Abstract
Modelling of autoresonant control of a loaded ultrasonic transducer is presented. Investigation of different control strategies is discussed. Numerical simulations were considered as the most appropriate method for analysis and a Matlab-Simulink computer model of a non-linear ultrasonic vibrating system with the possibility of autoresonant control was developed. The system controlled consists of two modules, the first of which is an electromechanical model of the ultrasonic transducer comprising a piezoelectric transducer and a step concentrator. The second module simulates an influence from the machining process. Coefficients of the electromechanical model were calculated through an identification process based on the real measurement of the ultrasonic transducer's vibration. The validity of the computer model of the ultrasonic vibrating system has been confirmed experimentally. Further, a numerical model of the autoresonant control of this system has been developed. The model allows exercise and comparison of different control strategies based on the feedback signal proportional to the displacement of the end of the concentrator (mechanical feedback) or on the signals proportional to the electrical characteristics of the piezoelectric transducer (electrical feedback). The results of the simulation are presented and discussed. To validate the results obtained through numerical simulations a prototype of an autoresonant control system was developed and manufactured. For all control strategies being modelled the machining experiments have been conducted with the control system. Experiments correlate well with the results of simulation.

Keywords
Ultrasonically assisted machining; active control of vibration; autoresonant control; phase control; modelling of ultrasonic transducer.

1. Introduction
Ultrasonically-assisted machining is superimposition of ultrasonic vibration on conventional machining processes such as turning, milling, drilling and other machining techniques, when the vibration is applied directly to a cutting tip [Markov, 1966; Kumabe, 1979]. Fig. 1 presents the typical set-up for ultrasonically-assisted turning. The ultrasonic transducer consists of piezoceramic rings clamped together with a wave-guide (concentrator) and a back section. A cutting tip is fixed in the tool holder installed at the thin end of the concentrator. The transducer is attached through its developed nodal cross section to the machine tool vertical slide. The workpiece is fixed by a three-jaw spindle chuck and is rotated universally by a lathe drive.

Fig. 1. Experimental set-up of ultrasonically-assisted turning.

When the high frequency electric impulses from an electronic amplifier are fed to the input of the piezo transducer it begins vibrating due to the piezoelectric effect. The vibration excites the longitudinal waves in the concentrator (which intensifies the amplitude of vibration in the direction of the thin end) and through it the vibration of the cutting tip.

The key problem in the promotion of ultrasonically-assisted machining is development of the proper adaptive control of the ultrasonic vibration. It was shown that frequency control (forced excitation with a prescribed frequency) is inefficient in achieving peak performance of ultrasonic cutting systems [Babitsky, 1998; Babitsky, Kalashnikov and Molodtsov, 2004; Astashev and Babitsky, 2007]. The main reasons for this are the non-linear behaviour of ultrasonic vibrating systems, when several regimes are possible with the same frequency applied, and the ill-defined nature of the ultrasonic process. The most advanced control method for overcoming these problems is *autoresonance* [Babitsky, Kalashnikov and Molodtsov, 2004; Astashev and Babitsky, 2007; Babitsky, 1995].
Autoresonant control is a self-sustaining excitation of a vibration mode at the natural frequency of a mechanical system, which maintains the resonant condition of oscillation automatically by means of positive feedback based on the transformation (phase shift, limitation) and amplification of the signal from a sensor. Depending on the choice of sensor, different control strategies can be used, which can be classified into two main types:

- Mechanical feedback, when the signal from the displacement (velocity) sensor attached to the end of concentrator or cutting tip is used for the control system.

- Electrical feedback, when the signal from the current sensor, measuring the electrical parameters of the piezoelectric transducer (current, power), is used in a control algorithm.

This paper is devoted to the investigation and comparison of control strategies based on mechanical and electrical feedbacks. The possible benefits and drawbacks of every control strategy will be revealed and considered. The method of investigation is by using numerical simulations, which requires the creation of a model of the ultrasonic vibrating system and a model of the control system.

