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SIMULATION OF ADAPTIVE CONTROL SYSTEM FOR SINGLE-CHANNEL PLANT WITH INPUT SATURATION

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Abstract
Article deals with simulation of synthesized by the hyperstability criterion adaptive control system for stationary dynamic plants with the input signal saturation, functioning under a priori uncertainty conditions. With the help of computational experiments, the quality of the control system operation under various initial conditions of the controlled plant is illustrated.

The results obtained in the article may be used in the practical implementation of the considered control system. In particular, they can be used for construction of the control systems for manipulation robots and other technical plants (for example, rotor systems) with the input signals restrictions.

Key words
adaptive control, priory uncertainty, filter-corrector, simulation, hyperstability criterion, *L*-dissipativity.

1 Introduction

In fact, in any automatic control system there is a restriction of the input signal, which can occur, for example, due to drive saturation. Despite this, in solving many problems of control systems synthesis, the presence of input constraints is often ignored. This circumstance can leads to a deterioration in quality or loss of efficiency of the control systems. Moreover, because of the incorrect synthesis of control algorithms, catastrophic consequences may arise [Andrievskiy, Kuznetsov and Leonov, 2014].

To date, a lot is known about solution of control problem of plants with input saturations. A variety of approaches to construct control systems are offered. In particular, in [Monopoli, 1975] an adaptive regulator for plant with hard saturation is offered. In [Feng, Zhang and Palaniswami, 1991] for the input constrained system a stability analysis is perform. In [Wang and Sun, 1992] for plant with input saturation a modified adaptive control scheme is proposed. In [Zhang and Evans, 1994] an approach to construction of continuous control system for class of the input saturated plants is offered. In [Yang, Calise and Craig, 2003] an adaptive output control scheme is proposed. In [Astrom and Hagglund, 2006] for input constrained plants advanced schemes with PID controllers are offered. In papers [Takagi, Nishida and Kobayashi, 2006; Takagi, Nishida and Kobayashi, 2007; Takagi, Oya, Wang and Kobayashi, 2010; Takagi, Oya, Wang and Kobayashi, 2009; Takagi, Zhuo, Oya and Wang, 2011; Takagi, Sato, and Oya, 2011], in the presence of input saturations, an adaptive approaches to construction of control systems for stable and unstable plants, as well as plants with relative order greater than one, are proposed. Finally, in [Eremin, 2016] with the help of the hyperstability criterion, an adaptive controller modification of the control system for stationary plant was suggested. This modification allowed to ensure the system working capacity in the presence of control signal limitations.

It is well known that one of the key stages of the practical implementation of analytically-designed control systems is their simulation [Musaev, Trofimov and Frolov, 2014; Pashchenko F., Durgaryan, Pashchenko A. and Belova, 2014; Gulyamov, Yusupbekov and Temerbekova, 2016].The importance of this stage, first of all, is due to the possibility of preliminary analysis of the systems operation when changing both external and internal conditions of their functioning. Since modeling is performed using not physical objects themselves, but their mathematical models, the time and costs for preliminary research are significantly reduced, thereby increasing the effectiveness of the theoretical results implementation. In addition, execution the computational experiments allows solving some specific problems, such as: selection of regulator parameters; choice of the sampling step for digital elements, and others.

In the present paper, the approach of [Eremin, 2006; Eremin, 2007a; Eremin, 2007b; Eremin, 2013] is used for analysis the adaptive system obtained in [Eremin, 2016] for various parameters of the controlled plant.

2 Mathematical Description of the Control System

It is assumed that the adaptive control system synthesized with the help of hypestability criterion consists of the following elements:

