INFLUENCE OF NONLINEARITY OF MODEL OF FREE-FLYING SPACE ROBOT ON ITS DYNAMICS WHEN CONTROLLING A MANIPULATOR

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Abstract

The paper considers a mechanical system that comprises a main body (carrier rigid body) with a swivel link to a system of carried bodies (multilink manipulator). A free-flying space manipulation robot (SMR) is an example of such a system. The paper introduces a nonlinear object model with discrete dynamics. The influence of the nonlinear terms (products of generalized velocities) of the object model on the control dynamics of manipulator operations is investigated. The paper defines the applicability conditions of a quasi-linear (simplified) SMR-model to syntheses of manipulator control algorithms for some operating modes of the space robot. A control algorithm was developed for the task of a payload installation on an orbital vehicle surface. The system simulations results confirm the efficiency of the suggested algorithms.

Key words
free-flying manipulation robot, control algorithm

1 Introduction

Free-flying space manipulation robots (SMR) belong to a new space technology class that comprises small vehicles, which are used for various open space jobs [Putz, 1999]. SMR-mechanical system comprises a carrier (main body), a three-link manipulator attached to the carrier by a swivel link and a payload (P) held by the manipulator's gripper. We denote the loaded SMR as SR-P. Kinematics of SR-P plane motion is shown in figure 1, where $q = (q_1 = X_0,\ q_2 = Y_0,\ q_3 = \vartheta,\ q_4 = \alpha_1,\ q_5 = \alpha_2,\ q_6 = \alpha_3)$ is the vector of the SMR-generalized coordinates; $q_1, q_2, q_3 \in q^0 = (X_0, Y_0, \vartheta)$ is the subvector of the coordinates that define the position of the SMR carrier in the inertia coordinate system (CS) *CYX* that is connected with the orbital station (OS); $q_4, q_5, q_6 \in q^\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is the subvector of the coordinates that define the manipulator configuration in bound CS oxy; $\bar{\alpha}_4$ is the time-invariant angle of the payload gripper; c_i, $i = \overline{1,4}$ are the centers of masses of the manipulator links and the payload, which are treated as one-dimensional bodies; $\rho_{oc} = (y_c, x_c)$ is the variable radius-vector that defines the position of SR-P center of mass (the point *c* in figure 1) in the bound CS; $\rho_{aA} = (Y_{aA}, X_{aA})$ is the controlled vector of the (typical) end point displacement $a = (y_a, x_a)$ of the payload from the point $A = (Y_A, X_A)$ of the OS; r_{sa} is the distance from the gripper payload point *s* to the payload's end point *a*; o_m is the root point of the manipulator, whose axes $o_m x_m$ and $o_m y_m$ are collinear with respect to axes of relative carrier CS. We assume that the manipulator is in its initial configuration when its three links are sequentially positioned along the axis $o_m x_m$. Also, we assume that the link *i* rotates in a positive direction if it rotates counter-clockwise with respect to the axes of the link *i*-1.

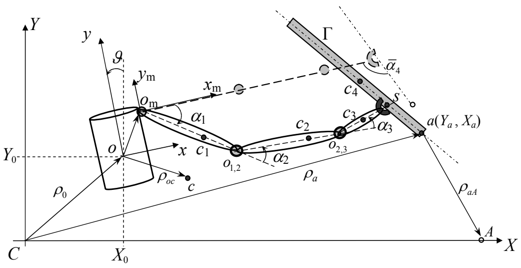

Figure 1. SMR configuration and systems of coordinates.

Free-flying manipulation robots basic designing ideas and principles were introduced in [Onego and Clingman, 1972, Popov, Medvedev and Yuschenko, 1979] quite some time ago. More recently, these ideas were advanced in [Lampariello, Agrawal and Hirzinger, 2003, Yoshida and Umetani, 1990] and partly implemented in, for example, [Oda, 1997].

However, regardless of recent advancements, a lot of problems related to a multifunctional nature of SMR have not been attempted as yet due to a lack of devices that could be used in practice.

The carrier mobility complicates the use of SMR in various servicing operations compromising the quality or even carrying out possibility of manipulator's tasks. One known approach to this problem [Popov, Medvedev and Yuschenko, 1979, Lampariello, Agrawal and Hirzinger, 2003] is to restrict the relation between the carrier inertia moment J_0 and the loaded manipulator inertia moment $J_{\mathrm{M+P}}$ as follows: $k_J = J_0 / J_{\mathrm{M+P}} \geq 10$.