2. Model of the ultrasonic vibrating system

The ultrasonic transducer is a complex continual system and creating a realistic model is a complicated task. Fig. 2 explains the process of simplification that allows the vibrating system to keep the important properties of the original whilst making the model accessible for simulation. Fig. 2(a) represents the ultrasonic transducer consisting of the piezoelectric transducer, concentrator and back section. Due to the existence of a nodal point between the piezoceramic rings in the working regime of the transducer, the back section of the transducer can be neglected for the modelling and an ultrasonic transducer can be substituted with the model consisting of one piezoceramic ring and a concentrator (see Fig. 2(b)). The left end of this structure is treated as unmovable. The strong filtering effect of the concentrator permits considering the model of the concentrator of the ultrasonic transducer as a two-degree-of-freedom (2-DOF) system, where the first and the second modes of vibration of the concentrator correspond to the first and the second modes of vibration of the 2-DOF system. Thus the ultrasonic transducer can be represented as the simplified model shown in Fig. 2(c). This model consists of two parts:

- Model of piezoelectric transducer
- Model of concentrator; for this investigation it is 2-DOF vibrating system.

Equations of motion for the system, shown in Fig. 2(c) can be written as:

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -c_1 (x_1 - \dot{x}_0) - k_1 (x_1 - x_0) + c_2 (x_2 - \dot{x}_1) + k_2 (x_2 - x_1) \\
    m_2 \ddot{x}_2 &= -c_2 (x_2 - \dot{x}_1) - k_2 (x_2 - x_1)
\end{align*}
\]  

From here

\[ F_0 = c_1 (\dot{x}_1 - \dot{x}_0) + k_1 (x_1 - x_0) \]  

is the force applied to the piezoelectric transducer from the concentrator, and \( x_0 \) is the displacement of the piezoelement, described by the following equation:

\[ x_0 = \frac{I_0 s^k}{S_0} F_0 + du, \]  

where \( S_0 \) is the area and \( l_0 \) is the thickness of a piezoceramic plate, \( s^k \) is the elastic compliance at constant electric field, \( x_0 \) is the amplitude of deformation for a single piezoceramic plate, \( u \) is the voltage supplied to the piezoceramic plates, and \( d \) is the piezoelectric charge constant [Morgan Electroceramics, 2008].

One-dimensional contact interaction between the ultrasonic transducer and a workpiece can be described with the help of a viscoelastic restraint, known as a Kelvin-Voigt model, Fig. 3.

The restraint is modelled schematically as a parallel working linear spring with stiffness \( k \) and a dashpot with damping coefficient \( c \). The initial gap between the ultrasonic transducer and a viscoelastic restraint is defined as \( \Delta \); negative \( \Delta \) corresponds to the initial interference. Such a model describes the dynamic loading of the ultrasonic transducer due to processing [Astashev and Babitsky, 2007]. Parameters of the ultrasonic transducer and the contact interaction have been chosen based on the identification process.
More details on modelling of the loaded ultrasonic transducer including general schematic of created Matlab-Simulink model and its experimental verification are given in [Voronina and Babitsky, 2008].

3. Model of the control system

In order to make possible the investigation of different control strategies, the model of the control system based on the principle of autoresonance [Astashev and Babitsky, 2007] has to be developed. Autoresonant control is a method based on phase control [Sokolov and Babitsky, 2001], which maintains the resonant regime of oscillation automatically by means of positive feedback using transformation (phase shift, limitation) and amplification of the signal from a sensor. It is based on the fact that during resonance the phase lag between the vibration of the working element (cutter) and the excitation force applied to the latter is constant. A general schematic of feedback is presented in Fig. 4.

![Fig. 4. General schematic of feedback.](image)

Depending on choice of the sensor, two different control strategies are possible: mechanical feedback, when the sensor attached to the end of the concentrator for measuring the mechanical characteristics of the oscillations (displacement, velocity or acceleration) is used for the control system, and electrical feedback, which uses the signal from any electrical sensor measuring the electrical characteristics of the piezoelectric transducer (current, voltage, power). Comparison of the amplitude-frequency characteristic for the displacement of the end of the concentrator with the amplitude-frequency characteristics for the current and power of the piezoelectric transducer has been carried out. The investigation showed that resonant peak of the current curve is shifted from the displacement curve and resonant peak of the power curve coincides with displacement very well. This means that using the signal from the current sensor for the control algorithm does not permit the maximum vibrations of ultrasonic system to be reached, which can have an impact on the performance of the control system. For the case when the power signal is employed in the control algorithm it is possible to reach the maximum amplitude of displacement as the resonant frequencies of power and displacement coincide. These findings will be further investigated in the next section.

