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DIAGNOSTICS NONLINEAR DYNAMIC SYSTEMS ON THE BASE OF GENERALIZED MULTIMODE MODELS

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Abstract
The complex dynamic systems (CDS) diagnostics on the basis of their generalized multimode models is submitted in that paper. For the description of a condition and CDS behaviour in the frameworks of multimode models concepts of state modes and behaviour modes are introduced respectively. It was showing that the CDS generalized multimode model, in the form of nonlinear oscillator assembly allows to according to the character of the generalized member change diagnose regular and chaotic regimes of nonlinear systems; determine a range of the system parameter change gas laser HF pumping system, providing uniform distributions of electronic density and stability of the gas discharge plasma regime.

Key words
Control of chaos, Modeling, Nonlinear systems

1. Introduction

A big increase in researches and practical use of complex dynamic systems (CDS) of a various nature (plasma, lasers, nonlinear radio-electronic devices and systems with dynamic chaos, etc.) results in the diagnostics methods development of their state and behaviour regimes [Potapov A.A., 2005] and [Arcimovich L.A., Sagdeev R.Z., 1979] and [Afanas'ev V.V., Polsky U.E., 2004]. At present there are three basic approaches to CDS mathematical description. CDS diagnostics is being carried out on the basis of these approaches:

1. CDS mathematical description by means of systems of the nonlinear differential equations which, as a rule, do not have analytical solution [Afanas'ev V.V., Danilaev M.P., Polsky U.E., 2008].
2. CDS reduction to thoroughly investigated nonlinear oscillators (Lorenz, Chua, Duffing, etc). CDS behaviour can be described on the basis of these oscillators [Afanas'ev V.V., Polsky U.E., 2004].
3. CDS description on the basis of their multimode models [Afanas'ev V.V., Polsky U.E., 2004] and [Afanas'ev V.V., Danilaev M.P., Polsky U.E., 2008]. Such conception assumes CDS splitting into a series of subsystems (modes). Use of the generalized multimode models, in our opinion, is especially expedient at diagnostics of CDS state and behaviour regimes with dynamic chaos.

The purpose of this paper is CDS diagnostics on the basis of their generalized multimode model.

2. Manuscripts

For the description of CDS state and behaviour in the framework of multimode models concepts of state modes and behaviour modes are introduced respectively [Afanas'ev V.V., Danilaev M.P., Polsky U.E., 2008]. The state mode is an uniform and indivisible part of the structure. The whole structure is defined by the state mode ensemble. The behaviour mode is the principle (or set of principles) resulting in variation of state mode ensemble in time. Mathematical formalization of the generalized multimode models depends on CDS physical nature, a mode of system partition into the modes, as well as on the intermode interaction. The value of intermode interaction (close or weak interaction) can be determined proceeding from the ratio interaction energy among modes (E_{ij}) to the mode energy (E_{ii}):

$\frac{E_{ij}}{E_{ii}} << 1$ - corresponds to weak interaction among modes;

$\frac{E_{ij}}{E_{ii}}$ W 1 - corresponds to close interaction among modes.

Series of non-linear systems, such as the plasma, the bound generators, the high-frequency (HF) gas laser pumping system, can be submitted, in the framework of multimode models, in such a way that the value of intermode interaction appears to be weak. Mathematical representation of such CDS can be formalized as a weak interaction nonlinear oscillator ensemble [Afanas'ev V.V., Danilaev M.P., Polsky

U.E.,Cencevicky, Usanov A.I., 2008]. Required dynamics of nonlinear systems has to be regular (HF gas laser pumping system), or chaotic (random-signal generators). Thus one of the most important practical problems of the CDS diagnostics is revealing of regular and chaotic regimes of their behaviour (with regular and chaotic behaviour modes).

Nonlinear systems with chaotic dynamics (Lorenz, Chua, Duffing, Van-der-Poll), describing a wide set of radiophysical CDS, can be presented in the form of generalized nonlinear oscillators [Afanas'ev V.V., Polsky U.E., 2004]:

$$\ddot{\theta} + D_\theta \cdot \dot{\theta} + S_\theta \cdot \theta = W_\theta , \qquad (1)$$

where θ is one of the chosen system variables (X, Y or Z), D_θ, S_θ are the constants defined by the parameters of nonlinear system, W_θ is the nonlinear function of variables $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$.

