CYBERNETICS AND PHYSICS, VOL. 2, NO. 1, 2013 , 10–14

INFORMATION TRANSMISSION BASED ON ADAPTIVE SYNCHRONIZATION OF CHAOTIC LORENZ SYSTEMS OVER THE DIGITAL COMMUNICATION CHANNEL

Boris Andrievsky

[1] Control of Complex Systems Lab., The Institute for Problems of Mechanical Engineering of the RAS,
[2] Mathematics and Mechanics Faculty, Saint Petersburg State University,
[3] Control Systems and Informatics Dept., The National Research University of Information Technologies, Mechanics and Optics, Saint Petersburg, Russia
boris.andrievsky@gmail.com

Abstract

In the paper the adaptive synchronization of chaotic Lorenz systems over the limited capacity digital communication channel is described and its application to information transmission, based on chaotic source modulation, is demonstrated. The simulation results for video signal transmission are presented.

Key words

Adaptive synchronization, chaos, channel coding.

1 Introduction

Starting from the works by [Pecora and Carroll, 1990; Rulkov and Tsimring, 1999; Boccaletti *et al.*, 2002; Lau and Tse, 2003; Xia *et al.*, 2004], to mention a few, the synchronization of chaotic systems becomes one of the most topical field of research in nonlinear sciences due to its various potential applications in secure communication, chemical and electrical engineering, information processing, etc., see [Andrievskii and Fradkov, 2004] for a survey. In [Yang Wu and Chua, 1996; Fradkov *et al.*, 2000; Fradkov *et al.*, 2002] various methods of the adaptive synchronization of chaotic systems are proposed and possibility of the adaptive synchronization for information transmission on chaotic carriers is demonstrated.

At present, the communication networks find escalating applications in many fields of communication, information processing, and technology. Due to the digital nature of the modern networks, the signals are quantized in value and time. The limitations of control and synchronization under constraints imposed by a finite capacity of communication channels have been widely studied in the control literature, see the surveys [Nair *et al.*, 2007; Andrievsky *et al.*, 2010; Matveev and Savkin, 2009] and references therein. Synchronization of nonlinear systems under data rate constraints is studied in [Fradkov *et al.*, 2008], where quadratic Lyapunov functions and the *passification method* [Fradkov, 1974; Andrievskii and Fradkov, 2006] were employed. In the mentioned works is assumed that the coupled systems have the so-called *Lurie form*: right-hand sides are split into a linear part and a nonlinearity vector depending only on the measured output. In the contrary of [Nair and Evans, 1997; Liberzon and Hespanha, 2005; Savkin and Cheng, 2007], [Fradkov *et al.*, 2008; Fradkov and Andrievsky, 2011] assumed that only master system output (instead of the components of the state vector) can be measured, and, consequently, based their synchronization scheme on transmission of the scalar signal rather than a full state vector. Application of these results to remote state estimation of nonlinear Lurie systems over the digital communication channel is given in [Fradkov and Andrievsky, 2009; Fradkov and Andrievsky, 2011] and extended to the case of delay in the channel in [Andrievsky and Andrievsky, 2012].

In the present paper the data transmission scheme of [Fradkov and Andrievsky, 2009; Fradkov and Andrievsky, 2011; Andrievsky and Andrievsky, 2012] is applied to a class of nonlinear (chaotic) systems which are not presented in the Lurie form. The fundamental assumption is that the system linear part satisfies the *Demidovich stability condition* [Demidovich, 1961; Pavlov *et al.*, 2004]. Then, the adaptive information transmission scheme of [Fradkov *et al.*, 2002; Andrievskii *et al.*, 2007] is applied to chaotic signal modulation/demodulation assuming that the modulator and demodulator are connected over the digital communication link with a finite capacity.

Although the transmission delay and the communication channel imperfections usually appear in practice, it is assumed here that the coded symbols are available at the receiver side at the same sampling instant as they are generated by the coder and transmission channel distortions are absent.