4. Numerical simulations and discussion

Based on the results set out in the previous sections, the following control strategies have been further investigated:

- Mechanical feedback, when the displacement was used in the control algorithm (displacement feedback).
- Electrical feedback, when the current signal was used for the control system. This strategy will be called current feedback.
- Electrical feedback, when a current signal was used as the control signal to generate excitation for the piezoelectric transducer and a power signal was used to define the actual performance of the system will be further called power feedback control.

Now the results of the control system simulation for three described types of control will be presented and discussed. In order to investigate the ability of the control system to keep the desired level of vibrations during the cutting process, the simulation of changes in the loading conditions (contact stiffness) was carried out step by step and the RMS value of the sensor signal was recorded. Simulation results for the case of mechanical feedback will be considered first. The control system uses the RMS value of the displacement signal as a control signal, which is shown in Fig. 5 as a solid line, the dashed line depicts the desired value of the RMS of the displacement signal. The desired value was defined as the RMS of the desired value of the amplitude of displacement of the loaded system.

![Fig. 5. RMS value of displacement signal – solid line, desired value of RMS - dashed line.](image)

Fig. 5 shows that the RMS value of the displacement was kept close to the desired level during the whole process of simulation. This result proves that an autoresonant control system based on mechanical feedback is able to maintain the level of vibrations during the process of cutting (in the conditions of the nonlinear load changing). The same simulation was repeated for the electrical feedback.
Current feedback when the RMS value of the current of the piezoceramic rings was used as a control signal will be considered first, Fig. 6.

![Image](image1.png)

**Fig. 6.** (a) RMS value of current – solid line, desired value of RMS -dashed line; (b) RMS value of displacement signal – solid line, desired value of RMS -dashed line.

In this test, in order to have a clear representation of what is happening with the oscillations of the system, the RMS value of the displacement, Fig. 6 (b), was observed together with the RMS value of the current, Fig. 6(a). From these figures we can see that the control system maintains the RMS value of the current, Fig. 6(a). However, the RMS value of the displacement, Fig. 6(b) deviates considerably from its desired value during the test.

The maximum deflection of the RMS value of the displacement from the desired value is 1.8\(\mu m\) (24%), which is noticeably higher than the maximum deflection for the displacement control (0.4\(\mu m\) (5%) from Fig. 5). This test shows that an autoresonant control system based on current feedback was unable to control the level of vibrations during this test.

Simulation results for the next case of electrical feedback when both the current and the power of the piezoelectric transducer are used in the control algorithm will be considered next. In this case the control system uses the current signal to generate excitation for the vibrating system by phase shifting and amplifying it, as in the case of current control. However in order to define the required amount of phase shift and amplitude, the control system uses the power signal. The same test on changing the contact stiffness value for the power feedback case will be considered now.

The RMS value of the power is presented in Fig. 7(a). To trace the vibrations of the system, the RMS value of the displacement was observed, Fig. 7(b). From this graph we can see that indeed the power control is able to keep the level of vibrations at the desired value. The maximum deflection of the RMS value of displacement from the desired value is 0.5\(\mu m\) (7%), which is much better than for the current feedback control (24%) and very close to the result of the mechanical feedback case (5%). This simulation shows that using the power signal for the control system considerably improves the results of electrical feedback control.

![Image](image2.png)

**Fig. 7.** (a) RMS value of power – solid line, desired value of RMS -dashed line; (b) RMS value of displacement signal – solid line, desired value of RMS -dashed line.

However, the following problem occurs with power feedback: any increase in the amplitude of the voltage supplied to the piezoelectric transducer also causes an increase in power. Thus, by maintaining the level of power we cannot control the level of vibrations (will be further considered in the next section).

A comparison of the performance of the autoresonant control based on the phase control algorithm with the conventional frequency control, regime without feedback, was undertaken when the system was excited with predefined frequency. The simulation showed that forced oscillations cannot control the level of vibrations at all. When the load was applied, the resonant frequency of the system was changed and, as it was excited with a different frequency, the oscillations were gradually damped.

5. Experimental results

The numerical investigation revealed the advantages and drawbacks of different control strategies and estimated the efficiency of each of them. To validate the results obtained through simulations a prototype of an autoresonant control system was designed and manufactured. For all the listed control strategies the turning experiments for different feed rates have been conducted with the control system. A lathe Harrison M300 was employed in the experiments as shown in Fig. 1. Spindle speed 125 rev/min and depth of cut 0.15mm were used. Samples of 50mm in diameter made of mild steel have been machined.

