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COMPUTING BOUNDS FOR THE DISTANCE OF FUNCTIONAL OUTPUT-CONTROLLABLE SYSTEMS REPRESENTING FIXED SPEED WIND TURBINE

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Abstract
This paper deals with the concepts of functional output-controllability character of a finite-dimensional linear dynamical system. And a method for computing the functional output-controllability consisting on the calculation of the rank of a certain constant matrix related to the system dynamics is introduced. The linear system under study is a fixed speed wind turbine (FSWT) formed by a squirrel cage generator connected directly to the grid. Due to the non-linear behaviour of such system, the linear system model is calculated by means of a Taylor's decomposition of the non-linear equations of the squirrel cage induction generator, being the system linearized around a steady state operating point. Finally, the study of the functional output-controllability of such system is done, and some boundaries are given to ensure functional output-controllability from some given operational values.

Key words
Functional output-controllability, squirrel cage induction generator, linear system, boundaries.

1 Introduction

In the control theory of continuous linear time-invariant dynamical systems the most frequently used mathematical model is given by the following system consisting of a differential state equation and an algebraic output equation

$$\left.\begin{array}{l}\dot{x}(t) = Ax(t) + Bu(t)\\ y(t) = Cx(t) + Du(t)\end{array}\right\} \qquad (1)$$

where x is the state vector, y is the output vector, u is the input (or control) vector, $A \in M_n(\mathbb{R})$ is the state matrix, $B \in M_{n\times m}(\mathbb{R})$ is the input matrix, $C \in M_{p\times n}(\mathbb{R})$ is the output matrix, and $D \in M_{p\times m}(R)$ is the feedthrough (or feedforward) matrix.

For simplicity we will write the systems as a quadruples of matrices (A, B, C, D).

For its analysis and solution, this system is usually described by the transfer function obtained by applying Laplace transformation to equation (1). It is obtained in the following form

$$\left.\begin{array}{r}s\dot{X} = AX + BU\\ Y = CX + DU\end{array}\right\}$$

$$C(sI - A)^{-1}B + D. \qquad (2)$$

A related problem to the control of the system is whether it is possible to steer the output following a previously assigned curve over any interval of time. The answer to this problem is given analyzing the functional output-controllability.

On the other hand, the recent increasing of wind power in the electrical network, makes interesting the study and ensure the functional output-controllability of Fixed-Speed Wind Turbines (FSWT), which can affect directly the behavior of power systems.

2 Functional Output-Controllability

Definition 2.1. *A system is functional output-controllable if and only if its output can be steered along the arbitrary given curve over any interval of time. It means that if it is given any output $y_d(t)$, $t \geq 0$, there exists t_1 and a control u_t, $t \geq 0$, such that for any $t \geq t_1$, $y(t) = y_d(t)$.*

Proposition 2.1 ([Chen, 1970]). *A system is functional output-controllable if and only*

$$\operatorname{rank} C(sI - A)^{-1}B + D = p$$

in the field of rational functions

A necessary and sufficient condition for functional output-controllability is

Proposition 2.2 ([Chen, 1970]).

$$\text{rank}\begin{pmatrix} sI-A & B \\ C & D \end{pmatrix} = n+p,$$

For systems in which the matrix D is the zero matrix in [García-Planas and Domínguez-García, 2013], a simple test to compute the functional output-controllability is obtained. In this section, a generalization of this test is presented.

For a linear continuous-time system, like (1), described by matrices A, B, C and D, the functional output-controllability matrix can be defined as.

Definition 2.2.

$$oC_{\text{f}}(A,B,C,D) = \begin{pmatrix} C & D & & & \\ CA & CB & D & & \\ CA^2 & CAB & CB & D & \\ \vdots & & \ddots & & \ddots \\ CA^n & CA^{n-1}B & \dots & CAB & CB\ D \end{pmatrix}.$$

and the following result is obtained.

Theorem 2.1. *The system (A,B,C,D) is functional output-controllable if and only if*

$$\text{rank}\, oC_{\text{f}}(A,B,C,D) = (n+1)p.$$

The null terms are not written in the matrix.

