BOUND STATES OF TOPOLOGICAL DEFECTS IN THE SYSTEM OF NON-LINEAR COUPLED GINZBURG-LANDAU EQUATIONS

A.B.Ezersky[1,2] and A.V.Nazarovsky[2]

(1) UMR CNRS 6143 « M2C » Université de Caen, 14000 Caen, France

(2)Institute of Applied Physics RAS, 46 Ul'janov Str. Nizhni Novgorod, 603950 Russia

Bound states of topological defects arising in a tetragonal lattice formed by two orthogonal standing parametrically excited capillary surface waves are investigated. A system of four coupled Ginzburg-Landau equations is proposed to model the bound states. Numerical modeling of this system gave solutions corresponding to the bound states observed in experiment.

Formation and destruction of bound states is a process determining the properties of spatio-temporal chaos of an ensemble of topological defects in nonequilibrium systems. Our earlier research revealed that by changing parameters of nonequilibrium, for instance by modulating them in time, one can control both the motion of individual topological defects and features of spatio-temporal chaos [1].

Dynamics of individual topological defects in roll and hexagonal structures is the best understood today. The present work is concerned with numerical computations of four coupled Ginzburg-Landau equations for amplitudes of waves forming a square lattice. The important point taken into account in numerical computations was coupling of counterpropagating waves caused by spatially homogeneous oscillations with a frequency twice as large as the wave frequency. Such a system of equations describes, for example, parametric excitation of surface capillary waves in a layer of viscous liquid (Faraday ripples):

$$\frac{\partial A_\pm}{\partial t} \pm v_g \frac{\partial A_\pm}{\partial x} + (c_0 + ic_1)\left(\frac{\partial^2 A_\pm}{\partial x^2} + \frac{\partial^2 A_\pm}{\partial y^2}\right) + \gamma A_\pm =$$
$$i(b_0 + b_1 B_+ B_-)A_\mp^* + i\Delta\Omega A_\pm - (1 + ic_2)\,|A_\pm|^2\, A_\pm + c_3\,|A_\mp|^2\, A_\pm + c_4(|B_+|^2 + |B_-|^2)A_\pm$$

(1)

$$\frac{\partial B_\pm}{\partial t} \pm v_g \frac{\partial B_\pm}{\partial y} + (c_0 + ic_1)\left(\frac{\partial^2 B_\pm}{\partial x^2} + \frac{\partial^2 B_\pm}{\partial y^2}\right) + \gamma B_\pm =$$
$$i(b_0 + b_1 A_+ A_-)B_\mp^* + i\Delta\Omega B_\pm - (1 + ic_2)\,|B_\pm|^2\, B_\pm + c_3\,|B_\mp|^2\, B_\pm + c_4(|A_+|^2 + |A_-|^2)B_\pm$$

Here, $A_{\pm}, B_{\pm}$ are wave amplitudes, $e^{i(\omega t \mp kx)}, e^{i(\omega t \mp ky)}$, v_g is the group velocity of the waves, $c_{1-4}, b_{0,1}$ are coupling coefficients, $\Delta\Omega$ is mismatch, and γ is the coefficient of linear dissipation.

The boundary conditions for the amplitudes corresponded to zero wave amplitude at side boundaries.

The first series of numerical experiments was aimed at finding bound states of topological defects in one pair of parametrically excited waves. We neglected the influence of the other pair of waves propagating normally to the first wave pair.

Numerical experiment demonstrated that solution of equation (1) may be a bound state of defects. Such a state is shown in Fig.1. The initial conditions were set to correspond to the topological defect belonging to the wave propagating to the right. Coupling due to pumping gave rise to a topological defect belonging to the wave propagating to the left. Increasing of group velocity results in increasing distance between the defects and the bound state will decay, which was verified in numerical experiment. The higher the group velocity, the longer was the distance between defects in the waves propagating in opposite directions. The defects were spaced apart in the direction of wave propagation along the *OX*-axis. Two topological charges forming a bound state moved as a whole.