The nonlinear right part of the equation (1) can be transferred to the left part of the equation and system can be presented in the form of generalized oscillators with nonlinear generalized free member (GFM) OS_θ or generalized dissipated member (GDM) OD_θ:

$$OS_\theta = S_\theta - \frac{W_\theta}{\theta}, \quad OD_\theta = D_\theta - \frac{W_\theta}{\dot{\theta}} \qquad (2)$$

Processes of laser generation are described by Maxwell equations, reduced in single mode approximation to the Lorenz equations system [Oraevsky A.N., 1980] for which the generalized free members OS_θ appear in the following form:

$$OS_X = \sigma - \frac{\sigma \cdot \dot{Z}}{b} + \frac{\sigma \cdot X^2}{b} + \frac{X \cdot \dot{X}}{b} - \sigma \cdot r$$

$$OS_Y = b + \sigma - \frac{b \cdot r \cdot X}{Y} - \frac{\sigma \cdot Y}{X} + X^2 - \frac{\sigma \cdot \dot{Y}}{X} \qquad (3)$$

$$OS_Z = b \cdot (1 + \sigma) - \frac{r \cdot X^2}{Z} + X^2 - \frac{\sigma \cdot Y^2}{Z},$$

where X depends on an amplitude of fluctuations of an electric field in a resonator, Y is determined by polarization of the medium, and Z is the number of the particles interaction with medium, σ, r, b are the parameters of the Lorenz system.

The generalized free member OS_X at system entry conditions, relative to the balance state, eventually makes quasiharmonic fluctuations. The amplitude of fluctuations and variation character OS_X depend on a mode prevailing in the system. Occurrence of a chaotic style is accompanied by the growth of the fluctuations amplitude OS_X with quasiharmonic fluctuations frequences. At a regular style, with the occurrence of the system of steady focuses in a phase space, the amplitude of fluctuations OS_X decreases. The character of the generalized free member variation in time OS_X (or its average value) can be a qualitative criterium, which allows us to diagnose a state of Lorenz dynamic system [Afanas'ev V.V., Danilaev M.P., Polsky U.E.,Cencevicky, Usanov A.I., 2008]. Thus, the diagnostics of a laser generation regularity is possible by means of the dynamics OS_X analysis of the submitted conception of the generalized nonlinear oscillator.

Conceptions in the form of the generalized nonlinear oscillators obtained on the basis of generalized multimode model allow us to determine the parameters of systems providing a required regular regime of their behaviour. We shall consider the definition of CDS parameters on the basis of conception in the form of generalized nonlinear oscillator with reference to nonlinear circuit of HF gas laser pumping system. The system of the equations describing such a nonlinear circuit in quasistationary regime when plasma exists in all discharge area is the following [Afanas'ev V.V., Danilaev M.P., Polsky U.E.,Cencevicky, Usanov A.I., 2008]:

$$\begin{cases} C_1 \cdot \dot{U}_1 = G \cdot (U_2 - U_1) - I_L - I_g, \\ C_2 \cdot \dot{U}_2 = G \cdot (U_2 - U_1), \\ L \cdot \dot{I}_L = -U_1, \end{cases} \qquad (4)$$

where C_1 is the output capacity of HF pumping system generator, C_2 is the capacity of the discharge chamber, L is the inductance of a connection circuit, R_1 is the equivalent output resistance of HF pumping system generator and laser discharge chamber, $g(U_2)$ is the nonlinear plasma resistance, U_2 is the output voltage of HF pumping system generator, $G = 1/R_1$, I_L is the inductance current, I_g is the nonlinear element current, $U_1 = U_{\mathrm{dc}} + U_{\mathrm{n}}$, U_{dc} is the voltage on the laser discharge chamber, U_{n} is the slowly changing noise voltage, $\dot{U}_1 \neq 0$; U_2 is the output voltage of HF pumping system generator.

The inductance value L of a connection circuit has been usually determined on the basis of the requirement of electron density uniform distribution in the laser discharge chamber. However more important is the estimation of inductance L influence on stability of work of the pumping system as, even at optimum value of this inductance, it is impossible to obtain uniform distribution of electronic density [Danilaev M.P., Pol'sky U.E., Usanov A.I., 2006].