Another approach considered in [Popov, Medvedev and Yuschenko, 1979, Lampariello, Agrawal and Hirzinger, 2003] and other papers is to determine whether the SMR carrier needs to be actively stabilized when performing a number of servicing operation.

These approaches have certain limitations. For instance, saving a payload that has accidentally detached from OS can violate the condition $k_J \geq 10$.

This paper considers specifics of the SMR mathematical model describing a mechanical system with discrete dynamics.

Also, the paper investigates the influence of nonlinear terms that are the products of generalized velocities on system dynamics as well as discusses applicability conditions of the simplified SMR model for the synthesis of control algorithms for specific working modes of space robots.

2 SMR model for the task of manipulator configuration control

Free-flying space robot is defined as the manipulation mechanical system on moving base (figure 1) that operates in the condition of weightlessness. Its mathematical model obtained on the basis of Lagrange's equations of the second genus can be written as follows [Rutkovsky, Sukhanov and Dodds, 1999]:

$$a_1^1\ddot{q}_1 + a_2^1\ddot{q}_2 + a_3^1\ddot{q}_3 + a_4^1\ddot{q}_4 + a_5^1\ddot{q}_5 + a_6^1\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^1 \dot{q}_j \dot{q}_i = F_x \operatorname{Cos}\vartheta - F_y \operatorname{Sin}\vartheta, \quad \vartheta \doteq q_3,$$

$$a_1^2\ddot{q}_1 + a_2^2\ddot{q}_2 + a_3^2\ddot{q}_3 + a_4^2\ddot{q}_4 + a_5^2\ddot{q}_5 + a_6^2\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^2 \dot{q}_j \dot{q}_i = F_y \operatorname{Cos}\vartheta + F_x \operatorname{Sin}\vartheta, \quad (1)$$

$$a_1^3\ddot{q}_1 + a_2^3\ddot{q}_2 + a_3^3\ddot{q}_3 + a_4^3\ddot{q}_4 + a_5^3\ddot{q}_5 + a_6^3\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^3 \dot{q}_j \dot{q}_i = M_\vartheta,$$

$$a_1^4\ddot{q}_1 + a_2^4\ddot{q}_2 + a_3^4\ddot{q}_3 + a_4^4\ddot{q}_4 + a_5^4\ddot{q}_5 + a_6^4\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^4 \dot{q}_j \dot{q}_i = M_{\alpha 1},$$

$$a_1^5\ddot{q}_1 + a_2^5\ddot{q}_2 + a_3^5\ddot{q}_3 + a_4^5\ddot{q}_4 + a_5^5\ddot{q}_5 + a_6^4\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^5 \dot{q}_j \dot{q}_i = M_{\alpha 2},$$

$$a_1^6\ddot{q}_1 + a_2^6\ddot{q}_2 + a_3^6\ddot{q}_3 + a_4^6\ddot{q}_4 + a_5^6\ddot{q}_5 + a_6^6\ddot{q}_6 + \sum_{i=1}^{6}\sum_{j=i}^{6} b_{ji}^6 \dot{q}_j \dot{q}_i = M_{\alpha 3}.$$

Here F_x, F_y are control forces acting along the axes of bound CS; M_ϑ is control moment of the system orientation; $M_{\alpha r}$, $r = \overline{1,3}$ are the drive moments of the corresponding manipulator links; a_i^k, b_{ji}^k are variable coefficients depending on the coordinate q; $i, j, k = \overline{1,6}$. These coefficients are calculated by the formulas that were obtained in [Rutkovsky and Sukhanov, 2000].

In the matrix form the SMR dynamics equation will be

$$\begin{bmatrix} A_{11}(q) & A_{12}(q) \\ A_{21}(q) & A_{22}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}^0 \\ \ddot{q}^\alpha \end{bmatrix} + \begin{bmatrix} B^0(q,\dot{q}) \\ B^\alpha(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} M^0(q^0,\dot{q}^0) \\ M_\alpha(?) \end{bmatrix}, \quad (2)$$

where $A_{ik}(q)$ are the matrices of the variable coefficients a_i^k, depending on the inertia moments of the carrier and manipulator with variable configuration, $B(q,\dot{q}) = \{B^0, B^\alpha\}^{\mathrm{T}}$ is the vector of centrifugal and Coriolis forces; $M^0(q^0,\dot{q}^0)$ is the vector of control action applied to the carrier and defining the translational and angular motions of the SMR; $M_\alpha(?)$ is so far unknown vector of the manipulator links drives depending on the goal of the concrete regime of the SMR operating.