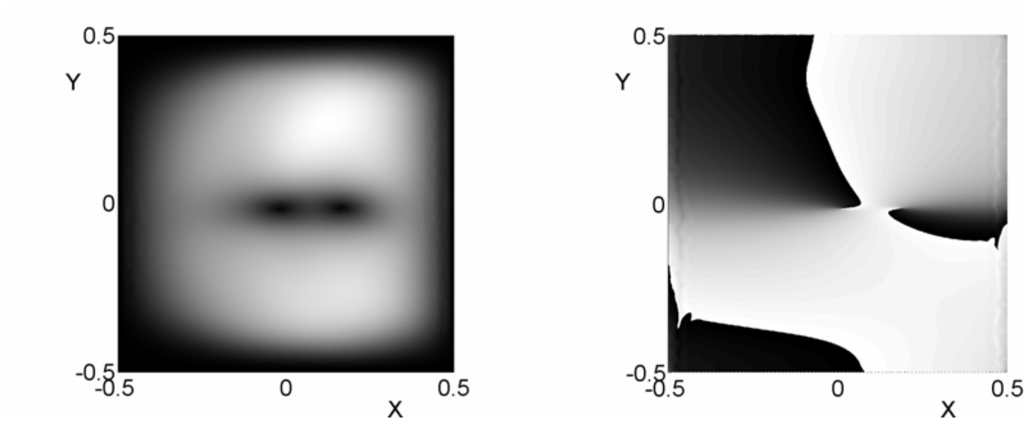

Fig.1. Fields of the product of amplitudes and phase difference of counterdirected waves obtained by numerical computations for $c_0 = -0.001, c_1 = -0.001$, $c_2 = 2, b_0 = 2$, $b_1 = 0, c_3 = -10, c_4 = 0, \Delta\Omega = 0$, v_g=0.3.

Our calculations revealed that with increasing group velocity, when the distance between individual charges became comparable with the size of the system, the trajectory of motions of a bound state could include motions of “climb” and “glide” type. The shape of the trajectory depended on position of the region where the topological defect had been formed. The bound state along the horizontal coordinate (“glide” motion) tended to shift to the nearest vertical boundary. Two bound states with opposite topological charges annihilated. As the bound states were approaching each other they did not rotate yet.

Such bound states were observed in experiment [2]. Note that numerical experiments with the equations used in our work were recently carried out in [3]. The authors of [3] did not find bound states like those observed by us both in numerical computations and in experiments. Possible reasons for this will be discussed below.

Numerical modeling of dynamics of defects in a square lattice formed by perpendicular standing waves using coupled Ginzburg-Landau equations disclosed a number of effects observed in experiments, namely, scattering of two bound states belonging to perpendicular modes and arising of pairs of defects having opposite charges.

Topological charges in the mode propagating along the OX-axis perturb the waves propagating along the OY-axis. These perturbations may be of two types (see the right-hand sides of the equations of system (1) for $B_{\pm}$): (i) nonlinear damping or nonlinear frequency shift $(c_4 B_{\pm}(\left|A_+^2\right| + \left|A_-^2\right|))$ and (ii) “efficient” pumping $(b_1 A_- A_+)B_{\mp}^*$. Perturbations of the first type do not introduce any phase changes, they only lead to amplitude modulation of waves $B_{\pm}$; whereas perturbations of the second type give rise to both amplitude and phase perturbations, with the topological charge being $\oint(\nabla\varphi_+ - \nabla\varphi_-)dl = 0$. Therefore, if waves A_+, A_- have topological charges like it is showed in Fig.1, they may induce topological charges in waves $B_{\pm}$ but their sum topological charge will be equal to zero. Consequently, interaction of normal waves, in contrast to interaction of waves propagating in opposite directions, does not lead to global restructuring of the entire phase field. This conclusion has been confirmed by experiments. Indeed, two bound states of topological defects belonging to normal modes may pass through each other. Such a process was revealed in calculations of system (1) consisting of 4 coupled equations. It was found that, when one bound state arises in each normal wave, these bound states pass through each other, without formation of a long-lived object. One can see in fig. 2 that in the region where one mode has a bound state (the dark spots corresponding to the amplitude drop down to 0), the wave intensity in the other mode grows (the bright spots caused by the amplitude increase). This is the result of nonlinear competition of normal modes that is the most pronounced for t=8,t=12.