The paper is organized as follows. The information transmission scheme by [Fradkov *et al.*, 2002; Andrievskii *et al.*, 2007] based on the adaptive synchronization of the Lorenz systems over the unlimited-band analogue communication link is briefly recalled in Sec. 2. Section 3 is devoted to transmission the output signal of the chaotic master system over the digital communication channel. The coding procedure employing a uniform quantizer and one-step memory coder is described in Sec. 3.1, the simulation results are presented in Sec. 3.2. Concluding remarks are given in Sec. 4.

2 Adaptive Synchronization of the Lorenz Systems Over the Analogue Communication Link

At first, let us recall the adaptive information transmission scheme of [Fradkov *et al.*, 2002; Andrievskii *et al.*, 2007] over the analogue (unlimited capacity) communication channel.

Consider the observer-based synchronization of chaotic nonlinear systems, which are not presented in the Lurie form. Let the drive system be described by the following equations:

$$\dot{x} = A(y)x + \varphi_0(y) + B\varphi(y)^{\mathrm{T}}\theta, \quad y = Cx, \qquad (1)$$

where $x \in \mathbb{R}^n$ is a vector of state variables, y is the scalar output, $A(y)$ is an $(n\times n)$-matrix, B is an $(n\times 1)$-vector, C is $(1\times n)$-matrix, $\varphi(y)$ is a continuous nonlinear function, θ is a transmitter parameter, representing an information signal (a message). The value of θ is unknown at the receiver side and should be recovered by a demodulator.

2.1 Modulated Chaotic Lorenz Generator

Let the master system (the modulator) be the following *Lorenz system* with a varying parameter $\theta(t)$ [Cuomo *et al.*, 1993; Kennedy and M. Ogorzalek, 1997]:

$$\begin{cases} \dot{x}_1 = \sigma x_2 - \sigma x_1, \\ \dot{x}_2 = -x_2 - x_1 x_3 + \theta(t) x_1, \\ \dot{x}_3 = -\beta x_3 + x_1 x_2. \end{cases} \qquad (2)$$

where β, σ are constants, known both at the modulator and demodulator nodes; parameter $\theta(t)$ depends on the information signal and should be recovered by the demodulator. Let the output signal $y(t) = x_1(t)$ be transmitted over the communication channel to the receiver.

Evidently, system (2) is a special case of (1) with the following components:

$$A(y) = \begin{bmatrix} -\sigma & \sigma & 0 \\ 0 & -1 & -y \\ 0 & y & -\beta \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad (3)$$
$$\varphi_0(y) = \mathbf{0}_{3,1}, \quad \varphi(y) = y, \quad C = [1, 0, 0].$$

It is clear, that all entries of matrix $A(y)$ are bounded for any bounded y and Assumption 4 of [Fradkov *et al.*, 2002, Theorem 3] holds. Let us find now a vector-function $k(y) \in \mathbb{R}^3$ so as the system $\dot{x} = (A(y) - k(y)C)\,x$, $y = Cx$ be asymptotically stable. Pick up $k(y) \equiv k = [0, -\sigma, 0]^{\mathrm{T}}$. Then the matrix-function $G(y) = A(y) - k(y)C$ is sum of a diagonal and a skew-symmetric matrix:

$$G(y) = \begin{bmatrix} -\sigma & \sigma & 0 \\ -\sigma & -1 & -y \\ 0 & y & -\beta \end{bmatrix}. \qquad (4)$$

Consider the system $\dot{x} = G(Cx)x$, with the matrix $G(y)$ given by (4), and introduce the Lyapunov function $V(x) = 0.5x^{\mathrm{T}}x$. Differentiating $V\big(x(t)\big)$ on t one obtains:

$$\dot{V}(x) = 0.5x^{\mathrm{T}}\big(G(Cx)^{\mathrm{T}} + G(Cx)\big)x$$
$$= x^{\mathrm{T}} \begin{bmatrix} -\sigma & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix} x = -\sigma x_1^2 - x_2^2 - \beta x_3^2.$$