The equations (1) or (2) are distinguished essentially from analogous equations for mechanical system of connected bodies because of the order of the SMR mathematical model is discretely changed [Rutkovsky and Sukhanov, 2001]. Really if it is not necessary to change the position of the manipulator *r*-th link with respect to the (*r*–1)-th link ($M_{\alpha r} \equiv 0$), its hinge is "necrosed".

This is realized through using of self-braking gears from engines to the links' drives. In the mathematical model it indicates that the angular velocity $\dot{\alpha}_r = 0$ and the corresponding equation (equation for α_r) in the system (1) is deleted. It is clear that in all other equations we consider $\dot{\alpha}_r = 0$ and $\ddot{\alpha}_r = 0$. Deleted equation is recovered when it is required to move the *r*-th link with respect to the previous one. At this as the initial conditions for the recovered equation are: $\dot{\alpha}_r(0) = 0$ and $\alpha_r(0) = \alpha_r(t^-)$, $\alpha_r(t^-)$ is the final value of the coordinate $\alpha_r(t)$ at the instant of the *r*-the link braking. Marked peculiarity shows that model (1) corresponds to the system with discretely changeable structure at arbitrary instants. In the view of theoretical mechanics these variations of the system structure indicate the loss one or some degrees of freedom. Mathematically this indicates lowering the order of the differential equations (1) on ($2\times k$) units, where k is the number of conditions $M_{\alpha r} \equiv 0$ acting simultaneously. This peculiarity of the model (1) must be taken into account by one means or another at the SMR dynamics simulation.

3 The influence of mathematical model nonlinear terms on the system dynamics

It is wellknown [Shahinpoor, 1987] that at low values of the links' rates $\dot{\alpha}_r \le \underline{\dot{\alpha}}_r$ the nonlinear terms $b^i_{ji}\,\dot{q}_j\dot{q}_i$ in system (1) play insignificant influence on the system dynamics. In this case control of the manipulator configuration (reconfiguration $q^{\alpha}_0 \to q^{\alpha}_*$) can be realized with the help of proportional algorithm by each link [10]. If the rates $\dot{\alpha}_r$ are comparatively high ($\dot{\alpha}_r > \underline{\dot{\alpha}}_r$) then self-influence of moving carrier and the links on each other is significant. This fact must be taken into account at the synthesis of the link control algorithms that will be nonlinear in this case.

Thus at the first stage of the link control algorithms synthesis it is necessary to determine the values $\underline{\dot{\alpha}}_r$ defining the boundary lower of which ($\dot{\alpha}_r \le \underline{\dot{\alpha}}_r$) we can consider the SMR dynamics as "slow" one. This allows to consider $b^i_{ji}\,\dot{q}_j\dot{q}_i = 0$ and further to synthesize the control algorithms choosing the independent control strategy by each link.

Let us consider the case when the control is realized by changing the position of the first and the second links ($r = 1.2$).

The idea of the boundary $\underline{\dot{\alpha}}_r$ calculating is as follows.

Nonlinear SMR model (2) can be rewritten in the form

$$A(q)\ddot{q} + B(q,\dot{q})\dot{q} = M_q \text{ , or}$$
$$\ddot{q} = A^{-1}(q)M_q - N(q,\dot{q}),$$
$$N(q,\dot{q}) = A^{-1}B(q,\dot{q})\dot{q}\,. \tag{3}$$

In scalar form equations (3) are

$$\ddot{q}_k = \left[\sum_{i=1}^{6} A^{-1}_{ki}(q)M_{qi}\right] - N_k(q,\dot{q}),\ k = \overline{1,6}\,. \tag{4}$$

Let us introduce the SMR simplified model. For this in (4) we will consider $N_k(q,\dot{q}) = 0$:

$$\ddot{q}_k = \sum_{i=1}^{6} A^{-1}_{ki}(q)M_{qi},\quad k = \overline{1,6}\,. \tag{5}$$

It is obvious the solution $q^{nl}_k(t)$ of nonlinear system (4) differ from the corresponding solution $q^l_k(t)$ of the simplified one (5). This difference increases with growth of the nonlinear function $N_k(q,\dot{q})$ influence on the SMR dynamics.