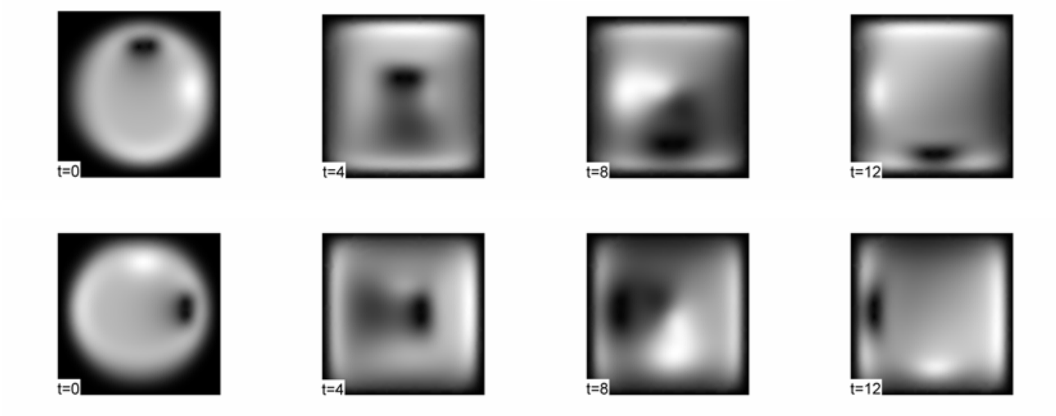

Fig.2 Fields of products of the amplitudes of two counterpropagating waves in perpendicular pairs. The upper row is for the mode with vertical wave number, and the lower one with horizontal wave number. The bound states belonging to the perpendicular modes pass through each other. The calculation was made for the following parameters: $\gamma=1$, $c_0=-0.001$, $c_1=-0.001$, $c_2=2$, $c_3=0.1\text{-}i2$, $c_4=-0.1+i2$, $v_g=0.12$, $\Delta\Omega=0$, $b_0=2$, and $b_1=2$.

However, we failed to find in numerical experiment bound state of four topological defects observed in [4]. A possible explanation is that system (1) is applicable at small supercriticality only, whereas this condition was not fulfilled in [4] when bound states of 4 defects were formed.

Results of our numerical calculations should be compared with computation results presented in [3]. There are several differences in numerical modeling. First, instead of two mutually orthogonal wave pairs, the authors of [3] investigated dynamics of two waves propagating in opposite directions along the OX-axis. Second, the basic difference between our two works is that periodic boundary conditions at side boundaries were used in [3], whereas we took zero boundary conditions. We believe that zero boundary conditions in numerical modeling are logically more justified if results of computations will be compared with physical experiment. There is usually no energy pumping near side walls, energy is only absorbed there. Zero boundary conditions for perturbations propagating in a nonequilibrium medium were observed, for instance, for spiral waves in the Coutte-Taylor flow [5], roll convection in a channel [6], and in other experiments. Periodic boundary conditions are more attractive in terms of mathematics for proving theorems, finding exact solutions, and so on, but they are not adequate to physical experiment.

Our solution in the form of a bound state of two defects in waves propagating along the OX-axis is not periodic along the Y-coordinate. Indeed, the number of spatial periods differs by unity in the wave $A_+e^{i(\omega t-kx)}$ at Y=-0.5 and at Y=+0.5. The same relation is true for the wave $A\_e^{i(\omega t+kx)}$. Hence, the bound state depicted in fig. 1 cannot be obtained in [3] at any boundary conditions. In [3] topological defects arise in pairs in each counterpropagating wave and have opposite topological charges in each pair. Such pairs live for finite time from the moment of their birth till annihilation. In our bound states defects are in different waves and cannot annihilate in principle.