1. Control plant with input saturation:

$$\frac{dx(t)}{dt} = Ax(t) + B[\sigma(u(t)) + f(t)], \quad (1)$$
$$y(t) = x_1(t),\ x(0) = x_0,$$

where $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \in R^n$ is the state vector; x_0 is the initial conditions vector; $A = (A_M + B_M c_0^T)$ is some matrix in the Frobenius form, the last row of which has the form $[a_0, a_1, ..., a_{n-1}]$; A_M is the Hurwitz matrix with given eigenvalues location; $B_M = Bk_0 = [0, ..., 0, b_M]^T \in R^n$, $B = [0, ..., 0, b_n]^T \in R^n$ are the constant vectors; $b_n = const > 0$; $b_M = b_n k_0$, $k_0, c_0 = [c_{01}, c_{02}, ..., c_{0n}]^T$ are number and vector with constant values respectively; $f(t) \in R$ is the external disturbance, at that $|f(t)| \leqslant f_0$, $f_0 = const > 0$; $y(t) \in R$ is the controlled plant output; $u(t) \in R$ is the control signal; $\sigma(u(t))$ is the nonlinear saturation function which has the form

$$\sigma(u(t)) = \begin{cases} \sigma_0, & u(t) > \sigma_0, \\ u(t), & |u(t)| \leqslant \sigma_0, \\ -\sigma_0, & u(t) < -\sigma_0, \end{cases} \quad (2)$$

where $\sigma_0 > 0$ is the known constant corresponding to the saturation level. The values of f_0 and the coefficients $a_{n-1}, a_{n-2}, ..., a_1, a_0, b_n$ are priori indefinite numbers whose values depend on a set of unknown parameters $\xi \in \Xi$; Ξ is the known bounded set.

2. Explicit reference model with two outputs:

$$\frac{dx_M(t)}{dt} = A_M x_M(t) + B_M r(t), \quad (3)$$
$$y_M(t) = x_{M1}(t),\ z(t) = g^T x_M(t),$$

where $x_M(t) = [x_{M1}, x_{M2}, ..., x_{Mn}(t)]^T \in R^n$ is the reference state vector; $y_M(t) \in R$, $z(t) \in R$ are main and auxiliary outputs respectively; $r(t) \in R$ is the command signal; g is the given vector.

3. Filter-corrector which is connected to the plant (1), (2) output:

$$\frac{dx_F(t)}{dt} = A_F x_F(t) + By(t), \quad (4)$$
$$z_F(t) = q_F^T x_F(t) + D_F y(t),$$
$$W_F(s) = q^T (sE_{n-1} - A_F)^{-1} B + D_F =$$
$$= \frac{g(s)}{(Ts+1)^{n-1}},$$

where $x_F(t) = [x_{F1}(t), x_{F2}(t), ..., x_{F(n-1)}(t)]^T \in R^{n-1}$ is the filter state vector; $z_F(t) \in R$ is the filter scalar output; A_F, D_F, q_F are matrixes and vector with given values; $W_F(s)$ is the filter transfer function; E_{n-1} is the unit matrix; T is the small time constant, value of which must be chosen from the special conditions [Eremin, 2006; Eremin, 2007a; Eremin, 2007b; Eremin, 2013]:

$$T < T_1 = \frac{0.93}{(n-2)a_{M1}}, \quad (5)$$
$$T < T_2 = \frac{0.465 a_{M1}}{(n-1)a_{M2}},$$

where $a_{Mi}, i = 0, 1, 2, .., (n-1)$ are coefficients of the polynomial $a_M(s) = det(sE - A_M) = s^n + a_{M(n-1)} s^{n-1} + ... + a_{M1} s + a_{M0}$.

4. Adaptive regulator with integral algorithms of self-tuning:

$$u(t) = k(t)r(t) - \sum_{i=1}^{n} c_i(t) x_{Fi}(t), \quad (6)$$
$$\frac{dk(t)}{dt} = \begin{cases} h_0 r(t)\tilde{v}(t)\tilde{\delta}(t), & |\tilde{v}(t)| > v_0, \\ 0, & |\tilde{v}(t)| \leqslant v_0, \end{cases}$$
$$\frac{dc_i(t)}{dt} = \begin{cases} -h_i x_{Fi}(t)\tilde{v}(t)\tilde{\delta}(t), & |\tilde{v}(t)| > v_0, \\ 0, & |\tilde{v}(t)| \leqslant v_0, \end{cases}$$
$$\tau \frac{\tilde{\delta}(t)}{dt} + \tilde{\delta}(t) = \delta(t), \tilde{\delta}(0) = 0, \tau = const > 0,$$
$$\delta(t) = \begin{cases} 1, & \forall[\sigma(u(t)) - u(t)]\tilde{v}(t) \geqslant 0, \\ \delta_0, & \forall[\sigma(u(t)) - u(t)]\tilde{v}(t) < 0, \end{cases}$$
$$\tilde{v}(t) = z_M(t) - z_F(t),$$

where $h_0, h_i, i = 1, 2, ..., n$ are arbitrary positive numbers; $v_0 = const > 0$ is the dead zone quantity; $\delta_0 = const < 1$.