It is expedient to consider the control vector in the equation (3) as a test impulse in our task and to choose it in the form

$$M_q = (M^0, M^{\alpha})^{\mathrm{T}}\text{, where } M^0 = (0)_{1\times 3}\,,$$
$$M^{\alpha} = M^{\alpha 1} = (M_{\alpha 1}, 0, 0) \text{ or}$$
$$M^{\alpha} = M^{\alpha 2} = (0, M_{\alpha 2}, 0) \tag{6}$$

where

$$M_{\alpha 1}, M_{\alpha 2} = \begin{cases} \bar{M}_{\alpha}\ \forall\, 0 \le t \le \tau,\ \bar{M}_{\alpha} = \text{const}, \\ 0\ \forall\, t > \tau. \end{cases} \tag{7}$$

Here $\tau = \text{const}$ is interval that is chosen from the condition of relatively boundedness of the coefficients $a^k_i(q), b^k_{ji}(q)$ changing with respect to their initial values $q^{\alpha}_0 = q^{\alpha}(0)$.

From all generalized coordinates $q = (q_1 = X_0,\ q_2 = Y_0,\quad q_3 = \vartheta,\ q_4 = \alpha_1,\ q_5 = \alpha_2,\ q_6 = \alpha_3)$ we take the coordinate $q_3 = \vartheta$ as observable one. It, on the one hand, is the most sensitive to the angular motion of the manipulator links α_r [Lampariello, Agrawal and Hirzinger, 2003] and, on the other hand, is one of the important coordinate for the SMR operating.

Thus as the nonlinear system (4) reaction on the control actions (6) and (7) we will consider the solution $\vartheta^r_{\mathrm{N}}(t),\ t \in [0,\tau],\ r = 1,2$, that is obtained on the computer. At this the number $r = 1,2$ defines the drive that is used for creation of the test pulse $M_{\alpha r}$. Similarly the reaction of simplified system (5) on the control action (6), (7) will be defined in the form $\vartheta^r_{\mathrm{L}}(t),\ t \in [0,\tau],\ r = 1,2$.

Now dynamic properties (ϑ-dynamics) of nonlinear system (4) and simplified system (5) can be evaluated with the help of the integral estimations

$$J^r_{\mathrm{N}} = \int_0^{\tau} \left|\vartheta^r_{\mathrm{N}}(t)\right| dt\ ,\ J^r_{\mathrm{L}} = \int_0^{\tau} \left|\vartheta^r_{\mathrm{L}}(t)\right| dt,\ r = 1,2\,. \tag{8}$$

The difference between the linear system (5) ϑ_{L}-dynamics and the ϑ_{N}-dynamics of the nonlinear system (4) can be defined by the error

$$e^r(\dot{q}) = \left|1 - \frac{J^r_{\mathrm{L}}}{J^r_{\mathrm{N}}(\dot{q})}\right|,\ r = 1,2\,. \tag{9}$$

In this expression generalized rates $\dot{q}$ dependence on the nonlinear system ϑ_{N}-dynamics was taken into account, including the dependence on manipulator rates $\dot{q}^{\alpha} = (\dot{\alpha}_r)^{\mathrm{T}}$, $r = 1,2$, or in the general case $r = 1,2,3$.

Being given by sequence of the test pulses $\bar{M}_{\alpha} = \mathrm{var}$ in the range from small values $\bar{M}_{\alpha\,\min}$ to significant ones $\bar{M}_{\alpha\,\max}$, that is, changing the drives rates $\dot{\alpha}_r$ in the range $(\dot{\alpha}_{r\min}, \dot{\alpha}_{r\max})$ it is possible to get the diagrams of the system (4) and the system (5) dynamics dependence on the moments $M_{\alpha r}$.

Let us introduce the restriction on the admissible error $e^r(\dot{q})$

$$e^r(\bar{M}_{\alpha r}) \le \underline{e}^r = \mathrm{const}\,. \qquad (10)$$

Then it is easy to determine the admissible values of the manipulator rates $\underline{\dot{\alpha}}_r$, that define the boundary of the domain $\dot{\alpha}_r \le \underline{\dot{\alpha}}_r$. Only in this domain the simplest algorithms of manipulator control that are synthesized on the basis of simplified model (5) can be used.