Then the following strengthened stability condition (the *Demidovich condition*, [Demidovich, 1961; Pavlov *et al.*, 2004]) for matrix $G(Cx)$ holds:

$$\lambda_i\big(G(y)^{\mathrm{T}} + G(y)\big) \le -\varepsilon < 0, \; y = Cx, \quad i = 1, 2, 3$$

for some $\varepsilon > 0$, where $\lambda_i(A)$ stand for eigenvalues of matrix A. This leads to an exponential asymptotic stability of the system $\dot{x} = G(Cx)x$ and, consequently, fulfillment of Assumption 3 [Fradkov *et al.*, 2002, Theorem 3].[1]

Assumption 1 [Fradkov *et al.*, 2002, Theorem 3] reads as: for any bounded initial condition $x(0)$ and any value of θ the state $x(t)$ is a bounded function, imposes restriction on variations of $\theta(t)$. This Assumption is supposed to be fulfilled. Validity of Assumption 2 (boundness of $\varphi(y)$ for any bounded y) for $\varphi(y) = y$ is evident.

Summarizing, Theorem 3 of [Fradkov *et al.*, 2002] may be used for designing the adaptation-based data transmission scheme for modulator (2).

2.2 The Demodulator Algorithm

The demodulator at the receiver's side is aimed to recover the information signal $\theta(t)$ based on the output signal $y(t)$ of the modulator (2), which is transmitted over the channel. As follows from [Fradkov *et al.*, 2002], the demodulator for Lorenz-based modulator (2) may be taken as

$$\begin{cases} \dot{\hat{x}}_1 = \sigma\hat{x}_2 - \sigma\hat{x}_1, \quad \hat{y} = \hat{x}_1 \\ \dot{\hat{x}}_2 = -\hat{x}_2 - y_r(t)\hat{x}_3 + \sigma e(t) + \hat{\theta}(t) y_r(t), \\ \dot{\hat{x}}_3 = -\beta x_3 + y_r(t)\hat{x}_2, \end{cases} \qquad (5)$$

[1] Evidently, for the considered case eigenvalues λ_i are as follows: $\lambda_1 = -2\sigma$, $\lambda_2 = -2\beta$, $\lambda_3 = -2$, therefore $\varepsilon = 2\min(\sigma, \beta, 1)$.

where $e(t) = y_r(t) - \hat{y}(t)$ can be referred as an *observation error*, y_r is the received signal; for the case of an ideal channel it is valid that $y_r(t) \equiv y(t)$.

For estimation of the unknown modulator parameter θ (and, thereby, for recovering the information signal), the following adaptation algorithm may be implemented in the demodulator:

$$\dot{\hat{\theta}} = \gamma\omega\hat{e} - \alpha(\theta - \theta_0), \;\; \hat{\theta}(0) = \hat{\theta}_0, \qquad (6)$$

where $\gamma > 0$ is an *adaptation gain*, $\alpha > 0$ is a feedback gain (design parameters), $\hat{\theta}_0$ is a "nominal" value of the estimated signal θ found *a priori*, the signals $\omega(t)$, $\eta(t)$ are generated by the following augmented filters:

$$\begin{cases} \dot{\Omega}_1 = \sigma\Omega_2 - \sigma\Omega_1, \\ \dot{\Omega}_2 = -\sigma\Omega_1 - \Omega_2 + y_r(t)\Omega_3 \\ \dot{\Omega}_3 = -\beta\Omega_3 + y_r(t)\Omega_2, \end{cases} \qquad (7)$$

$$\begin{cases} \dot{\eta}_1 = \sigma\eta_2 - \sigma\eta_1 - \Omega_1(t)\dot{\hat{\theta}}(t), \\ \dot{\eta}_2 = -\sigma\eta_1 - \eta_2 + y_r(t)\eta_3 - \Omega_2(t)\dot{\hat{\theta}}(t), \\ \dot{\eta}_3 = -\beta\eta_3 + y_r(t)\eta_2 - \Omega_3(t)\dot{\hat{\vartheta}}(t). \end{cases} \qquad (8)$$

$$\omega(t) = \Omega_1(t), \quad \hat{e}(t) = e(t) + C\eta(t). \qquad (9)$$

3 Synchronization and State Estimation Over the Limited-Band Communication Channel

3.1 Coding Procedure

Transmitted signal $z(t)$ is uniformly sampled with a certain sampling time T_s and represented as a discrete-time sequence $z[k] = z(t_k)$, where $t_k = kT_s$; $k = 0, 1, 2, \ldots$ is a discretization step number. Then the values of $z[k]$ are coded by means of the following coding procedure and transmitted over the communication channel.