3 Problem Statement

It is necessary to perform a simulation of the described system (1) (4), (6) in order to select the adaptive regulator (6) and filter-corrector (4), (5) parameters in such a way that at any level of uncertainty $\xi \in \Xi$ and any initial conditions $x(0)$ the following objectivities are satisfied:

$$\lim_{t\to\infty} |\Delta y(t)| = \lim_{t\to\infty} |y_M(t) - y(t)| \leqslant \Delta_y, \quad (7)$$
$$\lim_{t\to\infty} |k(t)| \leqslant \tilde{k}_0,$$
$$\lim_{t\to\infty} |c_i(t)| \leqslant \tilde{c}_{0i},$$

where $\Delta_y = const > 0$, $\tilde{k}_0 = const > 0$, $\tilde{c}_{0i} = = const > 0$ are small scalar numbers.

4 Computational Experiments

Let us consider the control problem for a single-channel third-order plant (1), (2) with input saturation, functioning under uncertainty conditions, the level of which is given by relations:

$$-4 \leqslant a_0 \leqslant 3.5,\ 0 \leqslant a_1 \leqslant 2, -5 \leqslant a_2 \leqslant -3, \quad (8)$$
$$1 \leqslant b_n \leqslant 1.5,\ 0.1 \leqslant f_0 \leqslant 2,$$

where the plant parameters have the following values:

$$a_0 = 1.5, a_1 = 1, a_2 = -2, b_n = 1.5, \sigma_0 = 10, \quad (9)$$

and the external disturbance is harmonic function

$$f(t) = 2\sin(0.03t). \quad (10)$$

The filter-corrector (4), which is described by equations

$$\dot{x}_{F1}(t) = x_{F2}(t), \quad (11)$$
$$\dot{x}_{F2} = -10^6 x_{F1}(t) - 2000 x_{F2}(t) + 10^6 y(t),$$
$$z_F(t) = x_{F1}(t) + x_{F2}(t) + 0.25\dot{x}_{F2}(t),$$
$$x_{F1}(0) = x_{F2}(0) = 0,$$

is connected to the plant (1), (2), (8) (10) output.

The desired dynamics of the plant (1), (2), (9), (10) and the required dynamics of the main contour (1), (2), (4), (9), (10) are determined by the explicit two-output reference model (3), which is described as follows:

$$\dot{x}_{M1}(t) = x_{M2}(t), \quad (12)$$
$$\dot{x}_{M2}(t) = x_{M3}(t),$$
$$\dot{x}_{M3}(t) = -8x_{M1}(t) - 12x_{M2}(t) - 6x_{M3}(t) +$$
$$+ 8r(t), x_{M1}(0) = x_{M2}(0) = x_{M3}(0) = 0,$$
$$r(t) = 1.2 + 0.5\sin(0.03t) + 0.6\sin(0.06t).$$

During the computational experiments parameters of the adaptive regulator (6) and the filter-corrector (4), (5), (11) with purpose of increasing the system speed were selected with following values:

$$g^T = [1\ \ 1\ \ 0.25], \quad (13)$$
$$h_0 = 3000, h_1 = 2500, h_2 = 1000, h_3 = 300,$$
$$k(0) = c_i(0) = 0, i = \overline{1,3},$$
$$v_0 = 0.001, \delta_0 = 0.0001, \tau = 8.$$

Simulation results of the considered system with zero initial conditions are shown on Figures 1–6.