Discussed above the procedure of the domain $\dot{\alpha}_r \le \underline{\dot{\alpha}}_r$ construction in which simplified model can be used for the manipulator control algorithms synthesis and for the dynamics SMR investigation using linearized model was realized in MATLAB.

In figure 2 the example of this task computer solving was shown. As the object of investigation was chosen the SMR with three-link manipulator and payload (figure 1). The influence of the third link mobility on the SMR dynamics in this example was not considered. Numerical values the SMR main parameters, that are required for coefficients a_i^k, b_{ji}^k, $i,j,k = \overline{1,6}$ calculation [Rutkovsky and Sukhanov, 2000] are determined in the table.

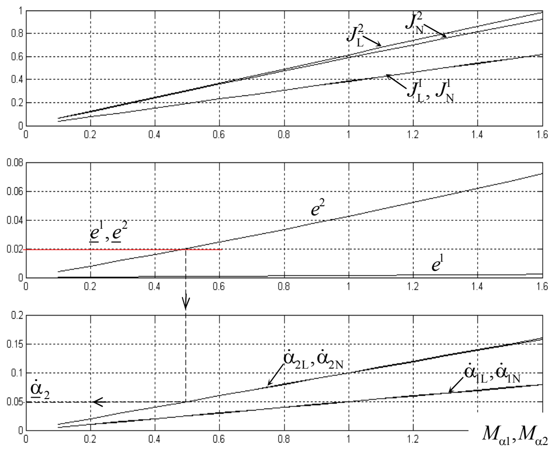

Figure 2. Computer solving results of the choice's task of manipulator link admissible rates.

Here at $p = 0$ the SMR carrier parameters are determined, at $p = 1,2,3$ the manipulator links parameters are determined and at $p = 4$ we have of the payload parameters.

The results of the investigation at $\tau = 5\,s$ (see (7)) are represented by three graphs. Along the horizontal axis the varying values of the manipulator drives moments $M_{\alpha 1}$, $M_{\alpha 2}$ are put aside.

Table

$m_p\,[kg],\ p = \overline{0,4}$	[300; 15; 10; 5; 20]
$J_p\,[kgm^2],\ p = \overline{0,4}$	[52; 5; 1,9; 0,07; 15]
$l_p\,[m],\ p = \overline{1,3}$	[2; 1,5; 0,4]
$r_p\,[m],\ p = \overline{1,4}$	[1,3; 0,8; 0,2; 0,3]
$(x_m, y_m)_p, [m],\ p=0$	[0,4; 0,6]
$(r_{sa}[m], \bar{\alpha}[\mathrm{rad}])_p,\ p=4$	[1,8; 0,6]

At the top integral estimations (8) for nonlinear and simplified systems are shown. Two cases of separately applied control actions $M_{\alpha 1}$, $M_{\alpha 2}$ were considered.

From the graphs J_{L}^r, J_{N}^r, $r = 1,2$, it follows that the difference between nonlinear and linear ϑ-dynamics is more when the second link moves ($M_{\alpha 1} = 0,\ M_{\alpha 2} \ne 0$) in compare with the case, when the first one moves ($M_{\alpha 1} \ne 0,\ M_{\alpha 2} = 0$). It is explained by more high values of the second link rates (see graphs below) because of more lower inertia of its virtual mass. As the consequence the value of the error e^2 (9) is more than the error e^1 and the difference between them increases with increasing of the rate $\dot{\alpha}_2$ (see the graphs in the middle). The desired restriction $\underline{e}^1, \underline{e}^2 \le 0{,}02$ for admissible difference between ϑ-dynamics of nonlinear and linearized systems permits to define boundary $\underline{\dot{\alpha}}_2 = 0{,}05\,\mathrm{rad/s}$. The graphs that corresponds to e^1 does not intersect the line $\underline{e}^1 = 0{,}02$. This fact indicates that there is no essential difference between ϑ-dynamics of nonlinear and linear systems at considered range of the control actions $(\bar{M}_{\alpha\min}, \bar{M}_{\alpha\max})$. Therefore we can use the simplified model (4) at all rates $\dot{\alpha}_1 \le \dot{\alpha}_{1\max} = \dot{\alpha}_1(\bar{M}_{\alpha\max})$.

