MINIMAX TERMINAL CONTROL PROBLEM IN HIERARCHICAL NONLINEAR DISCRETE-TIME DYNAMICAL SYSTEM

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Abstract
In this paper we consider the nonlinear discrete-time dynamical system consisting of several controlled objects which has two levels of control. Under investigation of it dynamical system we propose the mathematical formalization in the form of realization of two-level hierarchical minimax program terminal control problem and propose the general scheme for its solving. This work was supported by the Russian Basic Research Foundation, Grant 07-01-00008.

Key words
Hierarchical nonlinear discrete-time dynamical system, minimax terminal control problem.

1 Introduction

In this paper we consider the nonlinear discrete-time dynamical system consisting of several controlled objects which has two levels of control. One level (or first level) is dominating and the other level (or second level) is subordinate which have different criteria of functioning and are united a priori by determined information and control relations. We formulate the minimax program terminal control problem for processes in this two-level hierarchical discrete-time dynamical system and propose the general scheme for its solving. The results obtained in this report are based on [Krasovskii and Subbotin, 1988]–[Shorikov, 2005] and can be used for computer simulation and for designing of optimal digital controlling systems for actual technical, economic, and other multi-level control processes.

2 DESCRIPTION OF THE PROBLEM

At a given integer-valued time interval $\overline{0,\mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0)$ we consider the controlled multi-step dynamical system and it consists of $(n+1)$ objects ($n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers). The motion of object I which is a general object and controlled by dominating player P is described by the nonlinear discrete-time recurrent vector equation

$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)),\ y(0) = y_0, \quad (1)$$

and the motion of object II_i $(i \in \overline{1,n})$ which is a subsidiary object corresponding to index i and controlled by the subordinate player E_i is described by the following nonlinear discrete-time recurrent vector equation

$$z^{(i)}(t+1) = f^{(i)}(t, z^{(i)}(t), u(t), v^{(i)}(t), \xi^{(i)}(t)),$$

$$z^{(i)}(0) = z_0^{(i)}. \quad (2)$$

Here $t \in \overline{0, \mathrm{T}-1}$; $y \in \mathbf{R}^r$ and $z^{(i)} \in \mathbf{R}^{s_i}$ are the phase vectors of objects I and II_i respectively ($r, s_i \in \mathbf{N}$; for $k \in \mathbf{N}$, $\mathbf{R}^k$ is the k-dimensional Euclidean vector space of column vectors; $u(t) \in \mathbf{R}^p$ and $v^{(i)}(t) \in \mathbf{R}^{q_i}$ are the vectors of the control actions (controls) of players P and E_i respectively, restricted by the given constraints

$$u(t) \in \mathrm{U}_1,\ v^{(i)}(t) \in \mathrm{V}_1^{(\mathrm{i})};$$

$$\mathrm{U}_1 \subset \mathbf{R}^p,\ \mathrm{V}_1^{(\mathrm{i})} \subset \mathbf{R}^{q_i}\ (p,\ q_i \in \mathbf{N}); \quad (3)$$

where the sets U_1 and $\mathrm{V}_1^{(\mathrm{i})}$ are the finite sets of the spaces $\mathbf{R}^p$ and $\mathbf{R}^{q_i}$ respectively; vector-control $v(t)$ has the form $v(t) = (v^{(1)}(t), v^{(2)}(t), \cdots, v^{(n)}(t))' \in$

$\mathbf{R}^q$ $(q = \sum_{i=1}^{n} q_i)$; $\xi(t) \in \mathbf{R}^l$ and $\xi^{(i)}(t) \in \mathbf{R}^{l_i}$ are the vectors of non-controlling parameters (noises or simulation errors) of the objects I and II_i respectively, restricted by the following given constraints

$$\xi(t) \in \Xi_1,\ \xi^{(i)}(t) \in \Xi_1^{(\mathrm{i})};\ \Xi_1 \in \mathrm{comp}(\mathbf{R}^l),$$

$$\Xi_1^{(\mathrm{i})} \in \mathrm{comp}(\mathbf{R}^{l_i})\ (l,\ l_i \in \mathbf{N}); \tag{4}$$