Representation (4) in the form of generalized nonlinear oscillator (1) results in:

$$C_1 \cdot C_2 \cdot \ddot{U}_1 + OD(U_1, U_2) \cdot \dot{U}_1 + G^2 \cdot U_1 = 0 \qquad (5)$$

where $OD(U_1, U_2)$ - average for the period of HF generator fluctuations generalized dissipated member (GDM):

$$OD(U_1,U_2) = G\cdot C_2 - \frac{G^2\cdot U_2}{\dot{U}_1} - \\ -C_2\cdot\frac{F_g\left(U_2,L,\mathring{p}\right)}{\dot{U}_1} - \frac{C_2}{L}\cdot\frac{U_1}{\dot{U}_1}\quad . \tag{6}$$

Here p - a vector function of parameters of the gas laser digit chamber where distribution of electron density along the discharge chamber, geometrical parameters of the discharge chamber, structure of a gas mixture, type of cooling are taken into account. In the equation (5) only functional (6) in front of the first derivative is a variable. Therefore its change determines variation of energy of the whole system.

In the formula (6) for GDM it is possible to single out sign constant D_1 and oscillating D_2 members $OD = D_1 + D_2$ [Afanas'ev V.V., Danilaev M.P., Polsky U.E.,Cencevicky, Usanov A.I., 2008]:

$$D_1 = G\cdot C_2 - \frac{G^2\cdot U_2}{\dot{U}_1}, \\ D_2 = C_2\cdot\left(\frac{F_g\left(U_2,L,\mathring{p}\right)}{\dot{U}_1} + \frac{1}{L}\cdot\frac{U_1}{\dot{U}_1}\right). \tag{7}$$

Numerical experiment, where influence of inductance L on stability of HF pumping system according to GDM form is estimated, has been carried out for quid CO_2 laser described in the paper [Danilaev M.P., Polsky U.E., Usanov A.I., 2006]. Simulation has shown that GDM is oscillating, and $D_1 << D_2$. Optimum inductance value L_{opt} for close to uniform distribution of electronic density in the laser discharge chamber has been determined. Boundary values of inductance $\in\left(0,3...2,5\right) L/L_{opt}$ correspond to the excess limits of threshold value of the current in plasma I_p that results in plazma contraction. This range correlates well with the results received in the paper [Danilaev M.P., Polsky U.E., Usanov A.I., 2006]: $\left(0,2\,...\,5\right) L/L_{opt}$ at $L_{opt} \approx 0,4$ μHn. The carried out analysis of GDM dynamics has allowed us to specify a range of inductance at which requirements of uniform distribution of electronic density and stability of plasma regime will be carried out simultaneously. Thus, the carried out analytical and numerical researches correspond well to the known experimental data and confirm the efficiency of the suggested approach of diagnostics of nonlinear systems with chaotic dynamics on the basis of their generalized multimode models and their conceptions by the generalized nonlinear oscillators.

Suggested in the paper [Afanas'ev V.V., Danilaev M.P., Polsky U.E., 2008] CDS structural description on the basis of multimode models allows us to substantiate precise kinds of bases of the Fourier generalized series (FGS) used for finite-dimensional representation of mode ensemble of a state [Afanas'ev V.V., Polsky U.E., *2004*]:

$$\Phi\left(t\right) = \sum_{k=1}^{n} a_k f_k\left(t\right), \tag{8}$$

where f_k - the basic determined functions (state mode), a_k - weight coefficients $k=\overline{1,n}$, n - basis dimension. In this case conception (8) can be treated as a definition of the CDS mode structure. Every *k*–mode $k=\overline{1,n}$, is determined by a kind of the appropriate basic function f_k. Thus change of the generalized spectrum $\left\{a_k, k=\overline{1,n}\right\}$ reflects the change of the mode constitution of the analyzed structure. Such multimode approach in the generalized spectral analysis is suitable for linear - independent and orthogonal f_k, which minimize an error of finite-dimensional spectral representation [Afanas'ev V.V., Polsky U.E., 2004].