In order to prove this theorem an equivalence relation preserving the functional output-controllability is defined that permit to consider an equivalent simple reduced form for the system

Definition 2.3. *Two systems (A,B,C,D) and (A_1,B_1,C_1,D_1) are equivalent if and only there exist matrices $P, \in Gl(n;\mathbb{R})$, $R, \in Gl(m;\mathbb{R})$, $S, \in Gl(p;\mathbb{R})$, $V \in M_{m\times n}(\mathbb{R})$ and $W \in M_{n\times p}(\mathbb{R})$ such that*

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} P^{-1} & W \\ 0 & S \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} P & 0 \\ V & R \end{pmatrix}.$$

This equivalence relation coincides with the strict equivalence relation defined over pencils in the form $H(\lambda) = \begin{pmatrix} \lambda I - A & B \\ C & D \end{pmatrix}$. So the Kronecker canonical reduced form can be considered [García-Planas and Magret, 1999].

Proposition 2.3. *The functional output-controllability character is invariant under equivalence relation.*

Proof.

$$\begin{pmatrix} sI-A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} P^{-1} & -W \\ 0 & S \end{pmatrix} \begin{pmatrix} sI-A & B \\ C & D \end{pmatrix} \begin{pmatrix} P & 0 \\ -V & R \end{pmatrix}$$

Proof of the Theorem. It suffices to consider the Kronecker reduced form of the pencil associated to the system.

Corollary 2.1. *The system (A,B,C,D) is functional output-controllable if and only if*

$$\begin{array}{l} \text{rank}\begin{pmatrix} C & D \end{pmatrix} = p \\ \text{rank}\begin{pmatrix} C & D & \\ CA & CB & D \end{pmatrix} = 2p \\ \vdots \\ \text{rank}\begin{pmatrix} C & D & & & \\ CA & CB & D & & \\ CA^2 & CAB & CB & D & \\ \vdots & & \ddots & & \\ CA^i & CA^{i-1}B & \dots & CAB & CB\ D \end{pmatrix} = (i+1)p \\ \vdots \end{array}$$

Analogously to the case of triples of matrices (A,B,C) [García-Planas and Domínguez-García, 2013], this corollary provides an iterative method to compute functional output-controllability. Calling oC_f the matrices in the corollary, it is shown an example of a flowchart in Figure 1.

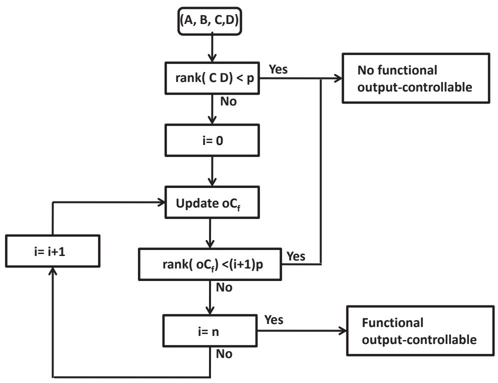

Figure 1. Flowchart showing the required iteration for functional output-controllability computation

3 A Lower Bound on the Distance to a Non Functional Output-Controllable System

The goal is to obtain a bound for the value of the radius of a ball which is neighborhood of a functional output-controllable element, containing only elements which are also functional output-controllable.

The distance considered is that deduced from the Frobenius norm. We recall that given a matrix $A = (a_{ij}) = \in M_{n\times m}(C)$, its Frobenius norm is defined as $\|A\|\sqrt{\sum_{ij} a_{ij}^2}$.

This norm leads to the natural definition of the norm of quadruples in M and the corresponding definition of the distance in M.

Definition 3.1. *Given a quadruple $(A, B, C, D) \in M$ its norm is defined as*

$$\|(A, B, C, D)\| = \sqrt{\|A\|^2 + \|B\|^2 + \|C\|^2 + \|D\|^2}$$

and the distance between the quadruples (A, B, C, D), (A', B', C', D') is defined as

$$d((A, B, C, D), (A', B', C', D')) = \\ \|(A - A', B - B', C - C', D - D')\|.$$

Finally, the distance between a functional output-controllable quadruple and the nearest non-functional output-controllable one is defined as

$$inf\|(\delta A, \delta B, \delta C, \delta D)\|$$

where $(\delta A, \delta B, \delta C, \delta D)$ is a quadruple such that $(A+\delta A, B + \delta B, C + \delta C, D + \delta C)$ does not satisfies the given property.