4 Investigation of SMR dynamics at installation payload in a point on the orbital station surface

In most cases the goal of the SMR using is the capture of the payload, closing with the orbital station (OS) and installation the payload in a point on the OS surface. At this for reasons of softy SMR and OS interaction after the stage of closing SMR must have the phase of hovering over the OS surface. In the context of the SMR carrier mobility it cannot be used the ordinary strategy of control by manipulator with motionless base that consists in planning and stabilization of the programmed trajectories [Shahinpoor, 1987].

Let the conditions $\dot{\alpha}_r \le \underline{\dot{\alpha}}_r$, $r = 1,2,3$, are fulfilled. In this case it is possible to use independent control by every manipulator link with the algorithm that is synthesized on the bases of simplified equations (5).

Let us consider the task of the payload installation in the point $A = (X_A, Y_A)$ on the OS surface that is realized with the help of the first link motion (figure 1). In this case it is assumed that the SMR initial position $q(t_0) \doteq q_0 = (q_0^0, q_0^\alpha)^{\mathrm{T}}$, $\dot{q}_0 = 0$ provides a possibility to carry out this required goal only at the expense of the first link position changing. We will consider that as a first approximation it is possible to neglect by changing of the carrier position that is $q_1, q_2 = \mathrm{const}$. The equation (5) with taking into account the restriction on the drive rate and the rotation angle of the link will be as follows

$$\ddot{\alpha}_1 = \frac{a_3^3}{a_3^3 a_4^4 - (a_3^4)^2} M_{\alpha 1}, \quad |\dot{\alpha}_1| \le \dot{\alpha}_{1\max}, \quad |\alpha_1| \le \alpha_{1\max}, \qquad (11)$$
$$\ddot{\vartheta} = -\frac{a_3^4}{a_3^3 a_4^4 - (a_3^4)^2} M_{\alpha 1}.$$

Taking into account that required goal in our case is $Y_{aA} = (Y_a - Y_A) \to \min$ let us introduce the equation of connection

$$Y_{aA} = Y_0 - Y_A + x_a(\alpha_1)\mathrm{S}\vartheta + y_a(\alpha_1)\mathrm{C}\vartheta \qquad (12)$$

where

$$\begin{aligned} x_a &= x_{\mathrm{m}} + l_1 \mathrm{C}\,\alpha_1 + l_2 \mathrm{C}(\alpha_1 + \bar{\alpha}_2) + \\ &\quad + l_3 \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-4}), \\ y_a &= y_{\mathrm{m}} + l_1 \mathrm{S}\,\alpha_1 + l_2 \mathrm{S}(\alpha_1 + \bar{\alpha}_2) + \\ &\quad + l_3 \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-4}), \end{aligned} \qquad (13)$$

$\mathrm{C}\alpha_i \doteq \cos\alpha_i$, $\mathrm{S}\alpha_i \doteq \sin\alpha_i$; $\bar{\alpha}_i = \mathrm{const}$; $\bar{\alpha}_{2-3} \doteq \bar{\alpha}_2 + \bar{\alpha}_3$;
$x_{\mathrm{m}}, y_{\mathrm{m}}$ are the coordinates of the point o_{m} (figure 1) in bound CS *oxy*.

If the SMR parameters are chosen correctly the condition $a_3^4 << a_3^3$ [Popov, Medvedev and Yuschenko, 1979] is fulfilled. In this case as it is evident from equation (11) for any moment $M_{\alpha 1}$ at the interval Δt of its operating the inequality $|\Delta\vartheta(t)| << |\Delta\alpha_1(t)|$ takes place.

Assuming the values $\mathrm{S}\vartheta \approx \vartheta$, $\mathrm{C}\vartheta \approx 1$ the equation (12) after substitution in it correlations (13) will be as follows

$$\begin{aligned} Y_{aA} &= Y_0 - Y_A + x_{\mathrm{m}}\vartheta + y_{\mathrm{m}} + l_1 \mathrm{S}\,\alpha_1 + l_2 \mathrm{S}(\alpha_1 + \bar{\alpha}_2) + \\ &\quad + l_3 \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-4}) \end{aligned} \qquad (14)$$