where for any $k \in \mathbf{N}$, $\mathrm{comp}(\mathbf{R}^{\mathbf{k}})$ is the set of all compact subsets of the space $\mathbf{R}^k$; for all fixed $t \in \overline{0, \mathrm{T}-1}$ and $i \in \overline{1,n}$ each from the vector-functions $f : \overline{0, \mathrm{T}-1} \times \mathbf{R}^r \times \mathbf{R}^p \times \mathbf{R}^q \times \mathbf{R}^l \longrightarrow \mathbf{R}^r$ and $f^{(i)} : \overline{0, \mathrm{T}-1} \times \mathbf{R}^{s_i} \times \mathbf{R}^p \times \mathbf{R}^{q_i} \times \mathbf{R}^{l_i} \longrightarrow \mathbf{R}^{s_i}$ are continuous by collection of the variables $(y(t), u(t), v(t), \xi(t))$ and $(z^{(i)}(t), u(t), v^{(i)}(t), \xi^{(i)}(t))$ respectively; the sets $f(t, \mathrm{Y}_*, u_*(t), v_*(t), \Xi_1) = \{f(t, y(t), u_*(t), v_*(t), \xi(t)),\ y(t) \in \mathrm{Y}_*,\ \xi(t) \in \Xi_1\}$ and $f^{(i)}(t, \mathrm{Z}_*^{(i)}, u_*(t), v_*^{(i)}(t), \Xi_1^{(\mathrm{i})}) = \{f^{(i)}(t, z^{(i)}(t), u_*(t), v_*^{(i)}(t), \xi^{(i)}(t)),\ z^{(i)}(t) \in \mathrm{Z}_*^{(i)},\ \xi^{(i)}(t) \in \Xi_1^{(\mathrm{i})}\}$ are convex sets of the spaces $\mathbf{R}^r$ and $\mathbf{R}^{s_i}$ respectively, for all fixed $t \in \overline{0, \mathrm{T}-1}$, $i \in \overline{1,n}$, convex set $\mathrm{Y}_* \in \mathrm{comp}(\mathbf{R}^r)$, convex set $\mathrm{Z}_*^{(i)} \in \mathrm{comp}(\mathbf{R}^{s_i})$, $u_*(t) \in \mathrm{U}_1$, $v_*(t) \in \mathrm{V}_1$ and $v_*^{(i)}(t) \in \mathrm{V}_1^{(\mathrm{i})}$.

We also assume that for all time moments $t \in \overline{0, \mathrm{T}}$ the phase vectors $y(t)$ and $z^{(i)}(t)$ of objects I and II_i $(i \in \overline{1,n})$ respectively, combined with the initial conditions in the relations (1) and (2) are restricted by the given following constraints

$$y(t) \in \mathrm{Y}_1,\ z^{(i)}(t) \in \mathrm{Z}_1^{(\mathrm{i})};$$

$$\mathrm{Y}_1 \in \mathrm{comp}(\mathbf{R}^r),\ \mathrm{Z}_1^{(\mathrm{i})} \in \mathrm{comp}(\mathbf{R}^{s_i}). \tag{5}$$

The control process in discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

In the field of interests of the player P are both possible states of object I and possible states of each objects II_i $(i \in \overline{1,n})$. And the player P also knows the formation principle of the controls $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ $(\forall\ t \in \overline{\tau, \vartheta - 1} :\ v^{(i)}(t) \in \mathrm{V}_1^{(i)})$ each of the player E_i $(i \in \overline{1,n})$ on the time interval $\overline{\tau, \vartheta}$, which will be described below.

It is assumed that in the field of interests of each player E_i $(i \in \overline{1,n})$ are only possible states of object II_i and for any considered time interval $\overline{\tau, \vartheta}$ he also knows realization of the control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ $(\forall\ t \in \overline{\tau, \vartheta - 1} :\ u(t) \in \mathrm{U}_1)$ of the player P at this time interval, which he can use for constructing his control $v^{(i)}(t) \in \mathrm{V}_1^{(i)}$ for every time moment $t \in \overline{\tau, \vartheta - 1}$.

Then considering these circumstances we will say that such possibilities of the behavior of player P combined with the objects I and II_i $(i \in \overline{1,n})$ are defined as the I level or the dominating level of the control process in considered system.

Let the mapping $\Psi_1^{(i)}$ for all $i \in \overline{1,n}$ we define by the following relation

$$\Psi_1^{(i)} :\ \mathrm{U}_1\ \rightarrow\ \mathrm{comp}(\mathrm{V}_1^{(i)});$$

$$\forall\ t \in \overline{0, \mathrm{T}-1},\ \forall\ u(t) \in \mathrm{U}_1,$$

$$v^{(i)}(t) \in \Psi_1^{(i)}(u(t)) \in \mathrm{comp}(\mathrm{V}_1^{(i)}), \tag{6}$$

where $\Psi_1^{(i)}(u(t))$ is convex polyhedron of space $\mathbf{R}^{q_i}$ for all $u(t) \in \mathrm{U}_1$ (here and below, the convex polyhedron is the convex cover of the finite set of vectors in the corresponding finite–dimensional Euclidean vector space). Therefore, it mean that choice of possible realization of control $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ by player E_i on time interval $\overline{\tau, \vartheta}$ at every time moment $t \in \overline{\tau, \vartheta - 1}$ constrained not only condition (6), but also constrained of the possible realization of control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ by player P, which communicate to player E_i $(i \in \overline{1,n})$.