The starting point to find a bound is the relationship between the norm of the associated matrix $oC_{\mathrm{f}}(A, B, C, D)$ to the quadruple (A, B, C, D) and the norm of this quadruple.

The difficulty to relate the norm of $oC_{\mathrm{f}}(A, B, C, D)$ to the quadruple (A, B, C, D) induce to consider the following matrix

$$M(A, B, C, D) = \\ \begin{pmatrix} A & B & -I & 0 & 0 & 0 & 0 & \dots & & 0 & 0 \\ C & D & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & A & B & -I & 0 & 0 & & & & \\ 0 & 0 & C & D & 0 & 0 & 0 & & & & \\ \vdots & & & & & & & & & & \\ & & & \dots & & & & A & B & -I & 0 \\ & & & \dots & & & & C & D & 0 & 0 \\ & & & \dots & & & & 0 & 0 & C & D \end{pmatrix} \\ \in M_{((n^2+(n+1)p)\times(n(n+1+m(n+1))}(\mathbb{C}).$$

The following theorem ensures that we consider this matrix to study the functional output-controllability.

Theorem 3.1.

$$\operatorname{rank} \begin{pmatrix} C & D & & & \\ CA & CB & D & & \\ CA^2 & CAB & CB & D & \\ \vdots & & \ddots & & \\ CA^n & CA^{n-1}B & & CB & D \end{pmatrix} + (n-1) =$$

$$\operatorname{rank} \begin{pmatrix} A & B & -I & 0 & 0 & 0 & 0 & \dots & & 0 & 0 \\ C & D & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & A & B & -I & 0 & 0 & & & & \\ 0 & 0 & C & D & 0 & 0 & 0 & & & & \\ \vdots & & & & & & & & & & \\ & & & \dots & & & & A & B & -I & 0 \\ & & & \dots & & & & C & D & 0 & 0 \\ & & & \dots & & & & 0 & 0 & C & D \end{pmatrix}$$

Proof. Making block elementary row and columns transformations we have

$$\operatorname{rank} \begin{pmatrix} A & B & -I & 0 & 0 & 0 & 0 & \dots & & 0 & 0 \\ C & D & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & A & B & -I & 0 & 0 & & & & \\ 0 & 0 & C & D & 0 & 0 & 0 & & & & \\ \vdots & & & & & & & & & & \\ & & & \dots & & & & A & B & -I & 0 \\ & & & \dots & & & & C & D & 0 & 0 \\ & & & \dots & & & & 0 & 0 & C & D \end{pmatrix} =$$

$$\operatorname{rank} \begin{pmatrix} I & & & & & & \\ & \ddots & & & & & \\ & & I & & & & \\ & & & C & D & & \\ & & & CA & CB & D & \\ & & & CA^2 & CAB & CB & D \\ & & & \vdots & & \ddots & \\ & & & CA^n & CA^{n-1}B & & CB \; D \end{pmatrix}.$$

Relating the norm of the associated matrix $M(A, B, C, D)$ to the quadruple (A, B, C, D) and the norm of this quadruple.

Theorem 3.2. *Given a non-functional output-controllable quadruple (A, B, C, D) a lower bound for the distance to the nearest non-functional output-controllable quadruple is given by*

$$\|(\delta A, \delta B, \delta C, \delta D)\| \geq \\ \frac{1}{\sqrt{n+1}} \sigma_{n^2+(n+1)p} M(A, B, C, D)$$

where $\sigma_{n^2+(n+1)p} M(A, B, C, D)$ denotes the smallest non-zero singular value of $M(A, B, C, D)$.

Proof. The functional output controllability of (A,B,C,D) implies that rank $M(A,B,C,D) = n^2 + (n+1)p$ and that if $(A+\delta A, B+\delta B, C+\delta C, D+\delta D)$ is not functional output controllable, rank $M(A + \delta A, B+\delta B, C+\delta C, D+\delta D) \le n^2 + (n+1)p$.