A *uniform quantizer* is defined as the following discretized map

$$q_{\nu,M}(z) = \begin{cases} \delta \cdot \langle \delta^{-1} z \rangle, & \text{if } |z| \le M, \\ M \operatorname{sign}(z), & \text{otherwise}, \end{cases} \qquad (10)$$

where *the quantizer range* M is a positive real number, ν is a nonnegative integer, $\delta = 2^{1-\nu}M$ is *the discretization interval*, z denotes the signal to be transmitted over the channel, $\langle\cdot\rangle$ denotes round-up to the nearest integer, $\operatorname{sign}(\cdot)$ is the signum function: $\operatorname{sign}(y) = 1$, if $y \ge 0$, $\operatorname{sign}(y) = -1$, if $y < 0$. Since the cardinality of the mapping $q_{\nu,M}$ image is equal to $2^\nu + 1$ then each symbol in the codeword contains $\breve{R} = \log_2(2^\nu + 1) = \log_2(2M/\delta + 1)$ bits. Taking into account sampling time T_s, bit-per-second rate R reads as $R = \breve{R}T_s^{-1} = \log_2(2^\nu + 1)T_s^{-1}$.

The *binary quantizer* has a form

$$q(z, M) = M \operatorname{sign}(z). \qquad (11)$$

Note that $\breve{R} = 1$ for quantizer (11). Then the corresponding bit-per-second rate R is as $R = T_s^{-1}$.

The static quantizer (10) is a part of the time-varying coder with memory [Nair and Evans, 2003; Brockett and Liberzon, 2000; Tatikonda and Mitter, 2004] utilizing the following procedure.

The output signal

$$\bar{\partial} z[k] = q_M(\partial z[k]) \qquad (12)$$

is represented as an $\breve{R}$-bit symbol from the coding alphabet and transmitted over the communication channel to the decoder. The following update rules are performed at the each step:

$$c[k+1] = c[k] + \bar{\partial} z[k], \quad c[0] = 0, \qquad (13)$$
$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \;\; k = 0, 1, \ldots, \qquad (14)$$

where $0 < \rho \le 1$ is the *decay parameter*, M_∞ stands for the limit value of M. The initial value M_0 should be large enough to capture all the region of possible initial values of z_0. For practice, to avoid computations of powers of ρ, it is advisable to calculate $M[k]$ in the following recursive form: $M[k+1] = \rho M[k] + m$, where $m = (1 - \rho)M_\infty$ and the initial condition is taken as $M[0] = M_0$.

Equations (11), (12), (14) describe the coder algorithm. A similar algorithm is used by the decoder: the sequence of $M[k]$ is reproduced at the receiver node utilizing (14); the values of $\bar{\partial} z[k]$ are restored with given $M[k]$ from the received codeword; the central numbers $c[k]$ are found in the decoder in accordance with (13). Then the decoder output $\bar{z}[k]$ is found as a sum of $c[k]$ and $\bar{\partial} z[k]$, $\bar{z}[k] = c[k] + \bar{\partial} z[k]$.

3.2 Simulation Results

Varying parameter $\theta(t)$ was represented in our simulations as $\theta(t) = r\big(1 + \vartheta(t)\big)$, where $r = 97$, $\vartheta(t)$ is a novel modulator parameter, and the demodulator algorithm (6)–(9) was represented with repect to ϑ by substitution. The following parameter values were taken for the simulations: $\sigma = 10$, $\beta = 8/3$, $\gamma = 0.75$, $\alpha = 0$. The coder parameters were taken as $M_0 = 50$, $M_\infty = 10$, $\rho = \exp(-20T_s)$.