Then, the collection n of players $E_i,\ i \in \overline{1,n}$ which will be called player E and objects $II_i,\ i \in \overline{1,n}$ controlled by them form II level or the subordinate level of control in considered system (which is subordinate to the I level or the dominating level of the control process).

It is also assumed that in this control process for all time moment $t \in \overline{0, \mathrm{T}}$ the player P knows all relations and constraints (1)–(6) and each player E_i knows (2)–(6) $(i \in \overline{1,n})$ for fixed value of the index i (the player E knows these relations and constraints for all $i \in \overline{1,n}$).

3 FORMULATION OF THE PROBLEM 1 AND GENERAL SCHEME OF THE SOLUTION THE PROBLEM 1

For fixed $k \in \mathbf{N}$ and integer-valued time interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau \leq \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi : \overline{\tau, \vartheta} \longrightarrow \mathbf{R}^k$ of the integer argument and by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$ that are nonempty and compact in the sense of this metric.

Using the constraint (3) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible controls $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ of the player P on the time interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ by the following relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot) :\ u(\cdot) \in \mathbf{S}_p(\overline{\tau, \vartheta - 1}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ u(t) \in \mathrm{U}_1\}.$$

Similarly, using the constraints (3) and (4) we define the following sets: $\Xi(\overline{\tau,\vartheta})$ is the set of all admissible errors of modeling dynamics of object I; $\mathbf{V}^{(i)}(\overline{\tau,\vartheta})$ is the set of all admissible controls of the player E_i; $\Xi^{(i)}(\overline{\tau,\vartheta})$ is is the set of all admissible errors of modeling dynamics of object II_i $(i \in \overline{1,n})$, and all these sets defined on the time interval $\overline{\tau,\vartheta}$.

We also introduce the sets

$$\mathbf{V}(\overline{\tau,\vartheta}) = \prod_{i=1}^{n} \mathbf{V}^{(i)}(\overline{\tau,\vartheta}),\ \hat{\Xi}(\overline{\tau,\vartheta}) = \prod_{i=1}^{n} \Xi^{(i)}(\overline{\tau,\vartheta}),$$

which are the sets of all admissible collections $v(\cdot) = \{v^{(1)}(\cdot), v^{(2)}(\cdot), \cdots, v^{(n)}(\cdot)\} \in \prod\limits_{i=1}^{n} \mathbf{V}^{(i)}(\overline{\tau,\vartheta})$ of program controls for company players $E_i, i \in \overline{1,n}$, or all admissible program controls $v(\cdot)$ of the player E, and all admissible collections $\hat{\xi}(\cdot) = \{\xi^{(1)}(\cdot), \xi^{(2)}(\cdot), \cdots, \xi^{(n)}(\cdot)\} \in \prod\limits_{i=1}^{n} \Xi^{(i)}(\overline{\tau,\vartheta})$ of errors of modeling dynamics of objects $II_i,\ i \in \overline{1,n}$ respectively, and each of them defined on the time interval $\overline{\tau,\vartheta}$.

Using constraints (3) and (6), for fixed admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ of the player P and for each index $i \in \overline{1,n}$ we define the set $\Psi^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)) \in \mathrm{comp}(\mathbf{S}_{q_i}(\overline{\tau,\vartheta-1}))$ of all admissible program controls $v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,\vartheta})$ of the player E_i on the time interval $\overline{\tau,\vartheta}$, corresponding admissible program control $u(\cdot)$ of the player P, by following relation

$$\Psi^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)) = \{v^{(i)}(\cdot):\ v^{(i)}(\cdot) \in \mathbf{V}^{(i)}(\overline{\tau,\vartheta}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ v^{(i)}(t) \in \Psi^{(i)}_1(u(t))\}.$$

We introduce also the set

$$\Psi_{\overline{\tau,\vartheta}}(u(\cdot)) = \prod_{i=1}^{n} \Psi^{(i)}_{\overline{\tau,\vartheta}}(u(\cdot)).$$

Let for admissible time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ the set $\hat{\mathbf{W}}(\tau) = \overline{0,\mathrm{T}} \times \mathbf{R}^r \times \prod\limits_{i=1}^{n} \mathbf{R}^{s_i}$ is the set of all admissible τ-positions $w(\tau) = \{0, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \overline{0,\mathrm{T}} \times \mathbf{R}^r \times \prod\limits_{i=1}^{n} \mathbf{R}^{s_i}$ of the player P $(\hat{\mathbf{W}}(0) = \{w(0)\} = \hat{\mathbf{W}}_0 = \{w_0\},\ w(0) = w_0 = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\})$.