The Eckart-Young and Minkowski theorem states that the smallest perturbation in the Frobenius norm that reduces the rank of a matrix A with rank $A = r$ from r to $r-1$ is $\sigma_r(A)$, the smallest non-zero singular value of A.

Noting that

$$M(A+\delta A, B+\delta B, C+\delta C, D+\delta D) = \\ M(A,B,C,D) + M(\delta A, \delta B, \delta C, \delta D)$$

where

$$\begin{pmatrix} \delta A & \delta B & 0 & 0 & 0\,0\,0 & \dots & & 0 & 0 \\ \delta C & \delta D & 0 & 0 & 0\,0\,0 & & & & \\ 0 & 0 & \delta A & \delta B & 0\,0\,0 & & & & \\ 0 & 0 & \delta C & \delta D & 0\,0\,0 & & & & \\ \vdots & & & & & & & & \\ & & & & & \dots & \delta A & \delta B & 0 & 0 \\ & & & & & \dots & \delta C & \delta D & 0 & 0 \\ & & & & & \dots & 0 & 0 & \delta C & \delta D \end{pmatrix}$$

and it is easy to prove that, for all quadruple (A,B,C,D),

$$\|M(\delta A, \delta B, \delta C, \delta D)\| \le \sqrt{n+1}\|(\delta A, \delta B, \delta C, \delta D)\|.$$

It suffices to compute.

$$\|M(\delta A, \delta B, \delta C, \delta D)\|^2 = \\ n(\|\delta A\|^2 + \|\delta B\|^2) + (n+1)(\|\delta C\|^2 + \|\delta D\|^2) \le \\ (n+1)\|(\delta A, \delta B, \delta C, \delta D)\|^2.$$

Therefore, the norm of the perturbation of the matrix $M(\delta A, \delta B, \delta C, \delta D)$ must be at least $\sigma_{n^2+(n+1)}(M(A,B,C,D))$,

Hence, a bound for the distance from (A,B,C,D) to the nearest non-functional output-controllable quadruple, taking into account above proposition is

$$\|(\delta A, \delta B, \delta C, \delta D))\| \ge \\ \frac{1}{\sqrt{n+1}}\|M(\delta A, \delta B, \delta C, \delta D)\| \ge \\ \frac{1}{\sqrt{n+1}}\sigma_{n^2+(n+1)p}(M(A,B,C,D)).$$

4 Modeling of FSWT

The global analyzed system is a wind power generator connected directly to the grid.

The linear system is defined by means of the squirrel cage induction generator differential equations and a first order mechanical system. The differential equations considered within this system are time dependant [Domínguez-García and García-Planas, 2011] [Ugalde-Loo, Ekanayake and Jenkins]. Its inputs are the voltage of the grid. Supposing the system to be in steady state, the system can be described as:

$$\begin{array}{l} \dot{X} = A_{FSWT}X + B_{FSWT}U \\ Y = C_{ci}X + D_{ci}U \end{array} \quad (3)$$

$$A_{FSWT} = \\ K_G \cdot \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & a_{15} \\ -\alpha_{12} & \alpha_{11} & -\alpha_{14} & \alpha_{13} & a_{25} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & a_{35} \\ -\alpha_{32} & \alpha_{31} & -\alpha_{34} & \alpha_{33} & a_{45} \\ K_\omega \bar{i}_{qr0} & -K_\omega \bar{i}_{dr0} & -K_\omega \bar{i}_{qs0} & K_\omega \bar{i}_{ds0} & 0 \end{pmatrix}$$

$\alpha_{11} = -\overline{R}_T\overline{X}_{rr}$, $\alpha_{12} = \alpha_{10}\overline{\omega}_s + \overline{X}_T\overline{X}_{rr}$, $\alpha_{13} = -\overline{R}_r\overline{X}_m$, $\alpha_{14} = -\beta_{r0}\overline{\omega}_s$, $\alpha_{31} = \overline{R}_T\overline{X}_m$, $\alpha_{32} = -\beta_{s0}\overline{\omega}_s - \overline{X}_T\overline{X}_m$, $\alpha_{33} = -\overline{R}_r\overline{X}_{ss}$, $\alpha_{34} = -\alpha_{20}\overline{\omega}_s$.

$$B_{FSWT} = \begin{pmatrix} \gamma\overline{X}_{rr} & 0 & 0 \\ 0 & \gamma\overline{X}_{rr} & 0 \\ \gamma\overline{X}_m & 0 & 0 \\ 0 & \gamma\overline{X}_m & 0 \\ 0 & 0 & \frac{1}{2H} \end{pmatrix}$$

$\gamma = \dfrac{-\omega_b}{\overline{X}_{ss}\overline{X}_{rr}\sigma}$.