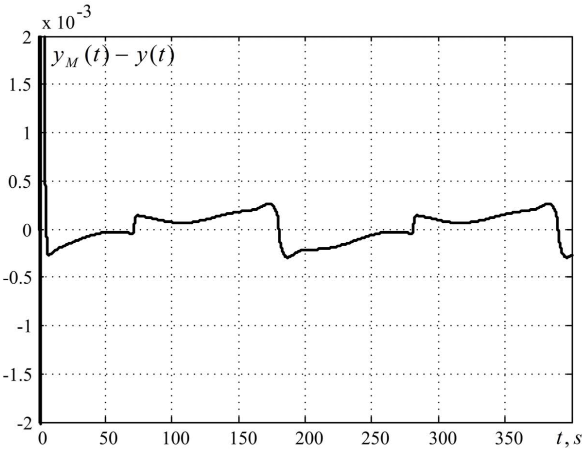

Figure 1. Tracking error in the system with zero initial conditions of the plant.

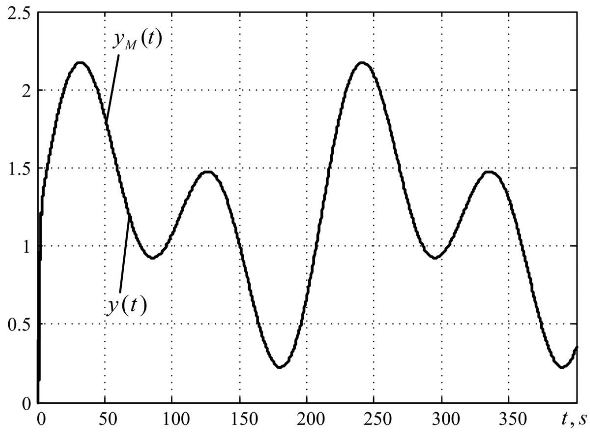

Figure 2. Output signals of the reference (12) and the plant (1), (2), (8)–(10) with zero initial conditions.

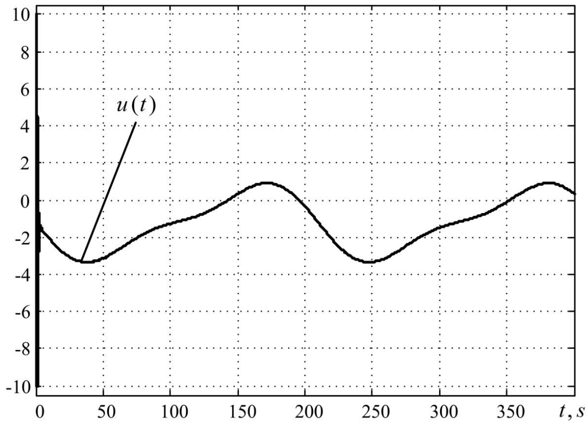

Figure 3. Control signal of the system (1), (2), (8)–(13).

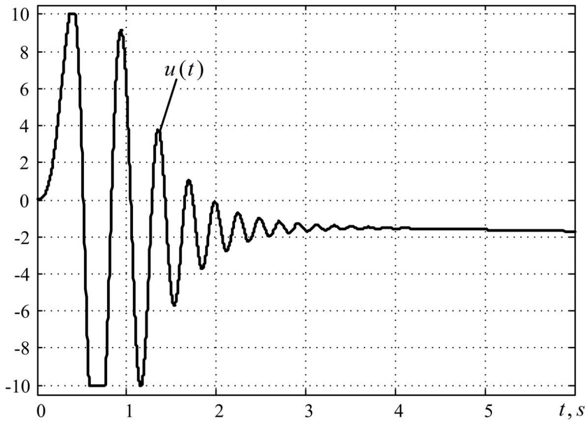

Figure 6. Control signal of the system (1), (2), (8)–(13) at time interval $t \in [0; 6]s$.