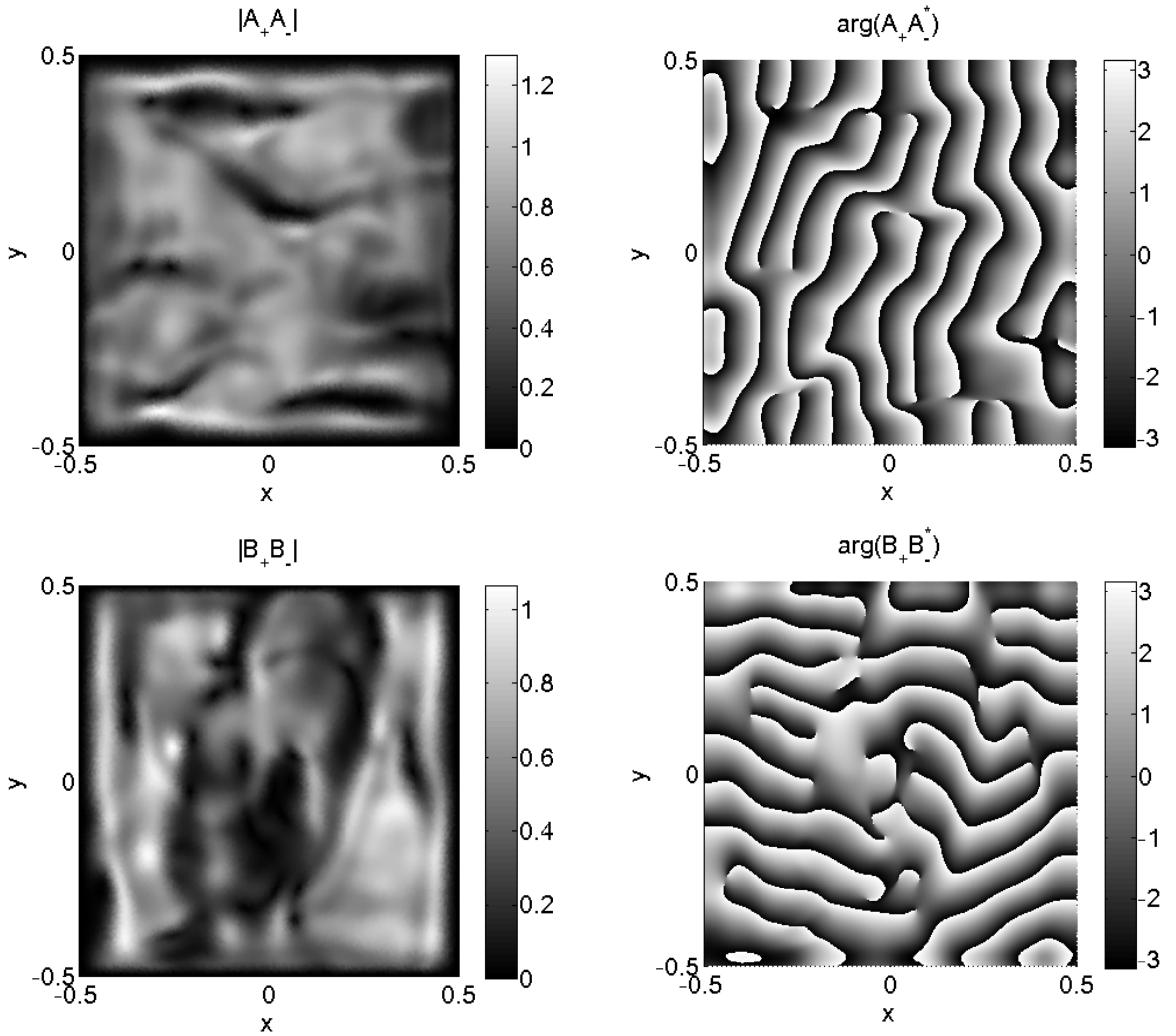

Fig. 3 Snapshots of amplitude and phase of chaotic fields A and B for $V_g = 0.1, c_0 = -0.001, c_1 = -0.001, c_2 = -5, \Delta\Omega = 0, c_3 = -0.1 + i2, c_4 = 0, b_0 = 2, b_1 = 2, \gamma = -1.$

Third, in our calculations the coefficient γ in eq. (1) was equal to +1 and corresponded to wave damping. Instability arose due to parametric forcing at double wave frequency. Parametric instability arose when the external force amplitude exceeded a threshold due to linear dissipation of the system. In [3] γ<0, which means the absence of linear dissipation and that instability is possible without parametric forcing. Even when the amplitude of parametric forcing is equal to zero $b_0 = 0$, there is energy pumping in the medium. It was shown in [3] that there may arise spatio-temporal chaos of appearing and annihilating pairs. We also made calculations for the case γ<0 under homogeneous boundary conditions. The results are given in fig. 3. We found that complex spatio-temporal dynamics arising in calculations is caused by interaction of long-lived bound states like the ones shown in fig. 1 rather than by formation of annihilating pairs.

Thus, dynamics of topological defects at parametric excitation of waves, formation of bound states of defects, and structure of spatio-temporal chaos depend significantly on boundary conditions. This effect may be of primary importance for numerical modeling of physical experiments.

REFERENCES

1. V.O. Afenchenko, A.B. Ezersky, S.V. Kiyashko, “Control of motion of topological defects generated in Faraday ripples”, Physics of Wave Phenomena, v.12 (2004), n..4. p.200-208.
2. A.B. Ezersky, S.V. Kiyashko, A.V. Nazarovsky, "Bound states of topological defects in parametrically excited capillary ripples", Physica D, v.152-153 (2001), p.310-324.
3. G.D. Granzow, H. Riecke, "Nonequilibrium defect-unbinding transition: defect trajectories and loop statistics", Physical Review Letters, v.87, n17 (2001), p.174502.
4. V.O. Afenchenko, A.B. Ezersky, S.V. Kiyashko, A.V. Nazarovsky, “Bound states of topological defects in parametrically excited capillary waves” Europ. J. Phys. (2007) (accepted for publication).
5. A.B. Ezersky, N. Latrache, O. Crumeyrolle, and I. Mutabazi, "Competition of spiral waves with anomalous dispersion in Couette-Taylor flow", Theor. Comput. Fluid Dyn. 18 (2004) p.85–95.
6. N. Garnier, A. Chiffaudel, and F. Daviaud, "Convective and absolute Eckhaus instability leading to modulated waves in a finite box", Phys. Rev. Lett. v. 88 (2002) 134501(1-4).