Where,

$K_G = \omega_b(\overline{X}_{ss}\overline{X}_{rr}\sigma)^{-1}$,
$K_\omega = \overline{X}_m(2HK_G)^{-1} = -\gamma$,
$\overline{R}_T = \overline{R}_s + \overline{R}_E$,
$\overline{X}_T = \overline{X}_E + \overline{X}_{tr}$,
$\overline{X}_{ss} = \overline{X}_{ls} + \overline{X}_m$,
$\overline{X}_{rr} = \overline{X}_{lr} + \overline{X}_m$,
$\sigma = \dfrac{1-\overline{X}_m^2}{\overline{X}_{rr}\overline{X}_{ss}}$
$\alpha_{10} = \overline{X}_{ss}\overline{X}_{rr} - s_0\overline{X}_m^2$,
$\beta_{s0} = \overline{X}_m\overline{X}_{ss}(1-s_0)$,
$\beta_{r0} = \overline{X}_m\overline{X}_{rr}(1-s_0)$,
$\alpha_{20} = \overline{X}_m^2 - s_0\overline{X}_{ss}\overline{X}_{rr}$,
$a_{15} = \overline{X}_m(\overline{X}_m\bar{i}_{qs0} - \overline{X}_{rr}\bar{i}_{qr0})$,
$a_{35} = \overline{X}_{ss}(\overline{X}_m\bar{i}_{qs0} - \overline{X}_{rr}\bar{i}_{qr0})$,
$a_{45} = \overline{X}_{ss}(-\overline{X}_m\bar{i}_{ds0} + \overline{X}_{rr}\bar{i}_{dr0})$,
$a_{25} = \overline{X}_m(-\overline{X}_m\bar{i}_{ds0} - \overline{X}_{rr}\bar{i}_{dr0})$.

Subindex 0 in some terms of the matrix A_{FSIG} indicates evaluation at the initial condition. Observe that the terms a_{i5} appear because the slip s is a function of $\overline{\omega}_r$. The effect of the transmission line has been considered through the following relations $\overline{v}_{ds} = \overline{v}_{d\infty} - \overline{X}_T\bar{i}_{qs} + \overline{R}_T\bar{i}_{ds}$ and $\overline{v}_{q\infty} + \overline{X}_T\bar{i}_{ds} + \overline{R}_T\bar{i}_{qs}$.

4.1 Output Variables Selection for C and D Matrices Definition

The active and reactive power delivered by the induction generator, and also the stator currents of the wind turbine have been chosen for such analysis.

4.1.1 Active and Reactive Power Selection The output system described as $Y = C_{c1}X + D_{c1}U$ can be written as follows:

$$\begin{pmatrix} \Delta Q_{ss} \\ \Delta P_{ss} \end{pmatrix} = \underbrace{\begin{pmatrix} v_{sd0} & -v_{sq0} & 0 & 0 & 0 \\ v_{sq0} & v_{sd0} & 0 & 0 & 0 \end{pmatrix}}_{C_{c_1}} \begin{pmatrix} i_{sq} \\ i_{sd} \\ i_{rq} \\ i_{rd} \\ \omega_r \end{pmatrix} + \underbrace{\begin{pmatrix} -i_{sd0} & i_{sq0} & 0 \\ i_{sq0} & i_{sd0} & 0 \end{pmatrix}}_{D_{c_1}} \begin{pmatrix} v_{sq} \\ v_{sd} \\ T_m \end{pmatrix} \quad (4)$$

where the subscripts 0 (as previously stated) represents the operating point selected to linearize.

4.1.2 Stator Currents of the FSWT In this case, the state variables have been selected as outputs variables to be analyzed. Then,

$$C_{c2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

and $D_{c2} = 0_{2\times 3}$

5 Functional Output-Controllability of FSWT

In the following subsections it is studied functional output-controllability of FSWT.