Then, for estimating the quality of the control process on the I level of control we define the following terminal functional

$$\alpha:\ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau,\mathrm{T}}) \times \mathbf{V}(\overline{\tau,\mathrm{T}}) \times \Xi(\overline{\tau,\mathrm{T}}) \times$$

$$\times \hat{\Xi}(\overline{\tau,\mathrm{T}}) = \mathbf{\Omega}(\overline{\tau,\mathrm{T}}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \quad (7)$$

and its values are defined by the following concrete relation

$$\alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \hat{\xi}(\cdot)) =$$

$$= \mu \cdot \hat{\gamma}(y(\mathrm{T})) + \sum_{i=1}^{n} \mu^{(i)} \cdot \hat{\beta}^{(i)}(z^{(i)}(\mathrm{T})). \quad (8)$$

Where by $y(\mathrm{T}) = y_{\mathrm{T}}(\overline{\tau,\mathrm{T}}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$ and by $z^{(i)}(\mathrm{T}) = z^{(i)}_{\mathrm{T}}(\overline{\tau,\mathrm{T}}, z^{(i)}(\tau), u(\cdot),\ v^{(i)}(\cdot), \xi^{(i)}(\cdot))$ we denote the sections of motions of object I and object II_i $(i \in \overline{1,n})$ respectively, at final (terminal) time moment T on the time interval $\overline{\tau,\mathrm{T}}$; $\mu \in \mathbf{R}^1$ and $\mu^{(i)} \in \mathbf{R}^1$ $(i \in \overline{1,n})$ are defined numerical parameters which satisfying following conditions:

$$\mu \geq 0;\ \forall\, i \in \overline{1,n},\ \mu^{(i)} \geq 0;\ \sum_{i=1}^{n} \mu^{(i)} = 1 - \mu. \quad (9)$$

Note, that the functionals $\hat{\gamma}$ and $\hat{\beta}^{(i)}$ for each $i \in \overline{1,n}$ are the convex for all vectors $y \in \mathbf{R}^r$ and $z^{(i)} \in \mathbf{R}^{s_i}$ and each of them meets the corresponding Lipschitz condition.

We denote by $\hat{\mathbf{G}}^{(i)}(\tau) = \overline{0,\mathrm{T}} \times \mathbf{R}^{s_i}$ the set of all admissible τ-positions $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \overline{0,\mathrm{T}} \times \mathbf{R}^{s_i}$ of the player E_i $(i \in \overline{1,n};\ \hat{\mathbf{G}}^{(i)}(0) = \{g^{(i)}(0)\} = \hat{\mathbf{G}}^{(i)}_0 = \{g^{(i)}_0\},\ g^{(i)}(0) = g^{(i)}_0 = \{0, z^{(i)}_0\})$, and by $\hat{\mathbf{G}}(\tau) = \overline{0,\mathrm{T}} \times \prod\limits_{i=1}^{n} \mathbf{R}^{s_i}$ the set of all admissible τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \overline{0,\mathrm{T}} \times \prod\limits_{i=1}^{n} \mathbf{R}^{s_i}$ for all company of the players $E_i,\ i \in \overline{1,n}$, or the player E, for II level of control process $(\hat{\mathbf{G}}(0) = \{g(0)\} = \hat{\mathbf{G}}_0 = \{g_0\},\ g(0) = g_0 = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\})$.

Then, for estimating the quality of the control process by each player E_i $(i \in \overline{1,n})$ on the II level of control process is define the corresponding following terminal functional, namely

$$\beta^{(i)}:\ \hat{\mathbf{G}}^{(i)}(\tau) \times \mathbf{U}(\overline{\tau,\mathrm{T}}) \times \mathbf{V}^{(i)}(\overline{\tau,\mathrm{T}}) \times$$

$$\times \Xi^{(i)}(\overline{\tau,\mathrm{T}}) = \mathbf{\Omega}^{(i)}(\overline{\tau,\mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbf{E}, \quad (10)$$

and its values are defined by the following concrete relation

$$\beta^{(i)}(g^{(i)}(\tau), u(\cdot), v^{(i)}(\cdot), \xi^{(i)}(\cdot)) =$$

$$= \hat{\beta}^{(i)}(z^{(i)}(\mathrm{T})), \tag{11}$$

where the terminal functional $\hat{\beta}^{(i)}$ is from relation (8).