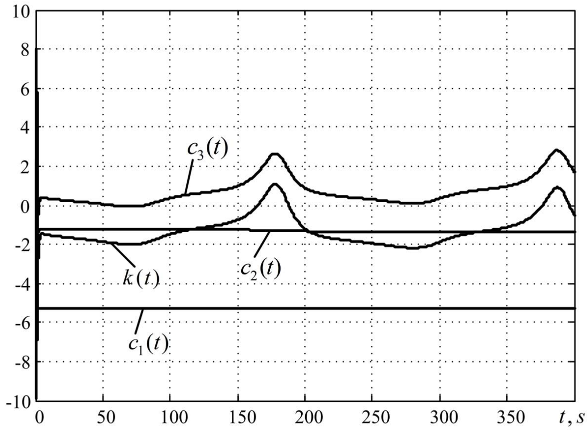

Figure 4. Tuning dynamics of the adaptive regulator (6), (13) coefficients.

On Figures 7–12 are shown the results of the similar system simulation but with non-zero initial conditions, whose values are given as follows:

$$x_0 = [x_{01}\ \ x_{02}\ \ x_{03}]^T = [0.3\ \ 0.01\ \ 0.02]^T. \quad (14)$$

The presented characteristics of the control system (1) – (14) illustrate that the appropriate selection of the adaptive controller (6), (13) coefficients values ensure the high speed and quality of the system operation for which the targets (7) are satisfied, both under zero initial conditions (Figures 1 – 6) and initial conditions other than zero (Figures 7 – 12).

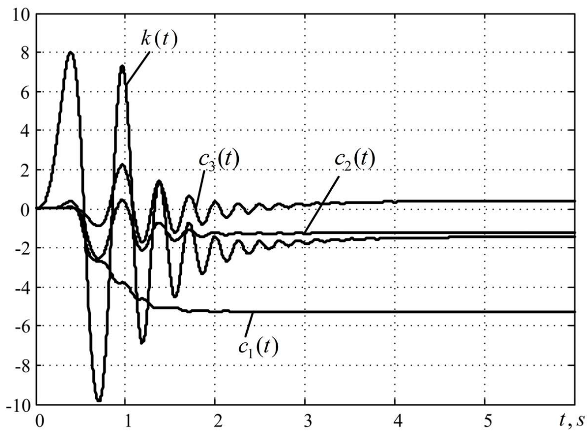

Figure 5. Tuning dynamics of the adaptive regulator (6), (13) coefficients at time interval $t \in [0; 6]s$.

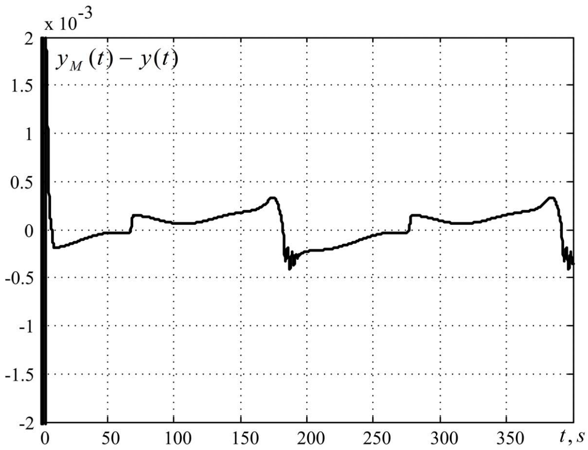

Figure 7. Tracking error in the system with the plant initial conditions (14).

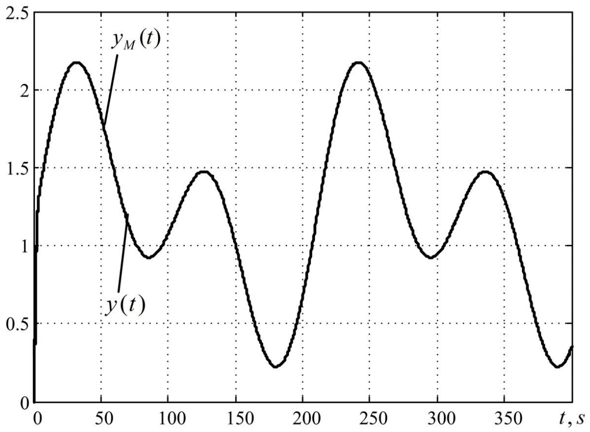

Figure 8. Output signals of the reference (12) and the plant (1), (2), (8) - (10) with initial conditions (14).