Applying the theorem 2.1 in the linearized system, it can be computed rank $oC_{\mathrm{f}}(A, B, C, D)$.

5.1 Case FSWT with Active and Reactive Power as Measured Outputs

Taking into account that i_{sd0} and i_{so0} can be not zero simultaneously, the matrix

$$D = \begin{pmatrix} -i_{sd0} & i_{sq0} & 0 \\ i_{sq0} & i_{sd0} & 0 \end{pmatrix}$$

has full row rank, then applying the test 2.1 it can be concluded that the system is functional output-controllable.

Notice that if the matrix D has full row rank the test finish in the first iteration, since the ranks of the iterative matrices differs exactly the rank of the matrix D.

Taking particular values it is possible to compute a bound ensuring functional output-controllability.

5.2 Case FSWT with Stator Currents of the Induction Generator as Measured Outputs

In this case the matrix D is the zero matrix, but matrix C has full row rank, and taking into account that $\gamma \neq 0$ and $\overline{X}_{rr} \neq 0$, the matrix

$$CB = \begin{pmatrix} \gamma \overline{X}_{rr} & 0 & 0 \\ 0 & \gamma \overline{X}_{rr} & 0 \end{pmatrix}$$

has full row rank and applying the test 2.1, as before it is concluded that the system is functional output-controllable.

Notice that if $D = 0$ but matrices C and CB have full row rank the test finish in the second iteration, since the ranks of the iterative matrices differs exactly the rank of the matrix CB from second iteration.

6 Computing Bounds for Particular Cases

Due to the fact that the linear system under study is derived from a non-linear system, it is important to determine the confidentiality of the linear system in comparison with the real one.

In order to be able to compute a boundary which ensures functional output-controllability characteristic of the system, some values for the symbolic parameters previously presented are used. Such parameters are defined in Appendix A

6.1 Case FSWT with Active and Reactive Power as Measured Outputs

If matrix D has full row rank, the system can be determined functional output-controllable. For that reason, a proper first boundary approximation can be obtained by computing the singular values of matrix D.

In order to calculate the boundary, the system is evaluated under two different operational cases: a wind turbine connected into a weak network and a wind turbine connected into a strong network. The initial conditions derived from those cases are introduced in Appendix A

6.1.1 Weak Network ($VA_{SC} = 16MVA$) In this particular case matrix D is

$$D = \begin{pmatrix} 0.527 & 0.791 & 0 \\ 0.791 & -0.527 & 0 \end{pmatrix}$$

and the smallest singular value of the matrix D is 0.9488.

It is worth to remark that if want to get a best bound, it should compute the singular values of the matrix M.

In this case, the smallest singular value of M is 0.6196 (computed using MATLAB). Then, the bound is

$$\frac{1}{\sqrt{6}} 0.6196 = 0.0253.$$

6.1.2 Strong Network ($VA_{SC} = 40MVA$) In this particular case matrix D is

$$D = \begin{pmatrix} 0.458 & 0.789 & 0 \\ 0.789 & -0.458 & 0 \end{pmatrix}$$

and the smallest singular value of the matrix D is 0.9201.

Analogously, it is worth to remark that if want to get a best bound, it should compute the singular values of the matrix M.

In this case, the smallest singular value of M is 0.7148 (computed using MATLAB). Then, the bound is

$$\frac{1}{\sqrt{6}}0..7148 = 0.2918.$$

6.2 Case FSWT with Stators Currents of the Induction Generator as Measured Outputs

In this case $D = 0$, but matrices C and CB has full row rank, taking into account C is a fixed matrix, then a proper first boundary approximation can be obtained by computing the singular values of matrix CB.

In our case

$$B = \begin{pmatrix} 87.0454 & 0 & 0 \\ 0 & 87.0454 & 0 \\ 84.9079 & 0 & 0 \\ 0 & 84.9079 & 0 \\ 0 & 0 & 0.1429 \end{pmatrix}$$

Then

$$CB = \begin{pmatrix} 87.0454 & 0 & 0 \\ 0 & 87.0454 & 0 \end{pmatrix}$$

and the smallest singular value of the matrix CB is 87.0454. Then, the nearest non-functional output-controllable system $(A + \delta A, B + \delta B, C + \delta C)$ is in such a way that $\|\delta(C \cdot B)\| > 87.0454$.