Note, that if we shall consider the functional

$$\gamma:\ \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{V}(\overline{\tau, \mathrm{T}}) \times \mathbf{\Xi}(\overline{\tau, \mathrm{T}}) =$$

$$= \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, \gamma) \longrightarrow \mathbf{E}, \tag{12}$$

and its values are defined by relation

$$\gamma(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) = \hat{\gamma}(y(\mathrm{T})), \tag{13}$$

and it functional estimate the quality on the II level of the control process for player P and dynamical system (1)–(6) on time interval $\overline{\tau, \mathrm{T}}$ by final phase states of the object I. Then if we consider the vector-functional $\delta = (\gamma, \beta^{(1)}, \beta^{(2)}, \cdots, \beta^{(n)})$ such that it define by relation

$$\delta:\ \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, \gamma) \times \prod_{i=1}^{n} \mathbf{\Omega}(\overline{\tau, \mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbf{E}^{n+1}, \tag{14}$$

and its $(n+1)$ values for admissible on the time interval $\overline{\tau, \mathrm{T}}$ realizations of all arguments are defined according to relations (10)–(13) and we can assert that functional α, which is defined by (7)–(9) and its convolution after using the scalar's method for vector functionals.

Then the aim of each player E_i $(i \in \overline{1, n})$ program control process on fixed time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ may be formulate in the following way. The player E_i $(i \in \overline{1, n})$ using his information and control possibilities has interest in such result of control process in dynamical system (1)–(6) on the time interval $\overline{\tau, \mathrm{T}}$ when functional $\beta^{(i)}$ which determined by relationship (10) and (11) for each admissible realizations of his τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \hat{\mathbf{G}}^{(i)}(\tau)$ $(g^{(i)}(0) = g_0^{(i)} \in \hat{\mathbf{G}}_0^{(i)})$ and program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of the player P on it time interval has minimal admissible value by means way using to choice his admissible program control $v^{(i)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$.

Then for realize it aim of the player E_i $(i \in \overline{1, n})$ we can formulate the following multistep program minimax terminal control problem by object II_i on the II level of the two level hierarchical dynamical system (1)–(6).

Problem 1. For fixed index $i \in \overline{1, n}$, time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, admissible on the II level level of the two level hierarchical dynamical system (1)–(6) realization τ-position $g^{(i)}(\tau) = \{\tau, z^{(i)}(\tau)\} \in \hat{\mathbf{G}}^{(i)}(\tau)$ $(g^{(i)}(0) = g_0^{(i)} \in \hat{\mathbf{G}}_0^{(i)})$ of the player E_i and admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of the player P on the I level of this control process it is required to find the set $\mathbf{V}^{(i,e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot)) \subseteq \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ program minimax controls $v^{(i,e)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of the player E_i corresponding the control $u(\cdot)$ of the player P and it set is determine by following relation

$$\mathbf{V}^{(i,e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot)) = \{v^{(i,e)}(\cdot):$$

$$v^{(i,e)}(\cdot) \in \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot)),\ c^{(e)}_{\beta^{(i)}}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot)) =$$

$$= \max_{\xi^{(i)}(\cdot) \in\ \mathbf{\Xi}^{(i)}(\overline{\tau, \mathrm{T}})} \{$$

$$\beta^{(i)}(g^{(i)}(\tau), v^{(i,e)}(\cdot), u(\cdot), \xi^{(i)}(\cdot))\} =$$

$$= \min_{v^{(i)}(\cdot) \in\ \mathbf{\Psi}^{(i)}_{\overline{\tau, \mathrm{T}}}(u(\cdot))} \ \max_{\xi^{(i)}(\cdot) \in\ \mathbf{\Xi}^{(i)}(\overline{\tau, \mathrm{T}})} \{$$

$$\beta^{(i)}(g^{(i)}(\tau), v^{(i)}(\cdot), u(\cdot), \xi^{(i)}(\cdot))\}\}, \tag{15}$$

where functional $\beta^{(i)}$ is defined by relations (10) and (11).