Some simulation results are shown in Figs. 1, 2, where the time histories of master system input $y(t)$, estimation error $e(t) = y(t) - \hat{y}(t)$, varying information signal $\theta(t) \in \{0, 1\}$ (a video signal), and its estimate $\hat{\theta}(t)$ for different T_s, ν, and $R = \log_2(2^\nu + 1)T_s^{-1}$ are depicted.

For obtaning the summarized characteristics of video signal transmission by means of the proposed adaptive data transmission scheme, the sequence of 200 pseudo-random values of the video singal $\theta(t) \in \{0, 1\}$ of width 5 s each has been taken for the simulations (therefore, each simulation run lasted 10^3 s). Recovered variable $\hat{\theta}(t)$ was averaged during the each time slot $[5k, 5(k+1))$ s, $k = 1, \ldots, 200$ and the result was

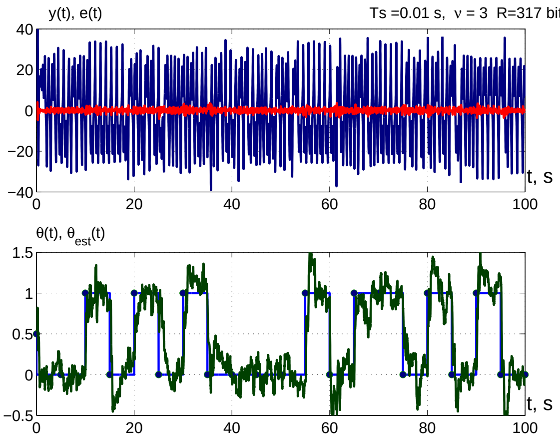

Figure 1. Time histories of $y(t)$, $e(t)$, $\theta(t)$, $\hat{\theta}(t)$ in the case of $T_s = 0.01$ s, $\nu = 3$, $R = 317$ bit/s.

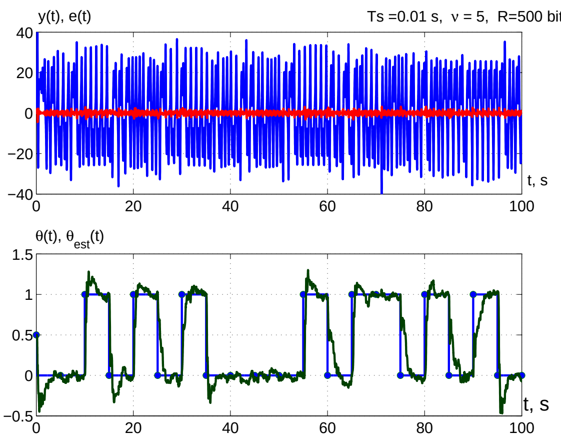

Figure 2. Time histories of $y(t)$, $e(t)$, $\theta(t)$, $\hat{\theta}(t)$ in the case of $T_s = 0.01$ s, $\nu = 5$, $R = 500$ bit/s.

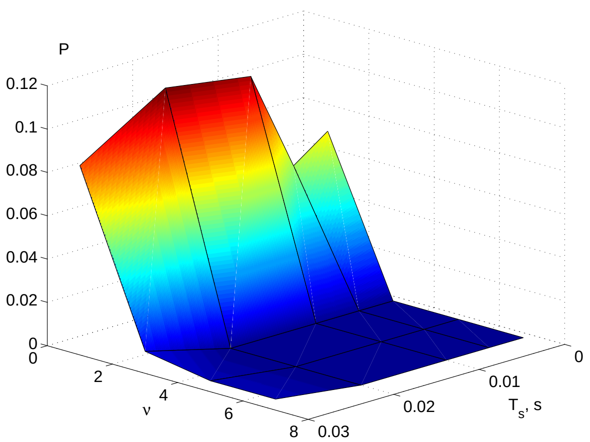

Figure 3. Error probability plot. Frequency of false identification P vs ν and T_s.

compared with the threshold equal to 0.5. This result was used to identify the actual value of $\theta(t) \in \{0, 1\}$. The error probability plot in the form of frequency P of faulse identification as a function on coder parameters ν and T_s is depicted in Fig. 3.