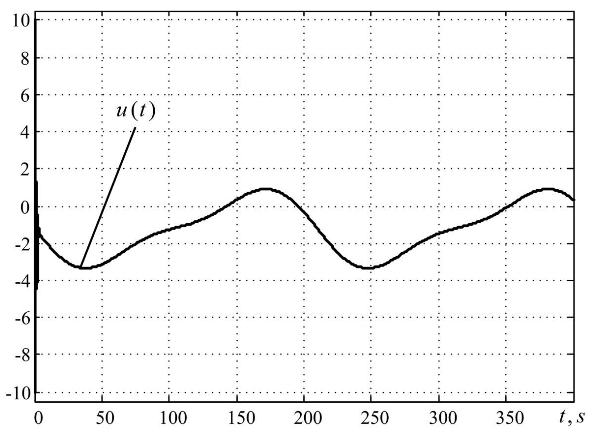

Figure 9. Control signal of the system (1), (2), (8) – (14).

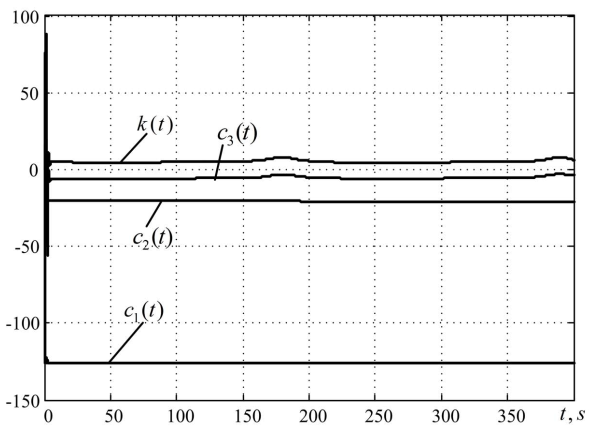

Figure 10. Tuning dynamics of the adaptive regulator (6), (13) coefficients.

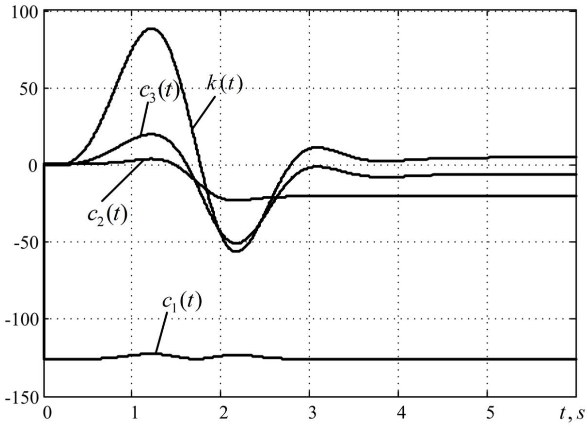

Figure 11. Tuning dynamics of the adaptive regulator (6), (13) coefficients at time interval $t \in [0; 6]s$.

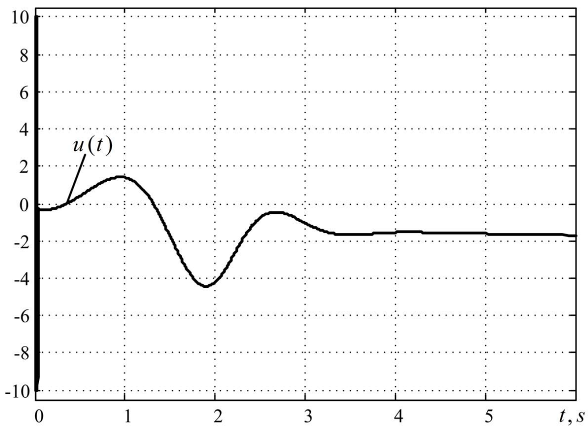

Figure 12. Control signal of the system (1), (2), (8) – (14) at time interval $t \in [0; 6]s$.

5 Conclusion

The simulation of adaptive control system for the class of stationary plants, which contains the input signal saturation, is considered. By computational experiments, it was shown that the adaptive regulator structure proposed in [Eremin, 2016] provided a sufficiently high quality of the system operation.

The obtained results can be useful for studying the control system operation when changing parameters for the purpose of their practical implementation.

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