We call the set $\mathbf{V}^{(e)}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot)) = \prod_{i=1}^{n} \mathbf{V}^{(i,e)}(\overline{\tau, \mathrm{T}}, g^{(i)}(\tau), u(\cdot))$ which formed due from solving of n problems 1 for $i \in \overline{1, n}$, the set program minimax controls of the player E on the II level of control process in dynamical system (1)–(6) and corresponding to it the value of the vector $c^{(e)}_{\beta}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot)) = (c^{(e)}_{\beta^{(1)}}(\overline{\tau, \mathrm{T}}, g^{(1)}(\tau), u(\cdot)), c^{(e)}_{\beta^{(2)}}(\overline{\tau, \mathrm{T}}, g^{(2)}(\tau), u(\cdot)), \cdot\cdot, c^{(e)}_{\beta^{(n)}}(\overline{\tau, \mathrm{T}}, g^{(n)}(\tau), u(\cdot)))' \in \mathbf{E}^n$ we call as the value of the result of the programm minimax control of the player E on the II level of control for this control process. It should be noted that the number $c^{(e)}_{\beta}(\overline{\tau, \mathrm{T}}, g(\tau), u(\cdot))$ is concrete value of the vector functional $\beta = (\beta^{(1)}, \beta^{(2)}, \cdots, \beta^{(n)})'$ which defined by relationship (10) and such that may be determine by following mapping

$$\beta:\ \prod_{i=1}^{n} \mathbf{\Omega}^{(i)}(\overline{\tau, \mathrm{T}}, \beta^{(i)}) \longrightarrow \mathbf{E}^n,$$

where for each index $i \in \overline{1, n}$ the value of the functional $\beta^{(i)}$ is defined by formula (11). Note, that we can use the vector functional β as quality test of behavior of the player E (or company of all players $E_i,\ i \in \overline{1,n}$) on the II level of the control process in situation when all players $E_i,\ i \in \overline{1,n}$ have common aim and they organize common coalition.

At the corresponding of the definitions and assumptions made above about parameters and information relations for dynamical systems (1)–(6), the aim of the player P which define the I level at realization of considered two-level program control process in its system on the time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$, respectively by the object I and $II_i,\ i \in \overline{1,n}$ may be formulate in the following way. The player P using his information and controls possibilities interested in such result of realization the program two-level control process for dynamical system (1)–(6) on the time interval $\overline{\tau,\mathrm{T}}$ when functional α determined by relation (8) for each admissible his τ-position $w(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = w_0 \in \hat{\mathbf{W}}_0)$ has minimal admissible value using choice his admissible program control $u(\cdot) \in U(\overline{\tau,\mathrm{T}})$ and program minimax control $v^{(e)}(\cdot) = \{v^{(1,e)}(\cdot), v^{(2,e)}(\cdot), \cdots, v^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot))$ of the player E (its forming by players $E_i,\ i \in \overline{1,n}$ from solving n problems 1 for $i \in \overline{1,n}$) which subordinate of the player P (where τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \hat{\mathbf{G}}_0)$ and defined the phase states of all objects $II_i,\ i \in \overline{1,n}$ on the II of control process at time moment τ is form from τ-position $w(\tau)$).

Below, for realization of it aim of the player P corresponding by I level of considered control process we formulate following programm minimax terminal control problem of the objects I and $II_i,\ i \in \overline{1,n}$ on the I level of control process in two-level hierarchical dynamical system (1)–(6).

Problem 2. For fixed time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ and admissible on the I level of the two-level hierarchical dynamical system (1)–(6) of the realization τ-position $w(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = w_0 \in \hat{\mathbf{W}}_0)$ of the player P it is required the set $\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,\mathrm{T}})$ of the program minimax controls of the player P which determine by following relation

$$\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) :\ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}}),$$

$$c_\alpha^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) = \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot))} \{$$

$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau,\mathrm{T}}) \\ \hat{\xi}(\cdot) \in \hat{\mathbf{\Xi}}(\overline{\tau,\mathrm{T}})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \hat{\xi}(\cdot))\} =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})} \ \min_{v^{(e)}(\cdot) \in \mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot))} \{$$

$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau,\mathrm{T}}) \\ \hat{\xi}(\cdot) \in \hat{\mathbf{\Xi}}(\overline{\tau,\mathrm{T}})}} \alpha(w(\tau), u(\cdot), v^{(e)}(\cdot), \xi(\cdot), \hat{\xi}(\cdot))\}. \qquad (16)$$