It is worth to remark, that if the parameters selected for the analysis implies that the smallest singular value of matrix CB is 0, the boundary must be computed using matrix M.

6.2.1 Weak Network ($VA_{SC} = 16MVA$) In this particular case matrix A is

$$A = \begin{pmatrix} 1.5058 & -369.1080 & 0.4661 & 345.9649 & 46.2303 \\ 369.1080 & 1.5058 & -345.9649 & -0.4661 & -282.1571 \\ -1.0525 & 360.1678 & 0.4769 & 337.5596 & 26.1976 \\ -360.1678 & -10.5254 & -337.5596 & 0.4769 & -170.7528 \\ -0.4812 & 0.1740 & -0.4511 & -0.3007 & 0 \end{pmatrix}$$

Analogously, it is worth to remark that if want to get a best bound, it should compute the singular values of the matrix M.

In this case, the smallest singular value of M is 0.0003 (computed using MATLAB). Then, the bound is

$$\frac{1}{\sqrt{6}}0.0003 = 0.00012247.$$

6.2.2 Strong Network ($VA_{SC} = 40MVA$) In this particular case matrix A is

$$A = \begin{pmatrix} 0.8614 & -362.5157 & 0.4661 & 345.8275 & 20.5053 \\ 362.5157 & 0.8614 & -345.8275 & 0.4661 & -143.3823 \\ -0.8399 & 345.2217 & -0.4769 & 337.4178 & 20.9843 \\ -345.2217 & -0.8399 & -337.4178 & -0.4769 & -236.5390 \\ 0.4734 & 0.1272 & -0.4511 & -0.2578 & 0 \end{pmatrix}$$

Analogously, it is worth to remark that if want to get a best bound, it should compute the singular values of the matrix M.

In this case, the smallest singular value of M is 0.0002 (computed using MATLAB). Then, the bound is

$$\frac{1}{\sqrt{6}}0.0002 = 0.00008165.$$

7 Conclusion

This paper has presented the concept of functional output-controllability and applied to a linearized system from the nonlinear equations representing the squirrel cage induction generator. Functional output-controllability has been determined using the A, B, C and D matrices. Moreover, the demonstration is made with a generic system. Therefore, it can be ensured not only for an example. Due to the functional output-controllability condition, it can be concluded that any output can be reached regulating the voltage inputs.

A System Under Study Parameters

The parameters of the squirrel cage induction generator used for the evaluation of the boundaries are the following: $V_b = 690V$, $S_b = 2MVA$, $f_b = 50Hz$, $\omega_b = 2\pi f_b$, $H = 3.5s$, $\overline{X}_{tr} = 0.05$, $\overline{R}_s = 0.00488$, $\overline{X}_{ls} = 0.09241$, $\overline{R}_r = 0.00549$, $\overline{X}_{lr} = 0.09955$, $\overline{R}_d = 0.2696$, $\overline{X}_{ld} = 0.0453$, $\overline{X}_m = 3.95279$ and $\overline{X}_{rm} = 0.02$.

$VA_{SC} = 16MVA$, $X/R = 10$, $\overline{Z}_E = S_b/VA_{SC}$, $\overline{R}_E = \overline{Z}_E/(1 + (X/R)^2)^{1/2}$, $\overline{X}_E = \overline{R}_E(X/R)$.

A.1 Weak Network ($VA_{SC} = 16MVA$)

The operating point of a weak network used for system linearization is $\bar{i}_{ds0} = -0.527$, $\bar{i}_{qs0} = 0.791$, $\bar{i}_{dr0} = -0.306$, $\bar{i}_{qr0} = 0.846$ and $s_0 = -0.0055$.

A.2 Strong Network ($VA_{SC} = 40MVA$)

The operating point of a strong network used for system linearization is $\bar{i}_{ds0} = -0.458$, $\bar{i}_{qs0} = 0.798$, $\bar{i}_{dr0} = -0.225$, $\bar{i}_{qr0} = 0.838$ and $s_0 = -0.0051$.

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