After double differentiation we will have

$$\begin{aligned} \ddot{Y}_{aA} &= x_{\mathrm{m}}\ddot{\vartheta} + \ddot{\alpha}_1[l_1 \mathrm{C}\alpha_1 + l_2 \mathrm{C}(\alpha_1 + \bar{\alpha}_2) + l_3 \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-4})] - \\ &- \dot{\alpha}_1^2[l_1 \mathrm{S}\alpha_1 + l_2 \mathrm{S}(\alpha_1 + \bar{\alpha}_2) + l_3 \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{S}(\alpha_1 + \bar{\alpha}_{2-4})]. \end{aligned}$$

In this equation it is possible to neglect by the last term because of the factor $\dot{\alpha}_1^2$ has the second order of smallness. As the result we will have

$$\ddot{\vartheta} = x_{\mathrm{m}}^{-1}[f_{\mathrm{C}}(\alpha_1)\ddot{\alpha}_1 - \ddot{Y}_{aA}] \qquad (15)$$

where

$$\begin{aligned} f_{\mathrm{C}}(\alpha_1) &= [l_1 \mathrm{C}\,\alpha_1 + l_2 \mathrm{C}(\alpha_1 + \bar{\alpha}_2) + \\ &\quad + l_3 \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-3}) - r_{sa} \mathrm{C}(\alpha_1 + \bar{\alpha}_{2-4})]. \end{aligned}$$

Taking into account the expression (15) the SMR dynamics equation (11) can be written as follows

$$\begin{aligned} \ddot{Y}_{aA} &= \frac{f_{\mathrm{C}}(\alpha_1)a_3^3 + x_{\mathrm{m}} a_3^4}{a_3^3 a_4^4 - (a_3^4)^2} M_{\alpha 1}, \\ \ddot{\alpha}_1 &= \frac{a_3^3}{a_3^3 a_4^4 - (a_3^4)^2} M_{\alpha 1}, \\ |\dot{\alpha}_1| &\le \dot{\alpha}_{1\max}, \quad |\alpha_1| \le \alpha_{1\max}. \end{aligned} \qquad (16)$$

Let us assume that coordinates Y_{aA} and $\dot{Y}_{aA}$ can be measured with the help of technical vision with add-in distance meter. Then the control by the manipulator coordinate α_1 and by the coordinate Y_{aA} in simplest case will be realized at using a continuous (or discrete) *PD*-algorithm

$$M_{\alpha 1} = -(k_1 Y_{aA} + k_2 \dot{Y}_{aA}). \qquad (17)$$

The signs of the coefficients k_1, k_2 are to be found using the procedure of search.

If we substitute the correlation (17) in the equation (16) the next equation will be obtained, that characterizes the SMR dynamics with respect to Y_{aA}:

$$\ddot{Y}_{aA} + k(\alpha_1)k_2\dot{Y}_{aA} + k(\alpha_1)k_1 Y_{aA} = 0. \qquad (18)$$

Control action for the minimization of the distance Y_{aA} is $k(\alpha_1)$. In order to have invariable dynamics of our system it is necessary to realize the tuning of the coefficients k_1 and k_2 to the expression

$$k_1 = \frac{n_1}{k(\alpha_1)}, \; k_2 = \frac{n_2}{k(\alpha_1)}, \; n_1, n_2 = \mathrm{const}. \qquad (19)$$

For safety operating SMR near the OS [Rutkovsky and Sukhanov, 2001] it is necessary to have monotonic character of the transient process that is the condition $n_1 \ge 2\sqrt{n_2}$ must be fulfilled.

Efficiency of the suggested type of control algorithm by two links of manipulator is illustrated by the oscillograms in figure 3.

It was used the SMR nonlinear model (1). The parameters of the SMR and payload are pointed out in the table (see section 3). The initial configuration of the manipulator is given by the angle $\alpha_{i=\overline{1,3}}=0$, $\alpha_4=110^\circ$. The initial value $Y_{aA}(0)=1m<l=\sum l_p$. This is the necessary condition for possibility to solve considered task ($Y_{aA}\to 0$) using the control by the first link and further by the second link of the manipulator. And it is given the restriction $|\alpha_1|\le\alpha_{1\max}=0{,}2\,\text{rad}$. The parameters of *PD*-algorithm (17) was chosen: $k_1=1{,}5;\quad k_2=4{,}0$.