Where the functional α defined by relations (7) and (8); τ-position $g(\tau) = \{\tau, z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{G}}(\tau)$ $(g(0) = \{0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = g_0 \in \hat{\mathbf{G}}_0)$ of the E formed due from τ-position $w(\tau)$ of the player P and determine the realization at time moment τ the phase states all the objects $II_i,\ i \in \overline{1,n}$ on the II level of control process at dynamical system (1)–(6) and the set $\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u(\cdot)) = \{v^{(e)}(\cdot) = \{v^{(1,e)}(\cdot), v^{(2,e)}(\cdot), \cdots, v^{(n,e)}(\cdot)\}\} \subseteq \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u(\cdot))$ admissible programm minimax controls of the player E for II level of considered control process at dynamical system (1)–(6) for all realizations τ-position $g(\tau) \in \hat{\mathbf{G}}(\tau)$ $(g(0) = g_0 \in \hat{\mathbf{G}}_0)$ of the player E and programm control $u(\cdot) \in \mathbf{U}(\overline{\tau,\mathrm{T}})$ of the player P formed from solving of the problems 1 for all values of the parameter $i \in \overline{1,n}$.

The set $\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,\mathrm{T}})$ which is forming from solving of the problem 2 we call the set of optimal programm minimax controls of the player P on I level of the control process at dynamical system (1)–(6) and corresponding to it the number $c_\alpha^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau))$ we call the value of the result of the programm minimax control for player P on the I level of this control process.

On the base of formulated the problems 1 and 2 we consider following problem.

Problem 3. For fixed time interval $\overline{\tau,\mathrm{T}} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \mathrm{T})$ and admissible on the I level of the control in two-level hierarchical dynamical system (1)–(6) realization the τ-position $w(\tau) = \{\tau, y(\tau), z^{(1)}(\tau), z^{(2)}(\tau), \cdots, z^{(n)}(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, z_0^{(1)}, z_0^{(2)}, \cdots, z_0^{(n)}\} = w_0 \in \hat{\mathbf{W}}_0)$ of the player P and admissible on the II level of the control process of it system the realization τ-position $g(\tau) \in \hat{\mathbf{G}}(\tau)$ $(g(0) = g_0 \in \hat{\mathbf{G}}_0)$ of the player E which formed due from the τ-position $w(\tau)$ and admissible realization of the optimal program minimax control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau))$ of the player P on the I level of it control process, which formed from solving the problem 2 is is required the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{\Psi}_{\overline{\tau,\mathrm{T}}}(u^{(e)}(\cdot))$ of the optimal program minimax controls $\hat{v}^{(e)}(\cdot) = \{\hat{v}^{(1,e)}(\cdot), \hat{v}^{(2,e)}(\cdot), \cdots, \hat{v}^{(n,e)}(\cdot)\} \in \mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot))$ of the player E for the II level of the control process and vector $c_\beta^{(e)}(\overline{\tau,\mathrm{T}}, g(\tau), u^{(e)}(\cdot)) = (c_{\beta^{(1)}}^{(e)}(\overline{\tau,\mathrm{T}}, g^{(1)}(\tau), u^{(e)}(\cdot)), c_{\beta^{(2)}}^{(e)}(\overline{\tau,\mathrm{T}}, g^{(2)}(\tau), u^{(e)}(\cdot)), \cdots, c_{\beta^{(n)}}^{(e)}(\overline{\tau,\mathrm{T}}, g^{(n)}(\tau), u^{(e)}(\cdot)))' \in \mathbf{E}^n$ of optimal value of the result of the program minimax control for the player E on the II level of control process for considered dynamical system corresponding the control $u^{(e)}(\cdot)$ of the player P and determine by relations:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u^{(e)}(\cdot))=\{\hat{v}^{(e)}(\cdot):\ \hat{v}^{(e)}(\cdot)\in$$

$$\in\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u^{(e)}(\cdot)),\ c_{\alpha}^{(e)}(\overline{\tau,\mathrm{T}},w(\tau))=$$

$$=\max_{\substack{\xi(\cdot)\in\Xi(\overline{\tau,\mathrm{T}})\\ \hat{\xi}(\cdot)\in\hat{\Xi}(\overline{\tau,\mathrm{T}})}}\alpha(w(\tau),u^{(e)}(\cdot),\hat{v}^{(e)}(\cdot),\xi(\cdot),\hat{\xi}(\cdot))=$$

$$=\min_{v^{(e)}(\cdot)\in\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u^{(e)}(\cdot))}\ \max_{\substack{\xi(\cdot)\in\Xi(\overline{\tau,\mathrm{T}})\\ \hat{\xi}(\cdot)\in\hat{\Xi}(\overline{\tau,\mathrm{T}})}}\{$$