Fig. 8 shows a schematic of the experimental setup used for the experiments with different control strategies. Contour 1 indicates the autoresonant control system. Contour 2 designates the arrangement used to record the experimental data.
Fig. 8. Experimental set-up used for experiments with different control strategies.

Fig. 9 represents the oscilloscope readings of the turning experiment with mechanical feedback control system. Solid line depicts the RMS of the inductive sensor’s output measuring vibration; dashed line illustrates the RMS value of the limiter’s output.

At the beginning of the experiments the lathe was switched off. At 50 sec the lathe was switched on and the feed rate 0.03 mm/rev was applied. We can observe the increase in the limiter’s output (dashed line); it is the reaction of the control system trying to compensate for the changes in the control signal, caused by the applied load. At 85 sec the feed was turned off and after setting up the value 0.1 mm/rev was turned on again at 95 sec. We can see that the limiter’s output was increased even more in this case. With switching off the feed at 125 sec the limiter’s output comes back to the previous value. At 145 sec the feed rate 0.2 mm/rev was setup and the amplitude of voltage supplied to the transducer was increased again. The increase in the limiter’s output in this case was almost twice as high as for the previous feed rate value. It can also be seen that the output of the inductive sensor was not changing during the experiment. This means that the control system was able to keep the level of vibrations stable in spite of considerable change in loadings.

In order to calibrate the inductive sensor output the velocity of the cutting tip oscillations was measured using the Polytec laser vibrometer and was recorded together with the output of the inductive sensor (see Fig. 10).

Fig. 9. Turning experiments with mechanical feedback control system; RMS value of the inductive sensor’s output – solid line, RMS value of limiter’s output – dashed line.

This experiment shows that the RMS value of inductive sensor 0.31 V corresponds to the RMS value of the laser vibrometer’s output 0.7 V. Taking into account the sensitivity of the laser vibrometer 1000 mm/s/ν the RMS value of the velocity of the cutting tip is 0.7 m/s, which gives the RMS value of displacement:

\[
x = \frac{\dot{x}}{2\pi f} = \frac{0.7}{2 \times 3.14 \times 18 \times 10^3} = 6.2 \mu m,
\]

where \( f = 18 kHz \) is the frequency of oscillation of the ultrasonic transducer.

The RMS value 6.2 μm corresponds to oscillations with the amplitude 8.8 μm, which is considerably low amplitude for freely vibrating transducer. The low amplitude of vibration can be explained by the Q-factor of the ultrasonic system.

Fig. 10. Inductive sensor calibration. RMS of the inductive sensor’s output- solid line, RMS of the cutting tip’s velocity- dashed line.

Fig. 11 represents the oscilloscope readings of the turning experiment with electrical feedback control system, when the output of the current signal was employed in the control algorithm. In this experiment output of the current signal was used as the actuating signal and as the control signal.

A solid line depicts the RMS of the current sensor’s output; a dashed line illustrates the RMS value of the limiter’s output. As in the previous experiment three different feed rates have been applied, these are: 0.03 mm/rev (at 85 sec), 0.1 mm/rev (at 110 sec) and 0.2 mm/rev (at 135 sec). For all 3 intervals when the feed was applied we can observe the increase in the limiter’s output (dashed line). This demonstrates that the control system is working to compensate for the changes in the control signal, caused by the applied load. We can also see that the output of the current signal (solid line) is not changing during the experiment. This shows the efficiency...
of the control system, as it is able to stabilize the amplitude level of the control signal.

Fig. 11. Turning experiments with current feedback control system; RMS value of the current sensor’s output – solid line, RMS value of limiter’s output – dashed line.

However, comparing the limiter’s output for this experiment with the same signal recorded for the mechanical feedback control system (see Fig. 9) we can see that the limiter’s output is changing within a much broader interval in the case of mechanical feedback. In the mechanical feedback case the limiter produces 2-3 times higher output for each feed rate value than for the current feedback control system. This observation let us doubt the appropriateness of the reflection of the ultrasonic system vibrations by means of the current sensor. To further investigate this case the same experiment was repeated again and in this case the output of the inductive sensor was recorded together with the limiter’s output (see Fig. 12).

Fig. 12. Turning experiments with current feedback control system; RMS value of the inductive sensor’s output – solid line, RMS value of limiter’s output – dashed line.