4 Conclusion

The adaptive synchronization of chaotic Lorenz systems over the limited capacity digital communication channel is described and its application to information transmission, based on chaotic signal modulation, is demonstrated. The data transmission scheme of [Fradkov *et al.*, 2008; Fradkov and Andrievsky, 2009; Fradkov and Andrievsky, 2011] is extended to a class of nonlinear (chaotic) systems which are not necessarily presented in the Lurie form with a scalar nonlinearity depending only on the measured output under the assumption that the Demidovich stability condition is fulfilled for the linear part of the system.

The adaptive information transmission scheme is applyed for chaotic signal modulation/demodulation assuming that the modulator and demodulator are connected over the digital communication link with a finite capacity. The simulation results for transmission of the video signal are presented and dependence of the signal transmission accuracy on the data rate is calculated. The simulation results demonstrate that *spreading factor*, which is defined as the number of chaotic samples sent for each bit, may be inadmissibly large for encryptions schemes, based on chaotic shift keying [Pecora and Carroll, 1990; Dedieu *et al.*, 1993; Sushchik *et al.*, 2000; Lau and Tse, 2003; Xia *et al.*, 2004; Kaddoum *et al.*, 2012] in the case when the digital communication channel is used for signal transmission.

Acknowlegements

The work was supported in part by the Russian Foundation for Basic Research (grant Nos. 11-08-01218, 12-08-01183, 13-08-01014) and Russian Federal Program "Cadres" (agreements 8846, 8855).

References

Andrievskii, B. and A. L. Fradkov (2006) Method of passification in adaptive control, estimation, and synchronization. *Autom. Remote Control* **67**(11), 1699–1731.

Andrievskii, B. R. and A. L. Fradkov (2004) Control of chaos: II, Applications. *Autom. Remote Control* **65**(4), 505–533.

Andrievskii, B. R., V. O. Nikiforov and A. L. Fradkov (2007) Adaptive observer-based synchronization of the nonlinear nonpassifiable systems. *Autom. Remote Control* **68**(7), 1186–1200.

Andrievsky, B. R., A. S. Matveev and A. L. Fradkov (2010) Control and estimation under information constraints: Toward a unified theory of control, computation and communications. *Autom. Remote Control* **71**(4), 572–633.

Andrievsky, B. and A. Andrievsky (2012) State estimation of non-autonomous Lurie systems over the limited-band communication channel with time delay and uniform quantization. *Cybernetics and Physics* **1**(4), 228–236.

Boccaletti, S., J. Kurths, G. Osipov, D. L. Valladares and C. Zhou (2002) The synchronization of chaotic systems. *Physics Reports* **366**(1–2), 1–101.

Brockett, R. W. and D. Liberzon (2000) Quantized feedback stabilization of linear systems. *IEEE Trans. Automat. Contr.* **45**(7), 1279–1289.

Cuomo, K. M., A. V. Oppenheim and S. H. Strogatz (1993) Synchronization of Lorenz-based chaotic circuits with applications to communications. *IEEE Trans. on Circuits and Systems II* **40**(10), 626–633.

Dedieu, H., M. P. Kennedy and M. Hasler (1993) Chaos shift keying: modulation and demodulation of chaotic carrier using self-synchronized chuas circuits. *IEEE Trans. Circ. Syst. II: Analog and Digital Signal Proc* **40**(10), 634–642.

Demidovich, B. P. (1961) Dissipativity of a nonlinear system of differential equations, part i. *Vestnik Moscow State Univiersity, Ser. Matem. Mekh. (in Russian)* **6**, 19–27.

Fradkov, A. L. (1974) Synthesis of an adaptive system for linear plant stabilization. *Autom. Remote Control* **35**(12), 1960–1966.

Fradkov, A. L. and B. Andrievsky (2009) State estimation of passifiable Lurie systems via limited-capacity communication channel. In: *35th Annual Conference of the IEEE Industrial Electronics Society, IECON 2009*. Porto, Portugal.