$$\alpha(w(\tau),u^{(e)}(\cdot),v^{(e)}(\cdot),\xi(\cdot),\hat{\xi}(\cdot))\}; \qquad (17)$$

$$\forall\, i\in\overline{1,n}:\ c_{\beta^{(i)}}^{(e)}(\overline{\tau,\mathrm{T}},g^{(i)}(\tau),u^{(e)}(\cdot))=$$

$$=\max_{\xi^{(i)}(\cdot)\in\Xi^{(i)}(\overline{\tau,\mathrm{T}})}\{$$

$$\beta^{(i)}(g^{(i)}(\tau),\hat{v}^{(i,e)}(\cdot),u^{(e)}(\cdot),\xi^{(i)}(\cdot))\}=$$

$$=\min_{v^{(i)}(\cdot)\in\Psi^{(i)}_{\overline{\tau,\mathrm{T}}}(u^{(e)}(\cdot))}\ \max_{\xi^{(i)}(\cdot)\in\Xi^{(i)}(\overline{\tau,\mathrm{T}})}\{$$

$$\beta^{(i)}(g^{(i)}(\tau),v^{(i)}(\cdot),u^{(e)}(\cdot),\xi^{(i)}(\cdot))\}. \qquad (18)$$

Note, that we may consider solutions of formulated problems 1–3 which in union are determined the problem of two-level programm minimax terminal control in hierarchical discrete-time dynamical system (1)–(6).

Then the general scheme of realization the process program minimax terminal control in two-level hierarchical dynamical system (1)–(6) for all fixed and admissible time interval $\overline{\tau,\mathrm{T}}\subseteq\overline{0,\mathrm{T}}$ $(\tau<\mathrm{T})$ and realization τ-position $w(\tau)=\{\tau,y(\tau),z^{(1)}(\tau),z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\}\in\hat{\mathbf{W}}(\tau)$ $(w(0)=\{0,y_0,z_0^{(1)},z_0^{(2)},\cdots,z_0^{(n)}\}=w_0\in\hat{\mathbf{W}}_0)$ of the player P on the I level of the control process and corresponding of it τ-position $g(\tau)=\{\tau,z^{(1)}(\tau),z^{(2)}(\tau),\cdots,z^{(n)}(\tau)\}\in\hat{\mathbf{G}}(\tau)$ $(g(0)=\{0,z_0^{(1)},z_0^{(2)},\cdots,z_0^{(n)}\}=g_0\in\hat{\mathbf{G}}_0)$ of the player E on the II level of the control process we can describe in the form of following sequence of actions:

1) for all fixed of the control $u(\cdot)\in\mathbf{U}(\overline{\tau,\mathrm{T}})$ of the player P on the I level of the control process and index $i\in\overline{1,n}$ from solution of the corresponding problem 1 its forming the set $\mathbf{V}^{(i,e)}(\overline{\tau,\mathrm{T}},g^{(i)}(\tau),u(\cdot))$ of the programm minimax controls of the player E_i and the number $c_{\beta^{(i)}}^{(e)}(\overline{\tau,\mathrm{T}},g^{(i)}(\tau),u(\cdot))$ which is the value of the result of the program minimax control for this player on the II level of the control process which corresponding of the control $u(\cdot)$ and satisfying of the relation (15); on the base of this elements and from solution of n problems 1 for all values of index $i\in\overline{1,n}$ we form the set $\mathbf{V}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u(\cdot))$ and vector $c_{\beta}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u(\cdot))$;

2) from solution of the problem 2 are forming the set $\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}},w(\tau))$ of the optimal program minimax controls of the player P on the I level of the control process and number $c_{\alpha}^{(e)}(\overline{\tau,\mathrm{T}},w(\tau))$ which is value of the result of the program minimax control of the player P on the I level of the control process and satisfying the relation (16);

3) for any optimal program minimax control $u^{(e)}(\cdot)\in\mathbf{U}^{(e)}(\overline{\tau,\mathrm{T}},w(\tau))$ of the player P on the I level of the control process from solution of the problem 3 are forming the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u^{(e)}(\cdot))$ and vector $c_{\beta}^{(e)}(\overline{\tau,\mathrm{T}},g(\tau),u^{(e)}(\cdot))$.

4 CONCLUSION

In conclusion we note, that the concrete algorithm for realization of two-level hierarchical minimax program terminal control process in discrete-time dynamical system (1)–(6) can be described on the base of the algorithms for solving program terminal control problem which are proposed in works [Shorikov, 1997] and [Shorikov, 2005].

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