It can be seen from Fig. 12 that the level of the inductive sensor’s output (solid line) drops each time when increase in the feed is applied (70-90 sec, 110-125 sec and 142-162 sec). This proves that the control system based on the current feedback is unable to control the level of vibration. Thus, the results of this experiment coincide with the results of simulation completed for the current feedback control system. They prove that when controlling the current of the piezoelectric transducer the level of vibrations of the ultrasonic transducer cannot be controlled. This confirms that the control system based on the current feedback is less efficient than the control system based on the mechanical feedback.

The oscilloscope readings of the turning experiment for power feedback control system are shown in Fig. 13. In this case the output of the current signal is used as the actuating signal for the positive feedback loop and the power signal serves as the control signal for the negative feedback loop.

A solid line depicts the RMS of the power sensor’s output; a dashed line illustrates the RMS value of the limiter’s output.

Fig. 13. Turning experiments with power feedback control system; RMS value of the power sensor’s output – solid line, RMS value of limiter’s output – dashed line.

As in previous experiments three different feed rates have been applied: 0.03 mm/rev (at 100 sec), 0.1 mm/rev (at 130 sec) and 0.2 mm/rev (at 155 sec). We can see that the control system in this case behaves in the same way as in previous experiments: the increase in the limiter’s output can be observed each time when increase in the feed is applied. This behaviour of the control system allows keeping the control signal (solid line) constant by compensating for the changes, caused by the applied load. The ability of the control system to stabilize the amplitude level of the control signal verifies its efficiency.

Again, comparing the limiter’s output for this experiment with the same signal recorded for the mechanical feedback control system (see Fig. 9) we can see that the limiter’s output is changing within a much narrower band here. It can also be observed, that the second and third increase in the limiter’s output have almost the same amplitude. This phenomenon has been initially discovered during the simulations completed with the model and can be explained due to dependence of the power of the piezoelectric transducer on the voltage supplied to the piezoelectric transducer.

At the beginning of the experiment, the desired level of power corresponds to the desired level of displacement. Application of the load requires an increase in the voltage supplied to the piezoelectric transducer, which also increases the power and the same level of power corresponds to the lower level of displacement now.
Fig. 14 shows oscilloscope readings of inductive sensor’s output (solid line) and the output of the power sensor (dashed line) observed during the same experiment with power feedback control system as described above.

We can see that the power sensor’s output is kept constant during the test. However, the signal from the inductive sensor decreases during the intervals of the load application (75-95 sec, 110-125 sec and 135-150 sec). Comparing the inductive sensor’s output for this experiment with the same signal obtained for the current feedback control system (Fig. 12) we can notice that they are different. For the current feedback case the drops in the level of the inductive sensor’s output are proportionally increasing with the increase in the applied load, whereas for the power feedback case they are not changing that much from each other. This again proves the dependence of the power of the piezoelectric transducer on the amplitude of the control signal supplied to it and demonstrates the reduced efficiency of the control system based on the power feedback for controlling the level of vibrations of the ultrasonic transducer. Thus, the results of this experiment coincide with the results of simulation completed for the power feedback control system.

6. Conclusions
Autoresonant control is the method of control of ultrasonically assisted machining, providing monitoring of ultrasonic vibrations in the most efficient way. It allows keeping the non-linear resonant mode of vibrations in ill-defined and time changing conditions. The efficiency of control depends on the feedback design, which in its turn relies on the sensor. The completed investigation revealed that the control system based on mechanical feedback provides the most efficient means of control. Advantages of mechanical feedback are linked to the location of the sensor. In the case of mechanical feedback, the sensor is placed near the cutting zone and provides the most reliable information about the dynamics of the machining process.

Electrical feedback is based on the sensor measuring the electrical characteristics of the piezoelectric transducer, which reflects the real vibrations of the ultrasonic system in an indirect way. The piezoelectric transducer is distant from the cutting zone and its electrical characteristics (current and power) are much less subject to the influence of the cutting process than are the mechanical characteristics. This explains the reduced efficiency of the control system with electrical feedback. The limited possibilities of electrical feedback can be improved by improving the correlation with the machining process. This can be done, for example, by introduction of an additional sensor measuring the load applied to the ultrasonic transducer. This would help to monitor the dynamics of the machining process and would improve the reliability of the electrical feedback.

References