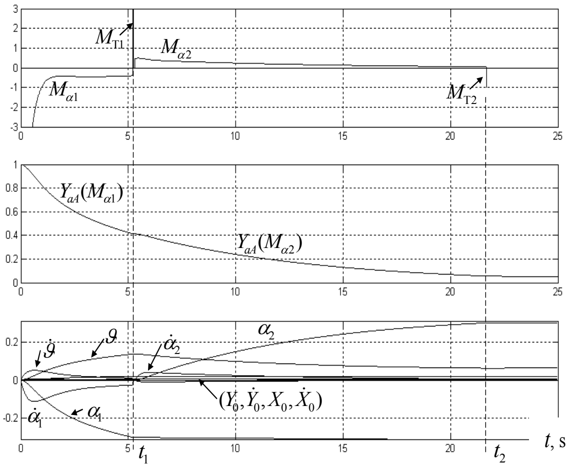

Figure 3. SMR dynamics at sequential control of the first and the second manipulator links.

From the oscillograms it is shown that at $0<t<5{,}2s$ the distance Y_{aA} monotonically decreases. At $t_1=5{,}2s$ the angle $\alpha_1(t)$ attains to its restriction $\underline{\alpha}_1=-0{,}3\,\text{rad}$ and braking moment M_{T1} is applied.

As the result all rates $\dot q(t_k)\to 0$ (because $\dot q(0)=0$) and coordinates became constant ($q(t_1)=\text{const}$, $Y_{aA}(t_1)=0{,}42\ne 0$). Further the distance Y_{aA} decreasing is realized using control by the second link. The control moment $M_{\alpha 2}=-(k_1^{'}Y_{aA}+k_2^{'}\dot Y_{aA})$, $k_1^{'}=k_1, k_2^{'}=k_2$ is created by corresponding drive. The distance Y_{aA} at the instant $t_2\approx 22s$ is decreased to admissible value $0{,}05\,m$ and the braking moment M_{T2} is applied.

5 Conclusion

Suggested results are only the initial ones that can be used for further consideration and solving more complicated and numerous tasks that the SMR will have to realize at service of the manned orbital station during the normal and emergency regimes of its operating.

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References

Lampariello R., Agrawal S.and Hirzinger G. (2003). Optimal Motion Planning for Free-Flying Robots. *International Conference on Robotics and Automation* (ICRA 2003). Taipei, Taiwan. P. 116-122.

Oda M. (1997): ETS-VII, A Rendezvous, Docking and Space Robot Technology Experiment Satellite. *Proc. 4-th Int. Symp. on AI, Robotics and Automation in Space* (I-SAIRAS'97), Tokyo. P. 59-62.

Onego G.T. and Clingman J.H. (1972). Free-Flying Teleoperator Requirements and Conceptual Design. *Proc. of the first National Conference on remotely manned systems*. California Inst. Technology.

Popov E.P., Medvedev V.S. and Yuschenko A.S. (1979). Free-flying manipulation robot computer control // *Proc. of the 8-th IFAC Symposium on automatic control in space*, Oxford, UK, Preprints, Pergamon Press. P. 295-301.

Putz P. (1999). Space Robotics. P. Ehrenfreund et all (Eds.) Laboratory Astrophysics and Space Research. Kluwer Academic Publishers. Printed in the Netherlands. P. 547-596.

Rutkovsky V.Yu., Sukhanov V.M. and Dodds S.J. (1999). Optimal mechanical structure design of a space robotic module as a manoeuvring transport unit. In: *Dynamics and Control of Structures in Space, IV*, (C.L. Kirk, R. Vignjevic, Ed.), Cranfield University, UK. P. 417-445.

Rutkovsky V.Yu. and Sukhanov V.M. (2000). Dynamic Model of Free-Flying Space Robot Engineering Module. *Automation and Remote Control*. V. 61. no. 5. Part 1. P. 749-767.

Rutkovsky V.Yu. and Sukhanov V.M. (2001). Some Questions of Safety and Economic Control by Space Robotic Module's Flight Near by Space Station // *Proc. of 15-th IFAC Symposium on Automatic Control in Aerospace*. Ed. G. Bertoni. Bologna/Forli, Italy. P. 61-66.

Shahinpoor M. (1987). A Robot Engineering Textbook. University of New Mexico. Harper&Row, Publishers, New York.

Yoshida K. and Umetani Y. (1990). Control of Space Free-Flying Robot. *Proc. of 29-th IEEE Conference on Decision and Control*, P. 97-102.