On the basis of such an approach in this paper the interrelation of wave numbers (k_i) with frequencies (ω_i) of the basic types of fluctuations in plasma has been determined. The equation for fluctuations in plasma is the following [Arcimovich L.A., Sagdeev R.Z., 1979]:

$$\frac{\partial^2 u}{\partial t^2} - s^2\left[1+\alpha\cos\left(\omega_0 t - k_0 x\right)\right]\frac{\partial^2 u}{\partial x^2} + \\ +L\left(k\right)u = 0, \tag{9}$$

where $u = u\left(x,t\right)$ - the generalized speed of particles in plasma; $\omega_0 = \omega_0\left(k_0\right)$ - frequency of pumping wave ($\omega_0 = \sum_{i=1}^{N}\omega_i$; $k_0 = \sum_{i=1}^{N}k_i$); $L\left(k\right)$ - the linear operator taking into account a deviation from the linear law of dispersion $\omega = sk$.

Spreading out the generalized speed of particles in plasma in harmonious Fourier series:

$$u\left(x,t\right) = \sum_{i=1}^{N} a_i \exp\left(j\omega_i t\right)\exp\left(jk_i x\right), \tag{10}$$

and providing preservation of stochastic invariant properties by limiting series by M members substituting (10) in (9), we receive a system of the nonlinear algebraic equations with regards to frequencies ω_i .

$$\omega_i^{\ 2} - \left(sk_i\right)^2\left(1+\frac{1}{a_i}\left(\frac{k_i}{s}\right)^2\right) \approx \\ \approx \left(sk_i\right)^2\alpha\cos\left(\omega_0 t - k_0 x\right), \\ i=\overline{1,M}\ . \tag{11}$$

The solution of this system allows us to reveal connection of the boundary values of the frequencies appropriate to the border of a steady mode of plasma behaviour with spatial plasma imperfections. The solution of a system (11) for two frequencies of plasma electron density oscillations ($M = 2$) is the following:

$$\omega_1 = \frac{k_1}{B(k_1,k_2)}\sqrt{C(k_1,k_2)\cdot A(k_1,k_2)};$$
$$\omega_2 = \frac{k_2}{B(k_1,k_2)}\sqrt{C(k_1,k_2)\cdot D(k_1,k_2)}. \qquad (12)$$

where

$$A(k_1,k_2) = 2s^2 + 4k_1^2 +$$
$$+s^2\alpha\left[2+2k_2^2t^2\left(k_2^2-k_1^2\right)+x^2\left(k_1^2+k_2^2\right)\right];$$
$$B(k_1,k_2) = 2 + s^2t^2\alpha\left(k_2^2+k_1^2\right);$$
$$C(k_1,k_2) = 2 + s^2t^2\alpha\left(k_2^2-k_1^2\right);$$
$$D(k_1,k_2) = 2s^2 + 4k_2^2 + \qquad (13)$$
$$+s^2\alpha\left[2+2k_1^2t^2\left(k_2^2-k_1^2\right)+x^2\left(k_1^2+k_2^2\right)\right];$$
$$\omega_0 = \omega_1+\omega_2;\; k_0 = k_1+k_2;$$
$$\cos(\omega_0 t - k_0 x) \approx 1 - \tfrac{1}{2}(\omega_0 t - k_0 x)^2;$$
$$a_{1,2} = \tfrac{1}{2}.$$

Substituting values of wave numbers appropriate to known particles distribution in the discharge chamber into expression (12), it is possible to determine frequencies of fluctuations. In the case of homogeneous distribution of plasma in the digit chamber the state $k_1 = k_2$ is satisfied, and $\omega_1 = 0, \omega_2 = 0$. An above mentioned range of the connection circuit inductance at which distribution of electronic density in the discharge chamber is close to homogeneous distribution corresponds to this case.

3. Conclusion

Thus, CDS generalized multimode model, in the form of nonlinear oscillator ensemble allows us to:

- diagnose regular and chaotic regimes of the laser generation according to the character of the generalized member change;
- determine a range of the system parameter change gas laser HF pumping system, providing uniform distributions of electronic density and stability of the gas discharge plasma regime on the basis of the GDM dynamics analysis;
- reveal connection between the wave numbers, appropriate to known distribution of electronic density and frequencies of fluctuations of plasma.

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