Fradkov, A. L., B. Andrievsky and R. J. Evans (2008) Adaptive observer-based synchronization of chaotic systems with first-order coder in presence of information constraints. *IEEE Trans. Circuits Syst. I* **55**(6), 1685–1694.

Fradkov, A. L., H. Nijmeijer and A. Y. Markov (2000) Adaptive observer-based synchronization for communication. *Int. J. Bifurcations Chaos* **10**(12), 2807–2813.

Fradkov, A. L., V. O. Nikiforov and B. R. Andrievsky (2002) Adaptive observers for nonlinear nonpassifiable systems with application to signal transmission. In: *Proc. 41st IEEE Conf. Dec. Control (CDC'02)*. pp. 4706–4711. IEEE Press, Piscataway, NJ.

Fradkov, A.L. and B. Andrievsky (2011) Passification based synchronization of nonlinear systems under communication constraints. In: *Prepr. 18th IFAC World Congress*. IFAC. Milano, Italy. pp. 6562–6566.

Kaddoum, G., J. Olivain, G. Beaufort Samson, P. Giard and F. Gagnon (2012). Implementation of a differential chaos shift keying communication system in GNU radio. In: *Int. Symp. Wireless Communication Systems (ISWCS), Paris, France*.

Kennedy, M. and *Eds*. M. Ogorzalek (1997) Special issue "Chaos control and synchronization". *IEEE Trans. Circuits Syst. I.*

Lau, F. C. M. and C. K. Tse (2003) *Chaos-Based Digital Communication Systems*. Springer-Verlag.

Liberzon, D. and J. P. Hespanha (2005) Stabilization of nonlinear systems with limited information feedback. *IEEE Trans. Automat. Contr.* **50**(6), 910–915.

Matveev, A. S. and A. V. Savkin (2009) *Estimation and Control over Communication Networks*. Birkhäuser. Boston.

Nair, G. N. and R. J. Evans (1997) State estimation via a capacity-limited communication channel. In: *Proc. 36th IEEE Conference on Decision and Control*. Vol. WM09. IEEE. San Diego, Califomia USA. pp. 866–871.

Nair, G. N. and R. J. Evans (2003) Exponential stabilisability of finite-dimensional linear systems with limited data rates. *Automatica* **39**, 585–593.

Nair, G. N., F. Fagnani, S. Zampieri and R.J. Evans (2007) Feedback control under data rate constraints: an overview. *Proc. IEEE* **95**(1), 108–137.

Pavlov, A., A. Pogromsky, N. van de Wouw and H. Nijmeijer (2004) Convergent dynamics, a tribute to Boris Pavlovich Demidovich. *Systems & Control Letters* **52**, 257–261.

Pecora, L. M. and T. L. Carroll (1990) Synchronization in chaotic systems. *Phys. Rev. Lett.* **64**, 821.

Rulkov, N. F. and L.S. Tsimring (1999) Synchronization methods for communication with chaos over band-limited channels. *Int. J. Circ. Theor. Appl.* **27**, 555–567.

Savkin, A. V. and T. M. Cheng (2007) Detectability and output feedback stabilizability of nonlinear networked control systems. *IEEE Trans. Automat. Contr.* **52**(4), 730–735.

Sushchik, M., L. S. Tsimring and A. R. Volkovskii (2000) Performance analysis of correlation-based communication schemes utilizing chaos. *IEEE Trans. Circuits Syst. I* **47**, 1684–1691.

Tatikonda, S. and S. Mitter (2004) Control under communication constraints. *IEEE Trans. Automat. Contr.* **49**(7), 1056–1068.

Xia, Y., C. K. Tse and F. C. M. Lau (2004) Performance of differential chaos shift-keying digital communication systems over a multipath fading channel with delay spread. *IEEE Trans. Circuits Syst. II* **51**, 680–684.

Yang Wu, C. Y. and L. Chua (1996) On adaptive synchronization and control of nonlinear dynamical systems. *Int. J. Bifurcations and Chaos* **6**, 455–471.