quad x_{j\min} + \varepsilon \;\geq\; x_j$} for left (17), $I_{3R\varepsilon}(u) = \{j \quad | \quad x_j \geq x_{j\max} - \varepsilon\}$ for right (17).

Condition 2. For any $(x, q) \in Y_\Delta$ the system of vectors $g_i = (\nabla x G_i(x, q),\ \nabla q G_i(x, q)),\ i \in I_{20}(x, q)$, $e_j,\ j \in I_{3L0}(x) \cup I_{3R0}(x),\ f_i = (\nabla x G_i(x, q),$ $\nabla q G_i(x, q))$, i=1,...,N_Q, is linearly independent.

Condition 2 means that the matrix $B(x, q)$ which rows are $g_i^T,\ i \in I_{20}(x, q),\ e_j^T, j \in I_{3L0}(x) \cup I_{3R0}(x)$, $f_i^T, i = 1, ..., N_Q$, is not rank-deficient. To test it is sufficient to fulfil transformations of Gaussian elimination method with choosing the resolving element on the i-th cycle as having maximum absolute value in the i-th row of the transformed matrix; the criterion of linear independence of rows is that the resolving element is nonzero for all rows. To evaluate the degree of independence of the vectors system b_i it is reasonable to scale it replacing b_i by $b_i' = b_i/||b_i||$. Numerical experience shows that for corresponding $B'(x, q)$ which rows are b_i' absolute values of resolving elements stay not very small and usually greater than 10^{-3}. The proof of Condition 2 validity may be effectuated in the same computational process as calculation of matrix $C(x, q)$ and demands not much more computations than determination of $C(x, q)$ since the number of additional rows to the left-side matrix in (22) for this goal is not greater than $N_X << N_Q$.

Theoretically for the problem in question linear dependencies between b_i are possible. Let k be the node with two entering branches (numbered as i and j) and one quitting branch (l) and let restrictions on them be $q_{i\min} \geq q_i \geq q_{i\max},\ q_{j\min} \geq q_j \geq q_{j\max}$, $q_{i\min}+q_{j\min} \geq q_l \geq q_{i\max}+q_{j\max}$. Then $q_l = q_i+q_j$ and if both q_i, q_j reach their low or high limits simultaneously then q_l reaches its corresponding limit as well, $i \in I_{20}(x, q),\ j \in I_{20}(x, q),\ l \in I_{20}(x, q)$ and $g_i + g_j = g_l$, so linear dependence may take place. This situation is, however, only formally possible and absolutely contradicts with real construction of MVN where consumers may not be ventilated successively! Then two theorems may be formulated which statements and proof may be derived from results of the paper [Valuev, 1990] as particular cases of more general propositions.

Theorem 1. *If the problem* (15)–(18) *satisfies the conditions 1 and 2 and the vector $y = (x, q)$ is optimal in it, then for each vector $w = (w_X, w_Q)$ satisfying conditions*

$$(G_{iy}(y),\ w) \leq 0,\ i \in I_{10}(y),$$

$$0 \leq w_{Xj},\ j \in I_{3R\varepsilon}(x); \quad w_{Xj} \leq 0,\ j \in I_{3R\varepsilon}(x),$$

$$(F_{ix}(x),\ w) = 0,\ i = 1, ..., N_Q,$$

the inequality is valid

$$(G_{0y}(y),\ w) \geq 0.$$

Theorem 2. *Let the problem* (15)–(18) *satisfies the conditions 1 and 2 and the sequence is generated with Algorithm Ω. Then the limit $y^*(\Lambda)$ of any converging subsequence $\{y^{(r)}, r \in \Lambda\}$ satisfy necessary optimality conditions of the theorem 1.*

As for most optimization problems containing nonconvex relationships the existence of the unique local minimum (that is the global one in this case) is not guaranteed.

4 Computational experience and practical applications

The proposed method was implemented and used for the solution of the optimum air flow distribution problems for a number of underground coal mines of the Donetsk coal basin. The conditions of ventilation change as a result of mining dynamics: aerodynamic resistances change slowly and permanently and the number of consumers may change after termination of work in a certain longwall. So for each mine the problem is being solved at least once a month. This fact results in a great computational experience. Usual dimension of the problem is: the number of fans 1, 2 or 4, the number of air consumers from 16 to 30, the number of diagonals 1 or 2, the number of branches from 390 to 470. Solving the 1$^{\text{st}}$ problem it is possible sometimes to meet the requirements of all the consumers while violation of them without regulation may reach 2–3 cubic meters per second. The number of iterations that allows reach the desired accuracy for the 1st problem may be from 5 to 15. Solving the 2$^{\text{nd}}$ problem it is possible usually for 3–5 iteration decrease the total energy consumption by fans to 1–3 %. As to time of computation, it ranges from several minutes on PC/386 to several seconds on typical modern PC of medium class.

References

Abramov, F.A., Tyan, R.B. and Potyomkin, V.Ya. (1992). *Calculation of mine ventilation networks*. Nedra, Moscow. (in Russian)

Ushakov, V.K. 1999 *Mathematical modelling of mine ventilation systems reliability and efficiency* Moscow, Moscow State Mining University Publishers (in Russian)

Kumar, G.V. Rao, G.V.K. and Kumar, B.R.(1998) Multiple Surface Fan Networks — An Algorithm for Finding Optimum Fan Operating Points. In *echnical Papers. Fifth International Mine Ventilation Congress* Johannesburg, 1992, pp. 327–333.

Wang, Y.J. (1999) Minimizing air power in a ventilation network using regulators in nonfixed branches. *Journal of the Mine Ventilation Society of South Africa*, **52**(2), pp. 39–43.

Avriel, M. (1976). *Nonlinear Programming: Analysis and Methods*. Prentice-Hall, Englewood Cliffs (N.J.)

Zukhovitskii, S.I., Polyak, B.T., and Primak, R.A. (1963) Algorithm for solving convex programming problems. *Doklady AN SSSR*, **153**(5), pp. 991–1000 (in Russian).

Lilić, N. and Kuzmanović, D. (1994) Optimization of Air Distribution in Mine Ventilation Networks. *Yugoslav Journal of Operations Research*, **4**(1), pp. 105–113 (in Russian).

Fiacco, A. and McCormic, G. (1968). *Nonlinear Programming: Sequential Unconstrained Minimization Technique*. Wiley, New York.

Pshenichnyi, B.N. and Danilin, Yu.M. (1975). *Numerical methods in extremal problems*. Nauka, Moscow (in Russian).

Demmel, J.W. (1997). *Applied Numerical Linear Algebra*. SIAM, Philadelphia.

H. Cross (1936). Analysis of flow in networks of conduits or conductors. *Bull. Ill. Univ. Engng. Exptl. Stn.*, (286), pp. 1–32.

Bazaraa, M., Sherall, H. and Shetty, C. (1993). *Nonlinear programming. Theory and Algorithms. 2^{nd} ed.*. Wiley, New York.

Valuev, A.M. (1990) Hybrid decomposition method for optimization problems with constraints of general type. *Trudy VNIISI. Models and methods of optimization*, (7), pp. 10–19 (in Russian).